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THE QUANTIFICATION OF UPPER BODY SYNERGIES:
A CASE COMPARISON FOR STROKE AND NON-STROKE VICTIMS

by

John O. Roylance

A thesis

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Committee Approval

To the Graduate Faculty:

The members of the committee appointed to examine the thesis of John O. Roylance find it satisfactory and recommend that it be accepted.

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November 19, 2015

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RE: regarding study number IRB-FY2016-119 : Quantification of upper body synergies: a case comparison for stroke and non-stroke victims

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Sincerely,

~~Ralph Baergen, PhD, MPH, CIP~~
~~Human Subjects Chair~~

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Abstract

The goals of this study was to create a system that would allow for the capture of the joint angles in the upper limb with some accuracy and then use that system to find if there was a pattern in synergies between stroke and non-stroke victims. The subjects were asked to perform five different movements all without any objects. The first movement was to point forward with the index and middle finger. The second movement was to manipulate a ball upon a table, look at the ball, and then return the ball to its position on the table. The third was to write the word “graphic” in the air as though they were writing on a board. The fourth movement was to open a door. And fifth was to pick up a cup and drink from it, then place the cup back on the table. All movements were captured on motion capture software using markers and camera system. This data was then used to extrapolate the angles of each joint. This was done with kinematics and two different math programs. The data underwent principle component analysis to remove any less important angular movement. This data was plotted and reconstructed to confirm any synergies that were found.

1. Introduction

1.1. Research Goals

The first goal in this research was to find a system that would allow for the capture of the joint angles in the upper limb with some accuracy. With this system data could be captured and then used to study the motion of the upper limb. The systems accuracy is important as well, if the system was not describing the motion of the upper limb then it could not be used to find joint movement. The second goal of this research is to discover if there are patterns in motion for the arm and the hand, and to identify if those patterns are different for both stroke and non-stroke victims. This is in hopes to find a solution in helping those recuperate from strokes and to help amputees with rehabilitation and future use of prosthetics. This research will help in the development of new devices to help in this rehabilitation.

1.2. Assumptions and Limitations

Not everything can be controlled in a lab situation, there are limitations in what can be done and recorded. There are several different tools used in this project all with their own limitations. Infrared cameras which are connected to a control box with long Ethernet cables. Markers, which are a 12.7 mm diameter ball made of a semi-reflective to infrared light material, attached to a black stand of similar diameter, which are placed on the subject to detect position in 3D space. The cameras capture of the markers was then sent to Vicon Nexus for marker analysis, then Mathematica for data analysis, then Matlab for numerical solving, and back to Mathematica for data analysis. All these components have different limitations and expected errors.

The infrared cameras were on tripods which allows for slight shaking of the cameras. The cameras have limited range, and need to be re-aimed even if they are moved less than millimeter. To avoid any error cameras were oriented before recording the different subjects and subjects were helped into the workspace. Thus it could be assumed that the cameras did not move and they remain aimed properly when testing.

The control box and cables bring in electrical signal errors. These errors are expected when working with anything electrical and create little amounts of noise in the data. With long cables the electrical signal has more for errors to develop. Also, the control box combines these signals from the eight different cameras and then sends these results to the computer through one cable. This compression of signals again can create some distortion. As nothing could be done to remove these small errors and as they are quite small, it was assumed they are negligible, thus they were neglected.

Markers are place on the skin and occasionally the clothes of the subject. These markers are attached with toupee tape when placed on the skin and Velcro if on fabric. There is an issue with this; kinematics tracks the skeletal system. So markers are influenced by the movement of the muscle and skin and clothing as well as the skeleton. Ideally these markers would work perfectly if attached directly to the bones; however, this cannot be done. So to compensate for this, markers were placed on the body where there is little muscle or skin interferences. An example is the scapula; the markers on the scapula were placed in the center of movement. This means that as the subject moves around the marker is placed ideally in the center of the area being located on the scapula. This would help obtain the movement of the scapula with average errors from all sides of movement.

Vicon Nexus is computer software designed to model motion that is captured from the camera system. In general the system works with full body or partial body gates. For custom models the software had to make assumptions in the natural movement of humans, assuming planar movement of segments of the model, and rigidity of the models segments and joints. These limitations would occasionally cause the markers to be labeled incorrectly. So to fix this time was spent in the 3D model given by Vicon Nexus GUI, graphical user interface, to correctly label every point on every frame, which is an instance of motion captured represented by positions of markers in 3D space at a time instance. As well as spending time to adding markers lost due to them not being seen by enough cameras. In the end depending on the data it would take an hour to correctly edit one hundred to one thousand frames. Over 60 hours were spent editing the data to provide the most accurate results possible. Vicon Nexus also had had a time stamp of ten milliseconds and an accuracy, according to the software, of one ten-thousandth of a millimeter. Using engineering standard we would assume that the accuracy of the positions of the markers was closer to a one hundredth of a millimeter.

If a marker is not available for any amount of frames, Vicon Nexus will create a mapping of the markers which can then be used to fill in that marker. There are four kinds of fillers for doing this all using the assumptions mentioned before. First is a spline fill which following the pattern of movement will fill in the missing marker in those frames. Next is a pattern fill which will use another marker's trajectory and its relationship to that marker to create a position for that marker to fill. Next is a ridged body fill which takes three markers and uses their relationship to that marker to fill in that marker. Lastly is a kinematic fill which uses a chosen segment to fill in the markers

place on the frames. Each of these fillers was considered in each set of data so whichever filler was chosen gave the most accurate results.

When doing the inverse kinematics several assumptions have to be made, namely a robotic model. Although it would be ideal to do the full kinematics of the human structure solving those kinematics would be neigh impossible at the moment. So a robotic model was used, which will be discussed later. One limitation however is in the solving of the kinematics. The data that was collected was made on a different model then the one solved for, which means the model does not match perfectly. This will lead to some error in the angles that are solved for, especially when the subject moves into superfluous positions. As such, when making calculations there had to be allowance for error, which was higher than normal to save time on solving the computations.

2. Background and Review Literature

The human body is made up bones, muscle, connective tissue, arteries and veins, and nerves. As discussed in [1] this complex mesh of sinew provides movement for humans. The bones provide structure for movement and the bones have muscles attached to them by tendons at different points to create different joint movements. The brain controls the muscles with nerve signals.

The skeletal system in the human body is what creates joint movement, see [1]. As with any structure or frame the skeleton has four types of joints found in it, sliding, rotational, hinged, and spherical. The glenohumeral joint found at the proximal end of the humerus is spherical joint. The Elbow is a hinge joint. Supination and pronation, rotation of the wrist, is considered a rotational and sliding joint as both are done to achieve the rotation. Flexion and extension, movement of the wrist bending the palm towards or away from the forearm, is a hinged and sliding joint. Ulnar and Radial deviation, movement of the wrist bending the hand towards the thumb or little finger, is a hinged and sliding joint. The palm consisting of the metacarpals two through five and the carpels is considered a sliding joint. The thumb, phalange one, has a spherical joint at the connection between the carpels and metacarpal. The metacarpophalangeal and interphalangeal joints found in the thumb are both hinged joints. The metacarpophalangeal and proximal and distal interphalangeal joints found in the other phalanges are all hinged joints.

Bones are not solid structures they are made up of several layers of different materials, see [1]. Although they are not exact layers, as they melt seamlessly between each layer, they will be explained as different layers. The first and outer layer is the

periosteum a layer of connective tissue. The next layer is compact bone, a dense layer of cells that provides rigidity to the bone. This layer is ductile yet brittle, due to it being very strong under compression and tension but not bending. A bone does not plastically deform naturally, so when it bends or twists too far it cracks or breaks. This is also true of compressive and tension extremes but it will shatter like soda lime glass when this happens. The last layer of bone is called trabeculae, spongy bone, this layer provides the strength of the bone, in this layer also resides bone marrow in the larger bones. The trabeculae layer is very ductile and comparable to a carbon fabric.

Skeletal muscles as discussed in [1] are made up of multiple fascicles, bundles of muscle fiber encased in epimysium, a strong outer layer which connects the muscles to tendons and protects the muscle fibers inside from rubbing other muscles and bones. These muscles contract themselves in order to create movement found in the skeletal joints. Muscles can only pull; another muscle on the other side of a joint must be present to bring a joint back to its original position. Every joint is controlled by two or more muscles.

Although much can be said regarding the nervous system it is sufficient to say nerves as discussed in [1] send chemical signals throughout the body to control the various muscles. The human brain is the controller for the body. Through the nerves it controls the muscles to contract; the muscles then move the bones to produce rigid movements. To study these movements they need to be recorded and reproduced, hence the need for motion capture.

Motion capture is a method of data acquisition for body function. As demonstrated in [2] the mapping of movement can be done with markers and an infrared

camera setup. The markers are design to reflect rays of infrared light that the cameras release. The cameras pick up that reflective light and then send to the computer. As shown in [2] the results of capturing motion in this manner can lead to definitive results. Thus a similar set up was taken in this report. The setup consists of an array of infrared cameras and markers on the trunk, arm, and hand of each participant. Vicon Nexus, the computer software, reads the images collected by the cameras to make a 3D profile of the information, see [3] for more information.

As will be mentioned later in this report, see section 3.3.2.1, the proper placement of these markers is essential to getting usable data. If the markers are too close to one another – a minimum distance of one centimeter – at most one marker in that area will show up at any given time in the Vicon Nexus GUI. One method looked into to help combat this problem were IMUs, Inertial Measurement Units.

IMU's are sensors used to determine direction, relative velocity, and relative acceleration [4]. These sensors signal would be sent first to a device called Lock by Vicon [3]. And then the information would be sent to Vicon Nexus and added to each according frame. From the data sheet this signal could be sent to an Arduino to get the electric signal translated to direction, velocity, and acceleration vectors and sent back to a data sheet and then to Mathematica. To take the data from the IMUs, they are connected to Lock with wires. This means twenty seven wires were cut to length, around twenty-five feet, and then attached to the IMUs. The wires were strung across the ceiling in wire holders, see Figure 1. Sadly the wiring of the IMUs interfered with the IMUs staying on the subject's finger and with the excessive amounts of electrostatic electricity in the room, the IMUs were not used.

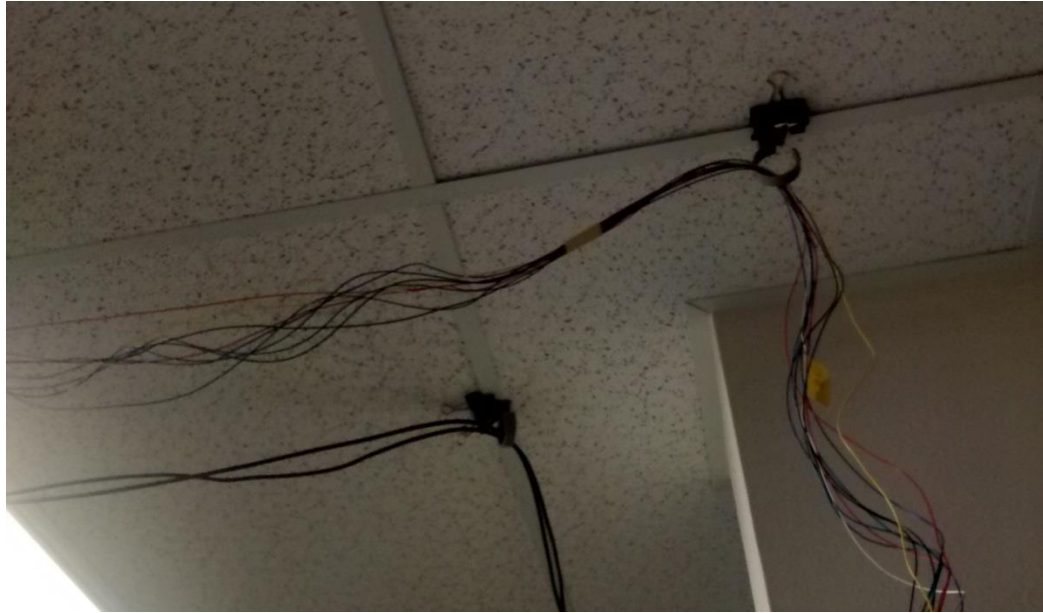


Figure 1: Part of the wiring which hangs from the ceiling, solid larger wires are the Ethernet cables for the cameras colored wires are for IMU's.

There are two forms of models in this project; first the model needed by Vicon Nexus, see section 3.3.2.1, and secondly the kinematic model. As mentioned earlier the human body is made of various different organic materials and this cannot be perfectly duplicated at the moment. This is mainly due to the large amounts of sliding and bending that can be done in bones due to its structure. Therefore a robotic model was formed to represent the data. A robotic model assumes that all segments are rigid, meaning no bending or sliding, and that the joints are a specific orientation and make. Thus a robotic model simplifies the joints and creates rigidity in the frame, see [5]. This new simplified model can then be used to figure out the joint angles.

In robotics there are three basic types of joints: prismatic, revolute, and spherical, see [6]. Each joint in the model is made up of one or more of these joint types. A prismatic joint creates a dislocation between both arms of the robot; a basic example of this is a piston, which extends and retracts to increase the reach of a robot or object that

lies on the extending arm. A revolute joint is a bending joint best described as a hinged joint, like a hinge it rotates about one planar direction. A spherical joint is best describes as a ball and socket joint, it can rotate in all three planar directions.

With the understanding of this new model the kinematics can be solved for, as found in [7], [8], [9], and [10] an algebraic solution to these new models can be formulated using either a homogeneous matrix or dual quaternion. A homogeneous matrix is a four by four matrix where: the first three by three parts of the matrix make up the rotational matrix; the four columns first through third components make up the position of the object; and the final row is a row of zeros with the final point being 1, in these cases, see eqn. 1.

$$[R_t] = \begin{pmatrix} & R & \Delta x \\ & & \Delta y \\ & & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad 1$$

Where $[R_t]$ is the matrix, R is a three by three rotation matrix, Δx , Δy , and Δz is the position or displacement in the x, y, and z direction respectively. A dual quaternion is made up of two, four parameter, vectors, the makeup of these two vectors are six parameters being rigid body displacement and the last two being algebraic constraints.

$$G_t = \begin{pmatrix} \sin\left(\frac{\phi}{2}\right) s_x \\ \sin\left(\frac{\phi}{2}\right) s_y \\ \sin\left(\frac{\phi}{2}\right) s_z \\ \cos\left(\frac{\phi}{2}\right) \end{pmatrix} + \varepsilon \begin{pmatrix} \sin\left(\frac{\phi}{2}\right) s_x^o + \frac{t}{2} \cos\left(\frac{\phi}{2}\right) s_x \\ \sin\left(\frac{\phi}{2}\right) s_y^o + \frac{t}{2} \cos\left(\frac{\phi}{2}\right) s_y \\ \sin\left(\frac{\phi}{2}\right) s_z^o + \frac{t}{2} \cos\left(\frac{\phi}{2}\right) s_z \\ -\frac{t}{2} \sin\left(\frac{\phi}{2}\right) \end{pmatrix} \quad 2$$

Where G_t is the matrix; ϕ is the angle; s_x , s_y , and s_z is the strew displacement in the x, y, and z directions respectively, s_x^o , s_y^o , and s_z^o is the strew axis moment in the x, y, and z

directions respectively; and ε is a dual unit. These eight parameters give information about a screw axis or axes and the angle of rotation about that axis or axes.

As found in [11], [12], and [13] this new kinematic model can be solved from the algebraic solution. This is done by comparing each component of an algebraic matrix to a solution matrix, and then solving for the various unknowns. This gives the solution for the screw axis and the angles or the position vector and angles. These angles can now be compared to discover synergies.

Synergies, as described in [14], can be used in depicting human motion. There are different forms of synergies as well, in robotics and in the medical field; note this report will focus on the robotics field. The different synergies have also been categorized into different types: power, precision, and rest; see [15]. Power poses are meant to hold objects of larger size and are for prolonged grasping and postures; an example of this is shoveling dirt or picking up a fence post. Precision poses are meant for small objects and activities that require quick movement; an example would be writing with a pencil or picking up a glass cup. The rest pose is not normally looked at but it is the pose or movement done at rest or in transition. This is helpful for improving the movement of those who have abnormal movement due to an ailment, disability, or deformity or in the robotics field in creating robots that are human like, see [16].

A stroke ruptures capillaries in the brain, depriving those sections of the brain around the rupture to be deprived of oxygen, in the end leaving those sections of the brain dead [17]. Depending on where the stroke impacts the brain it will cause the motion, of those impacted by the stroke, to become more rudimentary [17]. Primitives are

rudiments of synergies, see [14], [16], and [18], and with stroke victims these primitives are more easily seen, see [19].

3. Methodology

3.1. Sampling

The test group for this study was stroke victims and healthy comparisons. The victim of the stroke must have the stroke six or more months before to be able to be a part of the study. As the stroke victims have by that point in time adapted reasonably to the changes in their body. The healthy comparison is someone who is younger with similar physical build to the stroke victim. Both sets of subjects will perform five different motions designed to use different parts of their arm, wrist, and fingers. Both the stroke victim and their comparison will use the same arm, the afflicted arm of the stroke victim. In this report only three subjects were used and although this is not a complete sample number it is all that could be achieved this time.

3.2. Instrumentation

Most key to this project was the Vicon Nexus system. This system consists of eight infrared cameras, see Figure 2, Ethernet connections to a control box, and from the control box an Ethernet connection to the computer used. A focusing wand used for aiming and setting the coordinate planes for the software. Markers are attached with the toupee tape to the subject. There was also a suit which was used for those uncomfortable with taking off their shirt, which used Velcro to attach the markers. Lastly the Software used to comprehend the data being recorded was Vicon Nexus 2.2.



Figure 2: Image an infrared camera, this one is labeled 3 for reference in Vicon Nexus.

Kinematics and synergies were calculated using Wolfram Mathematica 8.0 and MATLAB R2015b. Mathematica is a math software which was used heavily in data processing and algebraic solutions. Matlab is another math software which was used heavily for its numerical solver which could handle the over and under constrained problems that Mathematica cannot handle.

3.3. Procedures

3.3.1 Vicon Nexus Set up

The setup for Vicon Nexus is not the same that is used in general for most people. It was only achievable due to the work that the Vicon Nexus staff did in making it work more stream line and efficiently for custom models. As this is the case this report will go in depth on how to setup the Vicon Nexus system without using their preset models. This explanation will assume that the cameras and the control box are already set up and everything reflective has been removed from the room, covered up, or moved out of sight of the cameras.

Step 1, starting the system. Although this probably may seem straight forward there are a few steps needed to get Vicon Nexus to run on a laptop. Firstly, plug in the system to the Ethernet port on the laptop. After this change the Ethernet Port Settings to the following in the Local Area Connection Properties. First in the network tab uncheck mark all connection types except “Internet Protocol Version 4 (TCP/IPv4)” then double click the connection name. In the new window click option labeled “Use the following IP address:” and enter in the IP address: 192.168.10.1 and Subnet mask as 255.255.255.0, the IP address may depend on your systems set up mentioned before, however, this is the system at Idaho State University. Note that the connection through the Ethernet port is only for Vicon Nexus to access the Hardware. Next, turn off any firewalls that are installed, for our devices, Windows Firewalls on the control panel which can be found by typing in the menu Windows Firewall. Once found click on the windows firewall option in the control panel; on the left side click the “Turn Windows Firewall on or off” option, then click on the box next to the red shield to turn off the fire wall. The firewall can be turned back on once the computer is connected to the cameras. At this point the cameras should begin to show up on the System Tab found in the Resource GUI, wait until all camera icons are green before moving on.

Step 2, aiming the cameras. The first thing to do is place the cameras in the room in a way that the whole workspace can be seen, see Figure 3 for example. To begin aiming the cameras, change the camera settings from “3D Perspective” to “Camera”, this is found in the top left of the middle GUI. Now click on the camera in the resource GUI that you want to aim. Also change in the resource setting that camera is set to “All” so that the markers on the aiming wand show up as white and gray balls. Then proceed to

the camera in question after placing the aiming wand in the center of the workspace at ground, bottom of workspace, depending on the cameras you have there may be different ways to aim them. On the cameras in the lab used, there were three knobs used in focusing. The knobs closest to the body and lens of the camera can be focused by going to full saturation or no saturation and moving them one at a time until the camera image focuses. When done correctly it looks as though you are hitting a point where the markers are shrinking and then getting larger, the ideal point is where they are optimally small. The final knob is then turned until all extraneous white noise is no longer visible. This is repeated with all the cameras in the room.



Figure 3: This is an example of a camera set up for this particular session, this is not the same camera set up used for this report as the cameras are set differently for each subject.

Step 3 Calibrating. Now that the cameras are aimed remove the wand from visible sight, and move to the Tools GUI to the “System Preparation” Tab. Now under the Mask Cameras section click the Start button and wait 5 to 10 seconds and then hit

End button, which is in the same spot. Now retrieve the wand and in the Calibrate Cameras section change the Wand to “5 Marker wand & L-Frame.” And change the Calibration Type to “Full Calibration.” If both are already set to the following; then hit Start. Highlight all the cameras and in the Middle GUI can be seen the progress of each camera in calibrating. Now spin the wand around in the workspace. In the lower right of the camera visual is a flashing pentagram. When a camera is being calibrated properly this will turn from red to green, once complete the pentagon will stop flashing and disappear. When the system is done calibrating place the wand again in the center of the workspace. If a subject is sitting in a chair a good place to place the wand is on the chair, remember to keep the wand level. Now below the Calibrate Cameras section is the “Set Volume Origin Section”, click start and then finish after it finds the Wand. Now there is one more step to the setup but it cannot be performed yet unless a model is available, and will be discussed below. At this point if using a preset model, then select it, and give it the required measurements.

3.3.2 Vicon Nexus Capturing

There are three parts to capturing data: model setup, capture itself, and fixing of the model. Each will be discussed below in detail.

3.3.2.1 Model set up.

A model in Vicon Nexus is made up of segments and links. This means that each segment needs a certain amount of markers and knowing that each segment is planar also makes the placement of these markers so crucial. The general marker placements for motion capture can be found in Appendix Table 3. Although this table covers the majority of moving parts of the hand, arm, and upper body, it cannot be used to

extrapolate all the data needed. Therefore a new marker placement was developed, see Table 1, to work with Vicon Nexus' segment scheme, and cover all segments of the body to be able to calculate all angles desired.

One key part of this marker placement is the fact the markers cannot be too close to each other. When cameras are 4 feet, 1.2 meters, away from the center of the work space makers have to be .4 inches, 1 cm, apart. This means the cameras have to be close together to capture the hand.

Another part is that Vicon Nexus uses planar patterns to help in modeling. Three markers are needed per segment as three markers make a plane. The distance issue mentioned before only allows one marker per segment was achievable for the finger. This was later modified to be a marker per joint and the end of the finger.

Another part of marker placement is Vicon Nexus expects certain markers to be present to help segment joints. These markers are the Sternum, GH, Elbow, Forearm, and CarpOr; these markers also appear in both Table 1 and A1. All these markers denote is the beginning of a joint. This is true for all segments including the free floating joint of the scapula. The difference in the way the scapula moves and the position of the origin marker, ScapOr, is that it resides in the center of movement based on the intraspinous fossa.

Table 1: List of marker names used and where they are placed on the body. Markers marked with a * are not on all subjects.

Label	Segment	Position
Sternum	<i>Sternum</i>	Superior Part of Sternum, L Frame
SternX*	<i>Sternum</i>	Sternum Right Superior Side, L Frame (Seems to only show on men)
SternY*	<i>Sternum</i>	Sternum Left Superior Side, L Frame (Seems to only show on men)
MedClav	<i>Sternum</i>	Mid Clavicle
DistClav	<i>Sternum</i>	Dorsal Clavicle
ScapOr	<i>Scapula</i>	Center of Scapula (average movement) Intraspinous fossa
ScapX	<i>Scapula</i>	Medial part Spine of Scapula (average movement)
ScapY	<i>Scapula</i>	Lateral part Spine of Scapula (average movement)
GH	<i>UpperArm</i>	Glenohumeral
UPArmOr	<i>UpperArm</i>	Lateral Mid Humerus, L Frame
UPArmX	<i>UpperArm</i>	Directed Proximal, L Frame
UPArmY	<i>UpperArm</i>	Directed Posterior, L Frame
Elbow	<i>Forearm</i>	Olecranon Process
Forearm	<i>Forearm</i>	Mid Ulna when wrist is abducted
ForearmOR	<i>Forearm</i>	Between Ulna and Radius Distal on arm Proximal to the styloid process
ForearmX	<i>Forearm</i>	Lateral edge Distal Radius (right arm) or Medial edge Distal Ulna (left arm)
ForearmY	<i>Forearm</i>	Medial edge Distal Ulna (right arm) or Lateral edge Distal Radius (left arm)
CarpOR	<i>Wrist</i>	Capitate (carpel)
CarpX	<i>Wrist</i>	Between Mid and Proximal portion of Metacarpal (2 if Right, 4 if Left)
CarpY	<i>Wrist</i>	Between Mid and Proximal portion of Metacarpal (4 if Right, 2 if Left)
TMCOr	<i>Phalange1</i>	Lateral edge Metacarpal 1 Middle portion, L Frame
TMCX	<i>Phalange1</i>	Floating toward lateral side of hand (either), L Frame
TMCY	<i>Phalange1</i>	Lateral edge Metacarpal 1 Proximal portion, L Frame
TMC*	<i>Phalange1</i>	Lateral edge Metacarpophalageal Joint (add if hands are large)
TProxPhala	<i>Phalange1</i>	Dorsal edge Interphalangeal Joint
TDistPhala	<i>Phalange1</i>	Dorsal edge end of Distal Phalange 1
IMC	<i>Phalange2</i>	Dorsal edge Metacarpophalageal Joint, Phalange 2
IProxPhala	<i>Phalange2</i>	Dorsal edge Proximal Interphalangeal Joint, Phalange 2
IMidPhala	<i>Phalange2</i>	Dorsal edge Distal Interphalangeal Joint, Phalange 2
IDistPhala	<i>Phalange2</i>	Dorsal Edge end of Distal Phalange 2
RMC	<i>Phalange4</i>	Dorsal edge Metacarpophalageal Joint, Phalange 4

RProxPhala	<i>Phalange4</i>	Dorsal edge Proximal Interphalangeal Joint, Phalange 4
RMidPhala	<i>Phalange4</i>	Dorsal edge Distal Interphalangeal Joint, Phalange 4
RDistPhala	<i>Phalange4</i>	Dorsal Edge end of Distal Phalange 4

The final part to the changes in marker placement took place due to the kinematics. In Vicon Nexus all the segments need three markers to determine a segment; the same is true in kinematics but to determine direction and position. The markers on L-frames with the ending X or Y and the markers ForearmOr and TMC1 are all additional markers to help with the kinematics. Noting that the markers on the finger are place the way they are to help with vector math, which was used instead of kinematics on the phalanges/end-effectors.

Now that the markers are on the subject, it is time to do a setup of the software model. First in the Tools GUI in the Capture tab, “label the section name in the Trial Name:”, and if more information is needed add a description of the trial or notes about the trial. Then the subject will stand or sit in the desired location for testing placing their hands on their knees or calf or raised up in front of then so all the markers are showing up on the GUI note that the software needs to be in 3D Perspective. Once all the markers are visible click on the start recording button in the Capture tab in the Tools GUI and after 50 or so frames have gone by, stop it with same button.

Now is the time to make the model for Vicon Nexus software to use. This is done by first going offline; click the button in the top of the Resource GUI labeled Go Offline. Now go to Data Management GUI found in the lower portion of the middle GUI/section. Click on the trial that was just recorded and open it, then in the upper part of the GUI click the four colorless balls button to run the reconstruct data. In the Resource GUI,

open the tab labeled Subjects. Directly under that there are three buttons, click the first one labeled “Blank Subject” and give it a name for the template.

In the Tools Section in the Subject Preparation tab, Scroll down if necessary, and under Create Segments: label the first segment, for instance for this project the segments were in order of Sternum, Scapula, UpperArm, Forearm, Wrist, Phalange1, Phalange2, and Phalange4. After labeling the segment, click in order the markers in that segment moving away from the trunk of the body; see Table 1 for the order use in this project. Now that the first segment is complete repeat this process with the rest of the segments. Next under “Link Segments:” select the “Ball Joint” and then “Link”. Now proceed to link each frame by clicking first on the first frame, then clicking on the following frames like a chain until all the segments are linked to each other. Note that a segment can have more than one child segment.

Now that all the segments are linked go to the Subjects tab in the Resource GUI and click the subject’s arrow to open it, then open the Markers likewise. Now re-label all the markers as shown in Table 1, or to whatever is desired, this can be done easily by double clicking the first marker, typing the name, then hitting the tab key on the keyboard, repeat this process until every marker is re-labeled. Each marker can then have its color changed to differentiate if a marker is labeled incorrectly later on. The color scheme used in this project was as follows: Sternum – Black, Scapula – Purple, UpperArm – Blue, Forearm – Green, Wrist – Pink, Phalange1 – Yellow, Phalange2 – Orange, and Phalange4 – Red. In each segment the colors started off dark then as they move down the list, see Table 1, these colors get progressively brighter. Once done, save

the subjects information and the file itself; to save the subject model right click on template name and save both of the files.

3.3.2.2 Capturing

Now that the model for Vicon Nexus is complete the capturing session can begin. As mentioned earlier go live with the recording, then label the trial and have the subject preform the desired action; when the subject is complete end the recording. Then review the recording by going into the data management and clicking on the recording. Run the reconstruct and label pipeline by clicking on the four colored bolls in the top bar of the GUI. Watch the trial, if too many markers are missing then repeat the trial; Vicon Nexus will automatically increase the trial number so relabeling is not necessary at this time. If the trial looks good, then repeat to process until all movements are done.

A note needs to be made about how Vicon Nexus saves the trial files. Firstly it saves 7 smaller files to each trial, once the reconstruction and labeling is run the files will begin to grow in size. The speed to which the file size grows is also not linear; it can be better approximated as a quadratic curve. With this said keep trials short, no longer than a minute, forty-five seconds if you use the Mathematica code below. A good length trial is between 500 and 1200 frames.

3.3.2.3 Correcting Model Data

As mentioned before Vicon Nexus Software makes several assumptions in the marker scheme of a model. In Figure 4, there is an example of a frame in which Vicon Nexus labeling scheme went off. In Figure 5, there is an example of ghosting, where markers appear where there are no markers placed, normally caused by shiny objects in the room or the cameras being shifted slightly. Note that when ghosting occurs because a

camera is shifted a lot more markers will show up than a shiny object, twenty to two thousand different markers. In Figure 6 is an example of the same frame with the markers labels where corrected. In Figure 7 is an example of the same frame but with the gaps filled in.

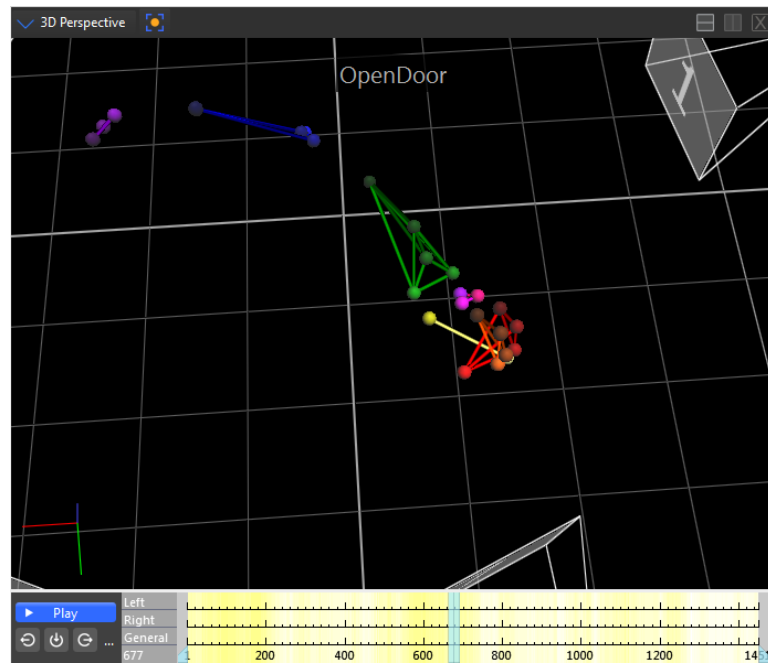


Figure 4: Not processed Frame 677 in Trial Open Door with SV1.

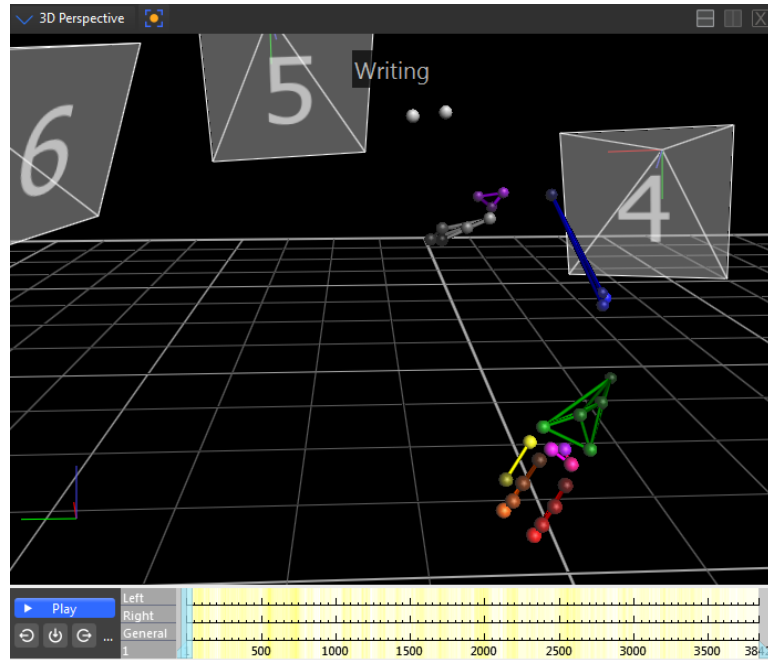


Figure 5: Frame 1 in trial Writing with SV1, an example of ghosting as seen with two markers making it look like there are eyes in the frame, most likely caused by a shiny object.

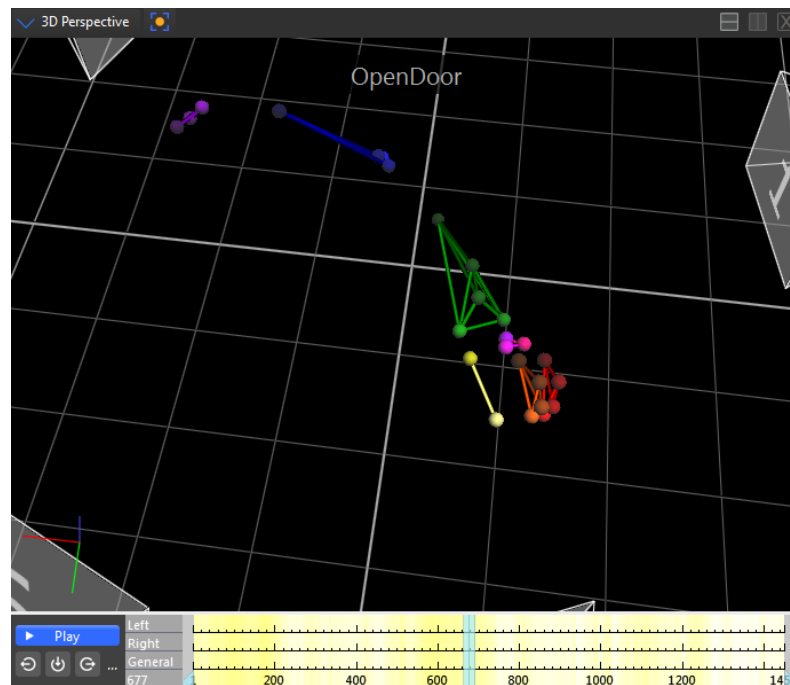


Figure 6: Corrected labeling of Frame 677 in Trial Open Door with SV1.

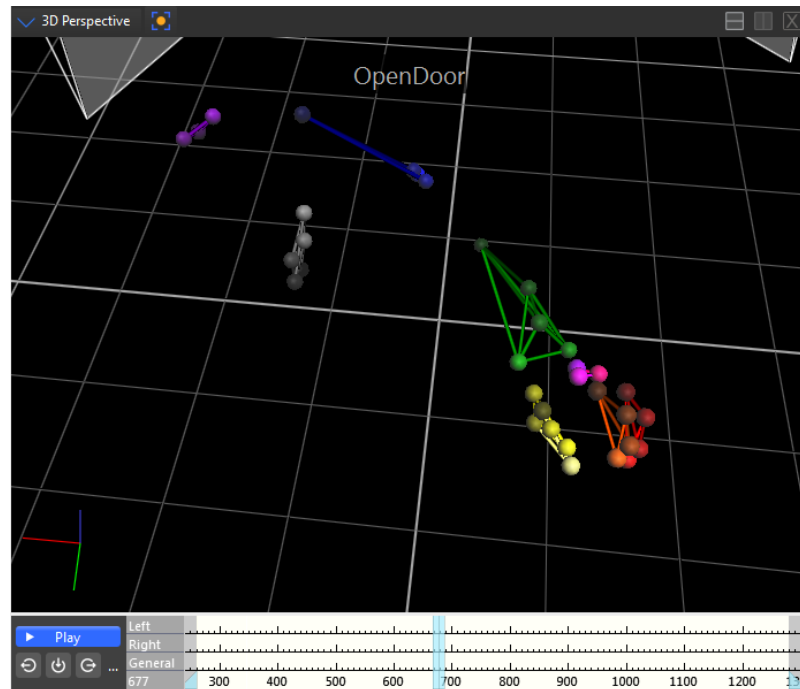


Figure 7: Gap filling of Frame 677 in Trial Open Door with SV1.

The process of fixing marker mislabeling is straight forward: go frame by frame and re-label each of the mislabeled markers. This is time consuming and a trial should be completing in one sitting breaking it into chunks adds more time. Before any labeling is done go to the Tools section in the Label/Edit tab and then under manual labeling click Forward, by default it is on Whole. The first trick of relabeling is right clicking markers mislabeled. When right clicked a list appears, near the bottom of the list are all the names of markers that are not showing, if the marker name is there click on it then and move on to another marker. If the name is not present, there are two options, the first is to find the mislabeled marker with that markers name and give it its proper name or to go into the Tools section in the Label/Edit tab pick the correct marker from the list, then click the appropriate marker.

Once relabeling is complete, gap filling can be started. Gap filling has four kinds of fillers: spline fill, pattern fill, rigid body fill, and kinematic fill. The kinematic fill will

not be discussed as it only works with Vicon Nexus' models. The spline fill uses only the pattern of the marker to determine where that marker needs to go; this should only be used when the marker is missing for a few frames and the movement is small. The pattern fill is the most used filler of the three, it determines the placement of the marker based off of another marker, and the plane it makes with that marker, which means it can be done with markers of other segments as well. The final filler the rigid body filler is very useful in areas that have more than three markers in them that do move relative with each other or can be considered a rigid frame. Although this is the most accurate of the three fills it cannot be used that often as three or more markers need to be available that make up a rigid body to fill in the one missing marker.

Filling is done in the Tools section in the Label/Edit tab. In the area labeled Gap Filling there is a box that has the trajectories and the number of gaps that the markers can be filled. By clicking on the marker the software will jump into the filling mode and what is next is for the user to choose the desired filler and then any markers that are necessary, once complete click "Fill" in that fillers section. This is repeated for all gaps and markers except those markers that have large gaps or do not show up near the beginning or end.

3.3.3 Mathematica Set up

Mathematica is a powerful math program used specifically for algebraic math and as an advance calculator. The data from Vicon Nexus is exported via an ASCII pipeline into a CSV file, comma delimited file, which can be read in excel. This file shows the markers with their corresponding XYZ values for each frame and needs to be setup in the Mathematica file. This file is edited in excel if necessary. An example of the

Mathematica code can be in the Appendix B: Mathematica Code and Mathematica Code Appended for Longer Trials. This code started with a setup and proceeds to different sections, groups of joints. This then is either exported to Matlab or has vector math done to solve for the angles. Finally the PCA of the data, Principle Component Analysis, is computed on the data. This code is set up to handle one trial at a time.

The setup of the code consists first of importing the raw data file into Mathematica. It is then spliced into marker sets of XYZ coordinates for each marker and then all but the distal phalanges is then changed into homogeneous matrixes of different segments. From here the homogeneous matrix is then orthogonalised, meaning the axes of the matrix are set to right angles. And finally the new matrix is plotted at each frame and then as a group in a single plot; this step is only done to visualize what was just done.

The code now splits into sections of joints: glenohumeral, elbow and ulna/radius rotation, wrist, thumb joint 1, index joint 1, ring joint 1, thumb joint 2 and 3, index finger joint 2 and 3, and ring finger joint 2 and 3. At this point either one of three things happens. First the data already set up as a homogeneous matrix is set equal to the components of an unknown matrix of variables, only done for spherical joints. Once this is set up it can be sent to Matlab to be solved. Second is ten points of the data, which are converted to a dual quaternion, are set up with a dual quaternion matrix of unknowns. This will be sent to Matlab and solved for the screw axes' values and then these values are plugged into the dual quaternion matrix of unknowns. With the new matrix now formed, the data can be set equal to zero with the data dual quaternion then sent to Matlab to be solved. The final option for the data is it can be solved directly for the joint angles using vector math, see eqn. 3.

$$\theta = ArcSin\left(\frac{|(M_2 - M_3) \times (M_2 - M_1)|}{|M_2 - M_1||M_2 - M_3|}\right) \quad 3$$

Once the angles are solved for in Matlab, they are sent back to Mathematica for data processing for results. First thing done is the results are converted to a format that Mathematica can work with. If necessary these values are also put through a trimetric filter to place them the between 180° to 0° or 90° to -90° . At this time as well they are organized in matrices for doing PCA. Finally the error, standard deviation in error and the eigenvalue and eigenvector are all solved for.

The last part of this code is the taking of all the results and finding any patterns. Firstly there are histograms of the angles to see common angle values. Secondly PCA is performed on the resulting data. This consists of first making a matrix of the resulting angle θ_{Sol} . Then the average is solved for θ_{SolAve} . Next the system is transposed $\theta_{SolTran}$; these are used to find the covariance matrix C_θ . The eigenvalue and eigenvector are solved for, *EigenVal* and *EigenVec* in the code or λ and μ respectively. At this time the eigenvectors with the dominant eigenvalues are chosen to be used in the reconstruction of the data X_{cor} . Once the data is reconstructed it can then be analyzed for synergies.

3.3.4 Matlab

Matlab is a powerful numeric solver, being able to solve over and under constrained equations. As such after Mathematica finishes up setting up the kinematic equations, these equations are sent to Matlab. Depending on the form of kinematics used to solve one of two things would be done: one, the position and angles could be solved all at once or second the axes would be solved for, then in another iteration the angles

would be solved for. In both cases a function and solver file would be created and the solver would be run, as there was no need to open the function file, unless the data did not translate over properly. The function file is setup to have all the equations set to zero and fsolve was use to solve as it has a good tolerance for noise. Matlab's fsolve is a nonlinear system solver, to learn more see [20].

3.4. Design and Analysis

This section of the report will take a more in-depth look into the process of finding synergies. There are a few steps as mentioned above that refer to this but there are distinct steps into finding these synergies. Firstly the angles have to be found. Second these angles need to have CPA done to remove any external noise from any non-dominant angles. Once this is done, a visual analysis of the results takes place to find any patterns in movement. Next is a reconstruction on the position and confirming it being a synergy.

Angles are calculated by inverse kinematics. The raw XYZ point data found can be converted into homogeneous matrixes, see eqn. 1, and later dual quaternions, see eqn. 2. These matrixes describe a joint, which is made of several angles, which can be described by a robotic model. These models are then equated to zero with the actual joint data and then solved in the numerical solver Matlab, see eqn. 4 and 5 and Mathematica Code.

$$[R_t]_{i,j} - RM_{joint_{i,j}} = 0 \quad 4$$

$$G_t - HM2dq(RM_{joint}) = 0 \quad 5$$

Where G_t is the matrix in eqn. 2; $[R_t]$ is the matrix in eqn. 1; RM_{joint} is the kinematic solution created from the raw data at a given data set, frame, and joint; i and j refers to

the row and column value of the matrix respectively; *HM2dq* is the command in Mathematica that converts a homogeneous matrix to a dual quaternion and is being used to denote that.

The CPA removes unwanted information about less important angles. The angles now calculated make up a new matrix θ_{Sol} , a matrix number of frames by angles large. This is used to calculate a covariance matrix C_θ , see eqn. 6 and Figure 8.

$$C_\theta = \sum_{i=1}^F [\theta_{Sol_i}^T - \theta_{ave_i}] \cdot [\theta_{Sol_j}^T - \theta_{ave_j}] \quad 6$$

With F being the number frames; T signifying a transpose and lastly i and j being which column is used and is also the number of angles. Following this the eigenvalues, λ , and eigenvectors, μ , are taken of this system. The eigenvalues can be used to determine what the most important eigenvectors of the system are, see Figure 9. This can be check with eqn. 7 to show which percentage of the system these eigenvectors affect the system.

$$\frac{\sum_{i=1}^v \lambda_i}{\sum \lambda_i} \geq 99.5\% \quad 7$$

With v being the number of eigenvalues considered. The weight, w , is then calculated using eqn. 8. To which this is used in eqn. 9 to find the reconstructed angle matrix, $\hat{\theta}_i$.

$$w_i = \sum_{i=1}^v [\theta_{Sol_i}^T - \theta_{ave_i}] \cdot \mu_i \quad 8$$

$$\hat{\theta}_i = \theta_{ave_i} + \sum_{i=1}^v w_i \mu_i \quad 9$$

A method similar to this can be seen in [21].

12.83	0.00	4.74	56.26	80.10	9.99	59.89	33.58	31.54	2.40	0.20	3.19	1.84	2.03	0.98	0.18	0.94	7.30
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.74	0.00	48.59	54.08	119.14	22.21	68.12	83.42	81.04	6.93	23.06	52.35	42.38	0.61	21.57	14.97	18.07	4.96
56.26	0.00	54.08	421.11	523.90	19.18	268.63	220.45	207.64	11.33	17.63	74.90	60.31	6.07	73.61	22.25	16.68	35.19
80.10	0.00	119.14	523.90	1021.79	13.27	297.43	428.40	391.42	29.12	51.43	134.93	111.51	9.68	111.30	46.38	40.96	40.30
9.99	0.00	22.21	19.18	13.27	45.64	129.37	2.49	6.40	1.92	10.94	35.31	26.16	0.69	10.81	7.24	14.09	6.21
59.89	0.00	68.12	268.63	297.43	129.37	801.44	110.86	87.66	18.60	42.45	92.28	77.35	17.12	24.64	29.75	49.65	63.93
33.58	0.00	83.42	220.45	428.40	2.49	110.86	432.30	382.42	31.70	43.53	147.42	83.75	2.11	63.43	36.39	22.52	13.79
31.54	0.00	81.04	207.64	391.42	6.40	87.66	382.42	352.03	27.29	42.89	136.81	82.22	1.73	57.24	33.25	23.00	10.12
2.40	0.00	6.93	11.33	29.12	1.92	18.60	31.70	27.29	9.47	3.91	7.88	3.62	0.26	1.19	2.20	0.42	2.16
0.20	0.00	23.06	17.63	51.43	10.94	42.45	43.53	42.89	3.91	16.62	24.06	24.74	2.21	12.40	9.34	10.50	7.84
3.19	0.00	52.35	74.90	134.93	35.31	92.28	147.42	136.81	7.88	24.06	118.69	63.03	2.51	44.80	20.42	22.30	7.39
1.84	0.00	42.38	60.31	111.51	26.16	77.35	83.75	82.22	3.62	24.74	63.03	55.62	4.64	33.98	17.37	22.17	13.75
2.03	0.00	0.61	6.07	9.68	0.69	17.12	2.11	1.73	0.26	2.21	2.51	4.64	4.83	6.43	2.77	1.96	6.57
0.98	0.00	21.57	73.61	111.30	10.81	24.64	63.43	57.24	1.19	12.40	44.80	33.98	6.43	115.43	25.67	13.70	5.54
0.18	0.00	14.97	22.25	46.38	7.24	29.75	36.39	33.25	2.20	9.34	20.42	17.37	2.77	25.67	10.41	7.81	3.73
0.94	0.00	18.07	16.68	40.96	14.09	49.65	22.52	23.00	0.42	10.50	22.30	22.17	1.96	13.70	7.81	12.13	7.33
7.30	0.00	4.96	35.19	40.30	6.21	63.93	13.79	10.12	2.16	7.84	7.39	13.75	6.57	5.54	3.73	7.33	21.51

Figure 8: Covariance Matrix, this matrix shows the relationship between the different joints by taking the magnitude of the original value and color coding the different cells where the darker colored cells are the more important angle relationships.

1981.18
848.73
325.24
128.61
85.96
41.26
30.93
21.13
12.36
7.56
5.22
3.63
2.70
2.06
1.77
1.47
0.62
0.00

Figure 9: Eigenvalues of the covariance matrix, with the relationships considered being colored.

Synergies are found through visual analysis of the reconstructed angle matrix and reconstructed models. The reconstructed angle matrix are plotted over the frames, see Figures A1-A15 in the Appendix A. The suspected synergies are found by looking at the

reconstructed plots and finding repeated patterns of the angles, see Figure 26 around frame 110 and 430. When the suspected synergies are found they are then reconstituted into a visual 3D model of the arm. This can then be used to tell if the suspected synergy is a synergy and what kind of synergy it may be if the motion shows up multiple times in other motions and subjects.

4. Results

The relationship of synergies has always been focused on the hand. As discussed in [16] and [14], the different synergies that show up are either classified as power or precision synergies. There is a more insightful pattern found in the synergies that were found. The elbow and glenohumeral joint movements are closely related to the type of synergy that is present. The movement around the wrist, namely: supination, pronation, flexion, extension, ulnar and radial deviation do not affect these types of synergies.

In power poses, the hand is structured in a way to create more strength. This can be seen in Figures A17, A18, A21, and A22, where the metacarpophalangeal and proximal interphalangeal joint, in phalanges two through five and metacarpophalangeal joint in phalange one dominate the motion. This is most likely due to the larger muscles that control in these joints particular the metacarpophalangeal joint in all the phalanges [1]. Now in the elbow and glenohumeral joint there are similar patterns in there motion, notably enlarging the work area. When picking up any object with any tool, it is easier to use a tool with large surface areas. As seen again in Figures A17, A18, A21, and A22, the glenohumeral joint juts out the elbow, and the elbow retains and approximate right angle for added strength. The reasoning behind this is it increases area, thus making it easier to hold objects.

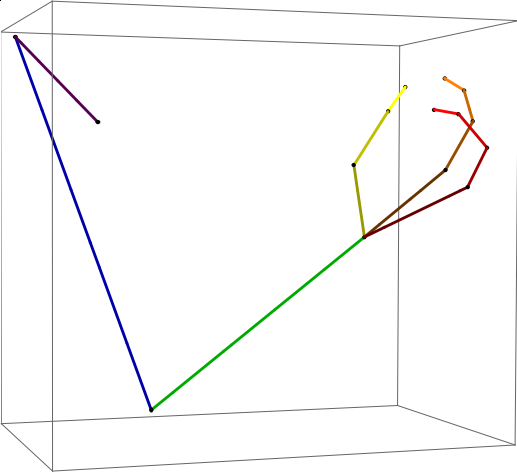
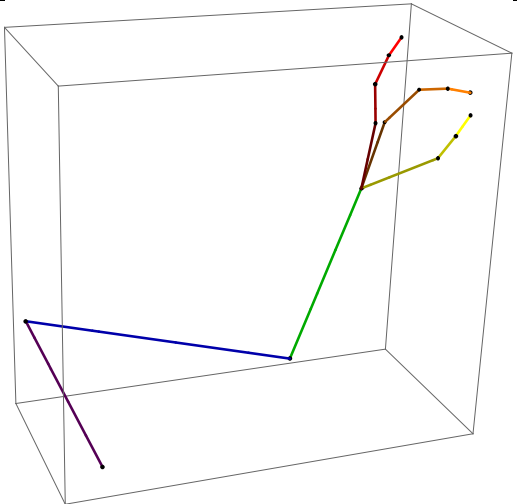
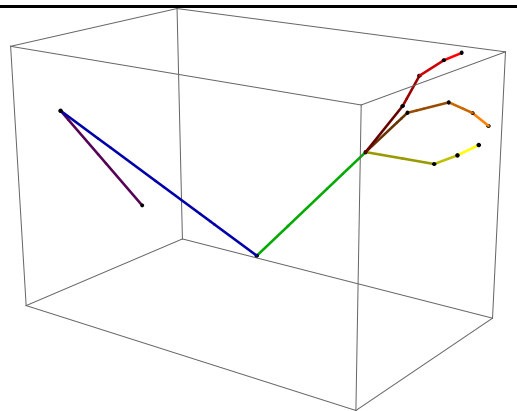
In precision poses, the hand is positioned in a way for more accurate movement. This can be seen in Figures A17, A19, and A20; where the proximal interphalangeal and distal interphalangeal joints in the index and ring finger do most of the moving. This is one reason why beginning musicians are instructed to keep these joints bent so they can more accurately play a piece of music. The glenohumeral joint has the elbow close to the body the elbow is generally obtuse or acute. The positions of the arm and the fingers are

kept in place by multiple muscle. As noted above the human muscle can only pull so with precision movements both pairs of muscles work together at similar forces to create a tense position. This can be compared to tying a tree down from multiple sides to keep it standing while the wind blows or laying concrete around a tall pole to keep it from tipping on any side.

The stroke victims do indeed show more simpler synergies which are more easily found as well. SV1's data actually had the least amount of errors in the inverse kinematics; again reiterating the fact that a stroke victim's motion is less complex than those who have not had a stroke. With SV2's hand data in general was too convoluted to be used in much of the research, however, their arm data does often show power poses and when the hand is usable, it is stuck in a power pose. This is most likely due to the over compensation that SV2 had to use to overcome the issues with their arm due to the stroke caused by a lack of continual therapy.

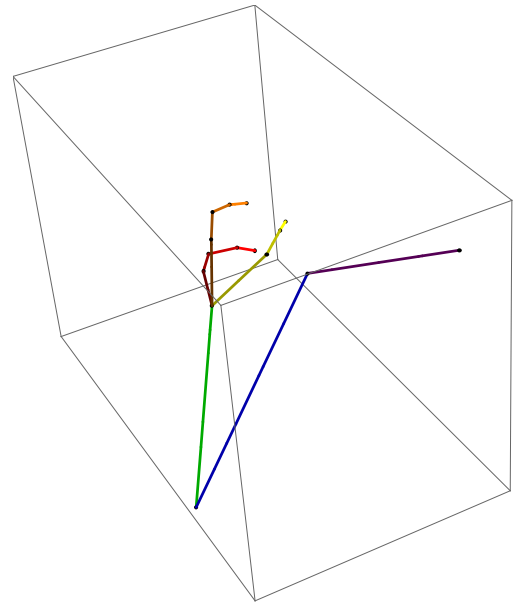
Finally all the synergies that were found are listed on Table 2 and are all on national list of synergies stated by [16] and [14] when referring to the hand portion of the synergy. These synergies are listed first by power poses then by precision poses, and will go in order of frequency they are seen. These images are simplified so that each limb or segment is a line. The lines' colors represent the segment or limb; a purple line dictates the upper torso starting from the sternum; the blue line is the upper arm; the green is the forearm; the yellow colored lines are different segments of the thumb; the orange colored lines determine the different segments of the index finger; and the red colored lines indicate the different segments of the ring finger.

Table 2: Synergies found in this project

Synergy Type	Trials Found	Image of Synergy
Precision	SV1 Ball Manipulation NS2 Ball Manipulation NS2 Open Door	
Precision	SV1 Writing NS2 Drinking NS2 Writing	
Precision	SV1 Writing NS2 Open Door NS2 Writing	

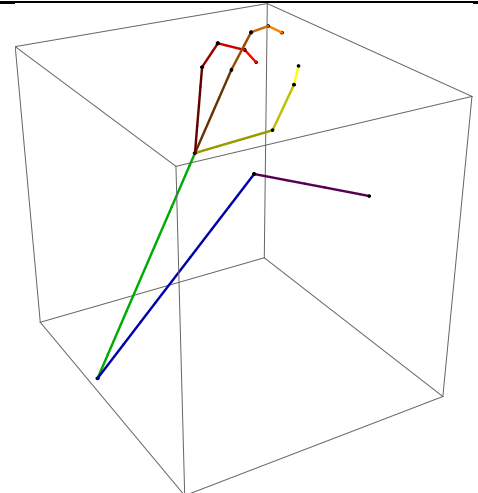
Power

SV1 Drinking
SV1 Open Door
SV2 Drinking
NS2 Drinking
NS2 Open Door



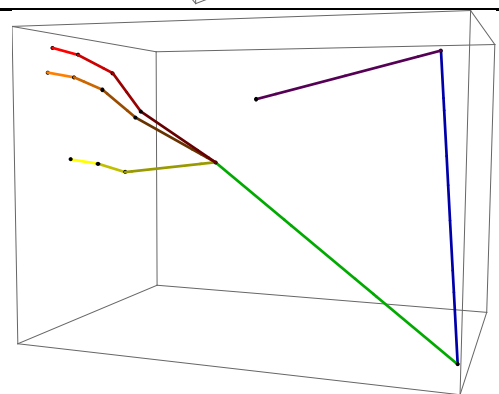
Power

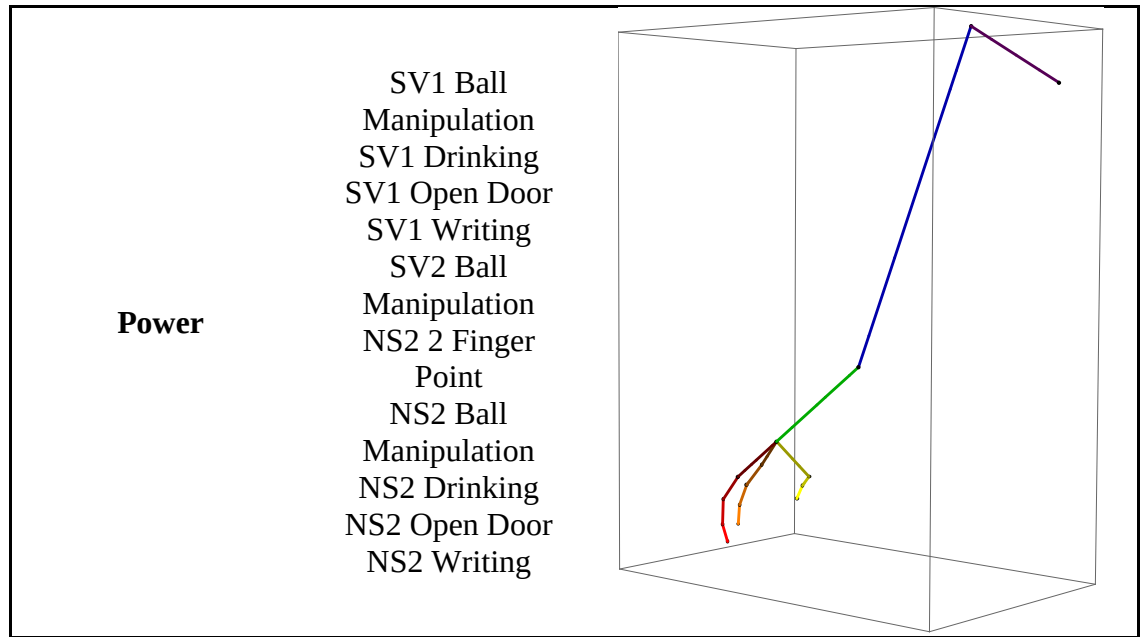
SV1 Drinking
SV1 Open Door
SV2 Open Door
SV2 Writing
NS2 Drinking
NS2 Open Door
NS2 Writing



Power

SV1 Open Door
SV2 Open Door
NS2 Open Door





5. Conclusion

5.1. Research Finding

The report has covered a system in which motion can be captured and the process of finding motion synergies. The system used to capture the motion of the upper limb is the Vicon system; using infrared cameras, markers, and Vicon Nexus software. The correct markers placement was figured out over several months of testing. The data that is used to find the synergies of this report were collect over a series of trials with three different subjects. This data was then processed with inverse kinematics and vector math to find the joint angles. CPA was then preformed on the angles and the new data matrixes were plotted. The plotted data allowed patterns to be found in the motion and these patterns were then confirmed with known synergies to determine if a synergy was found.

The synergies that were found are already defined hand synergies in [16] and [14], with the addition to the arm down from the glenohumeral joint. These synergies are defined by their commonality and their type, either power or precision. The synergies found, see Table 2, show that synergistic movements are also found in the arm not just the hand. This research also shows that stroke victims do have limited movement. Not all of the precision synergies that were found were present at the same parts of motion in non-stroke counterparts.

The synergies of the arm can be defined as the follows. First, power synergies are designed to hold weight or resist force. To resist force the arm will stick out from the torso due to the glenohumeral joint. Second the elbow is at its strongest point which is pulling towards the body; about ninety degrees give or take twenty five degrees. The

precision synergy is the opposite of the power synergy; the elbow is close to the trunk or on top of the trunk. The elbow itself is an acute or obtuse angle.

Lastly, are the differences and similarities in the stroke and non-stroke victims' synergies. This can easily be summated as stroke victims lack the use of many precision synergies. SV1 had the presence of precision synergies however it occurred less often, and was often replaced with power synergies. SV2 had no precision synergies present, possibly due to the stroke occurring many years prior. Now stroke victims show all the power poses and often as well, more often than the non-stroke subject. In SV2 only power synergies were found and they rattled the movement. SV1 showed some precision synergies but often when NS2 showed these synergies, SV1 would show power synergies.

5.2. Future Research Possibilities

This research can easily be continued in several different fields of study. Firstly in the robotics field the study of synergies can lead to more competent robots that can interact better with their environment. It can help in biomedical engineering in the creation of more capable prosthetics, that can perform the movements that the amputee or disfigured may no longer be able to perform in a natural way. In the health sciences it can be used to better understand and help in rehabilitation of amputees, disfigured, and stroke victims. The goal in these particular fields would be different but the end goal is the same, to better the lives of those who need help.

The work that should be continued from this study is the differences in stroke and non-stroke victims, and the synergies of the arm. Key to the study of the differences in synergies is the lack of precision synergies in stroke victims, as mentioned before precision synergies are either not present or not commonly used. Research in the arm

synergies should include more explicated classification of the synergies and their relationship with hand synergies.

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Appendix

Appendix A, Figures and Tables

Table 3: marker placement for general motion capture with adjusted markers for fingers.

Marker Placement				
Area	Positions			Notes
Thorax				
	C7 Vertebra	dorsal		7 vertebra from the skull, place "on" vertebrae
	T8 Vertebra	dorsal		15 vertebra from the skull, place "on" vertebrae
	Sternum	caudal		At the end, bottom, of the sternum
	Suprasternal Notch		in notch	The notch at the "top" of the sternum, on the rib cage
Clavicular				
	Acromioclavicular Joint	dorsal		Joint on top of the shoulder on the joint formed by the highest point on the scapula and clavicle
	Sternoclavicular Joint	ventral		On the clavicle at the connection point to the sternum
Scapular				
	Angulus Inferior	caudal		Lowest portion of the scapula
	Angulus Acromialis	later dorsal		Portion of scapula that juts out and to the back of the body at the shoulder
	Processus Coracoideus	ventral		Portion of scapula that juts out and to the front of the body at the shoulder
	Trigonum Spine Scapular	mid-point		Portion of scapula that juts out and to the back of the body on the higher portion of the scapula
Humerus				
	Glenohumeral Rotation Center	Estimated by Ticlical Axis Method		The "ball" of the humerus, estimated per person, normally at a 45° to the shoulder on the ball
	Lateral Epicondyle	caudal		End of the Humerus perpendicular to the rotation of the joint on the inside of the arm

	Medial Epicondyle	caudal		End of the Humerus perpendicular to the rotation of the joint on the outside of the arm
Forearm				
	Ulnar Styloid	caudal		In parallel to the fingers on the end of the Ulna (pinky side of wrist)
	Radial Styloid	caudal		In parallel to the fingers on the end of the Radius (thumb side of wrist)
Hand				
	Processus Styloideus OS Metacarpal 3	dorsal		At the connection of the "middle finger" metacarpi and the wrist bones
	Metacarpi Phalanges	distal	below metacarpophalageal joint	Below the knuckle on the back side of the hand laying on the bone
	Proximal Phalanges	dorsal	centered	Centered on the portion of the finger following the knuckle and the 1st joint on the back side of the hand
	Intermediate Phalanges	dorsal	centered	Centered on the portion of the finger following the 1st joint and the 2nd joint on the back side of the hand
	Distal Phalanges	dorsal	distal	Residing on the last portion of the finger (fingernail area)

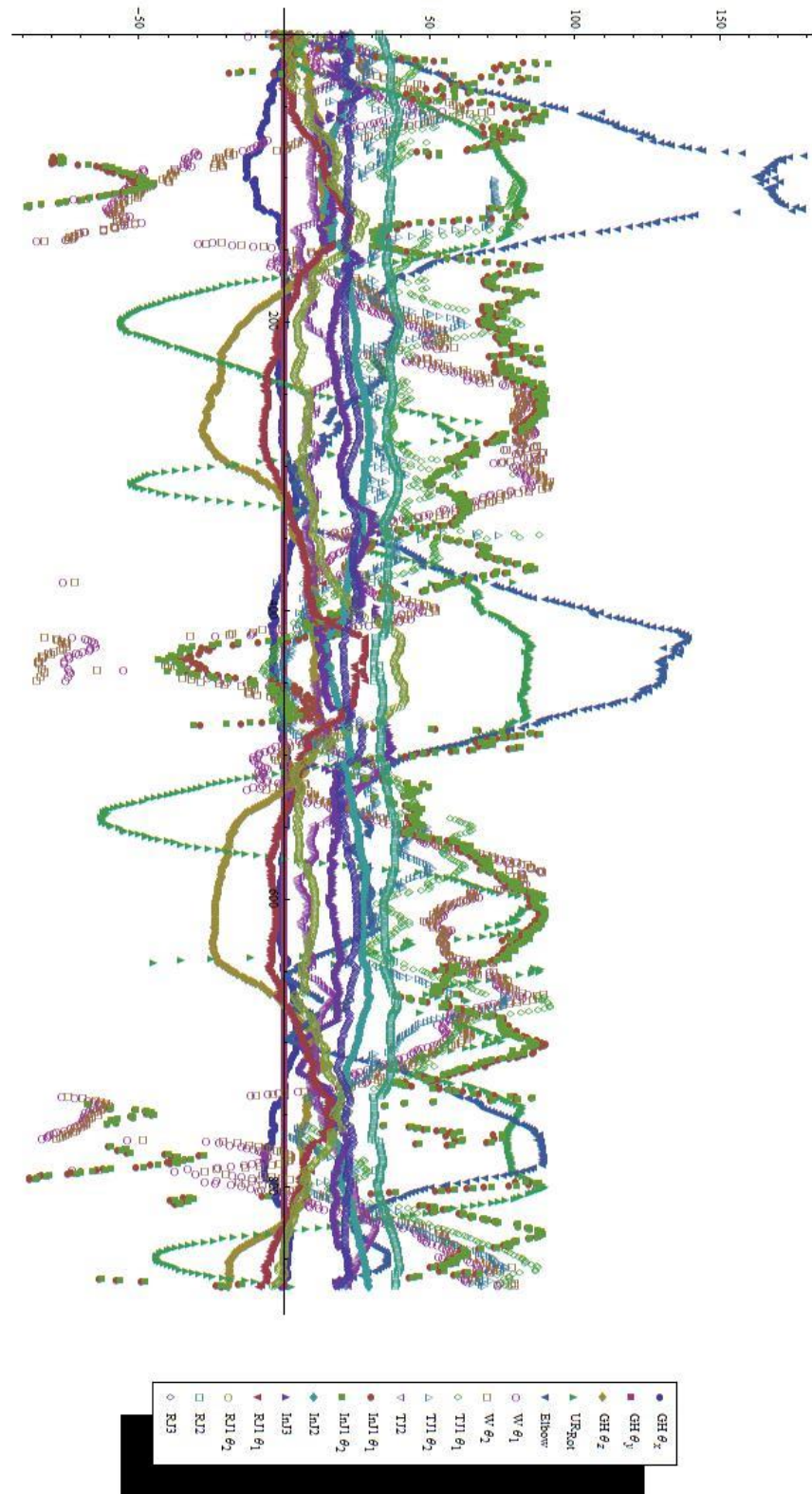


Figure 10: reconstructed angles of SV1 2 Finger Point, in degrees, at each frame.

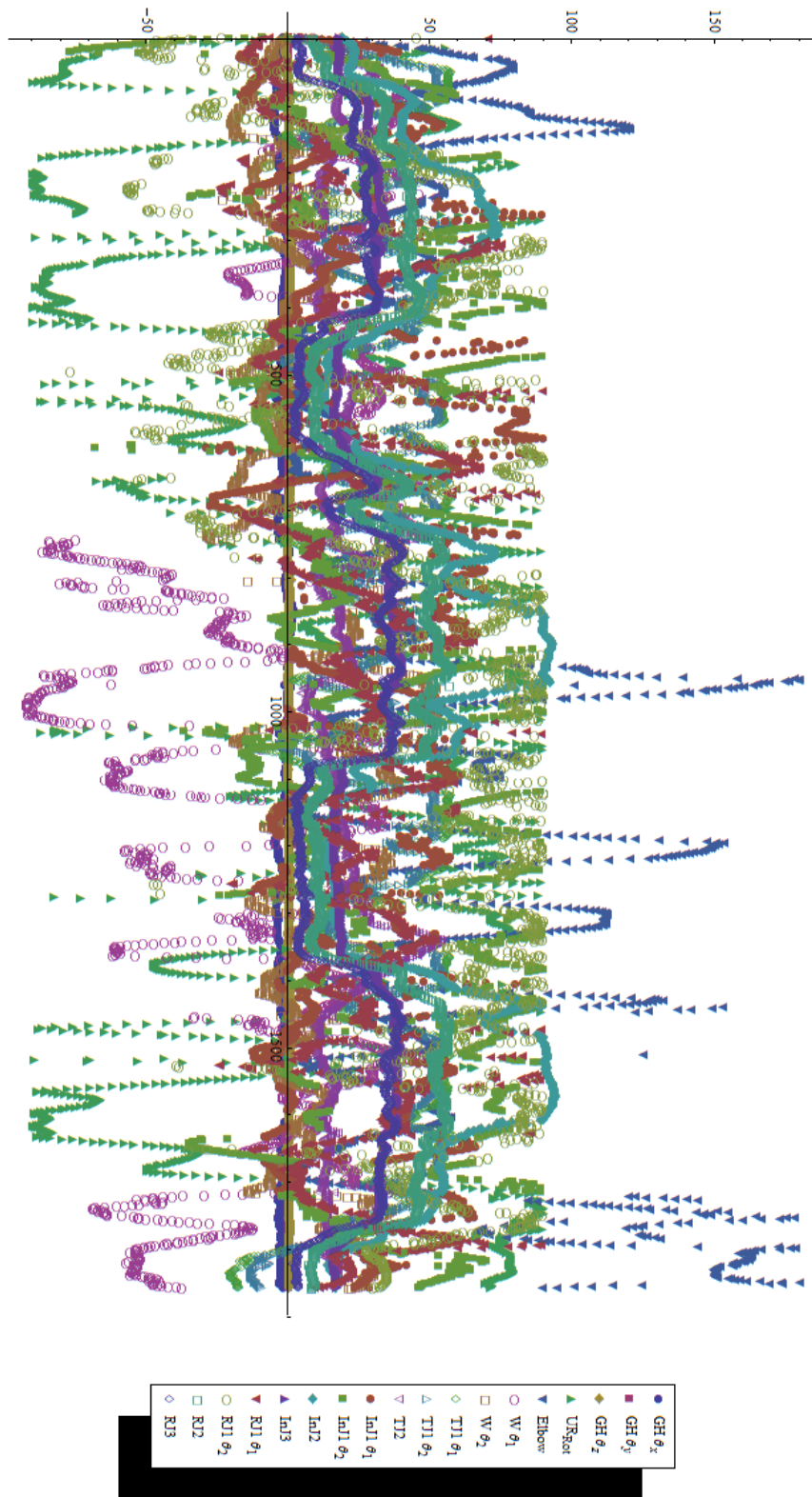


Figure 11: reconstructed angles of SV1 Ball Manipulation, in degrees, at each frame.

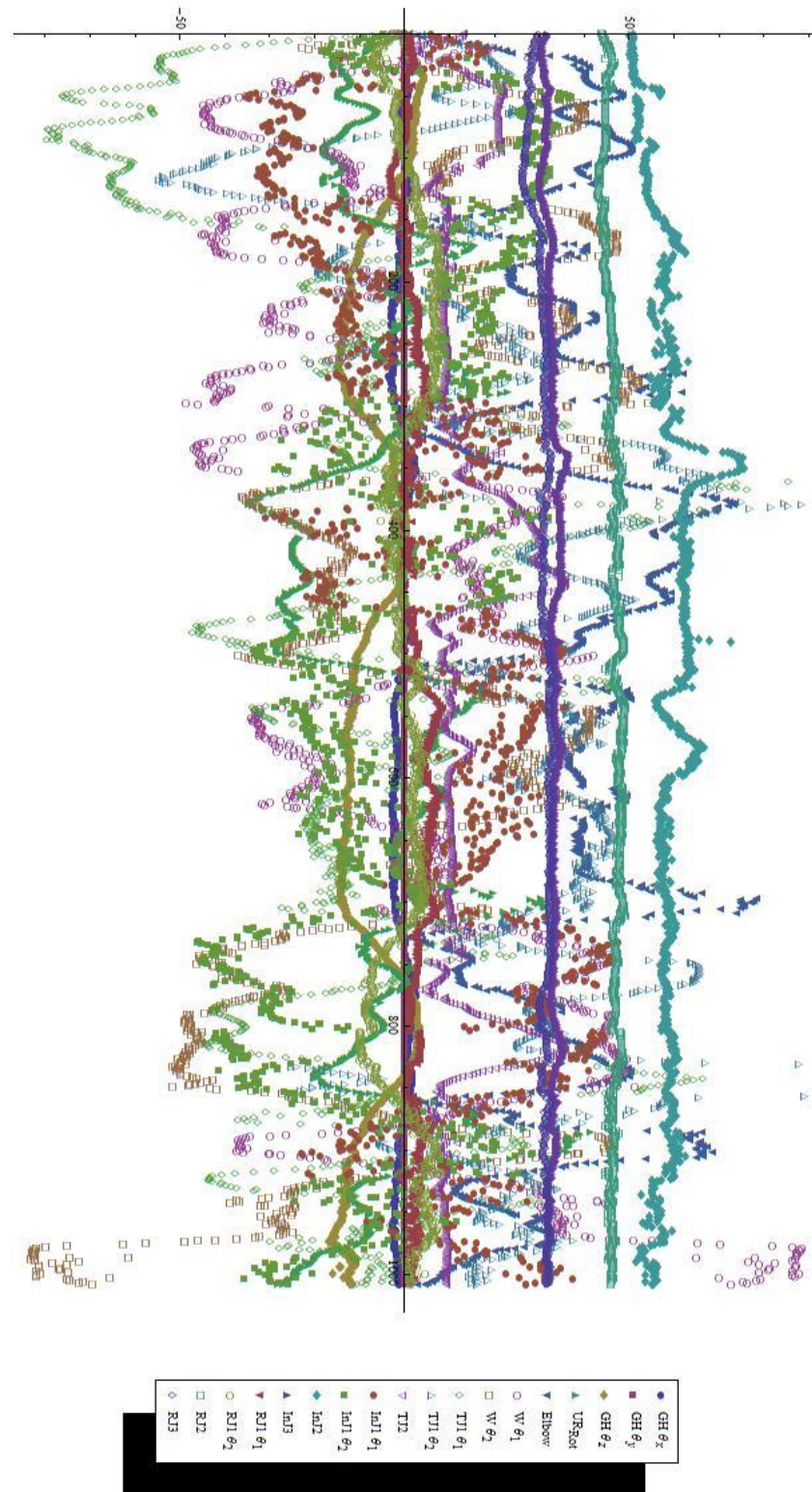


Figure 12: reconstructed angles of SV1 Drinking, in degrees, at each frame.

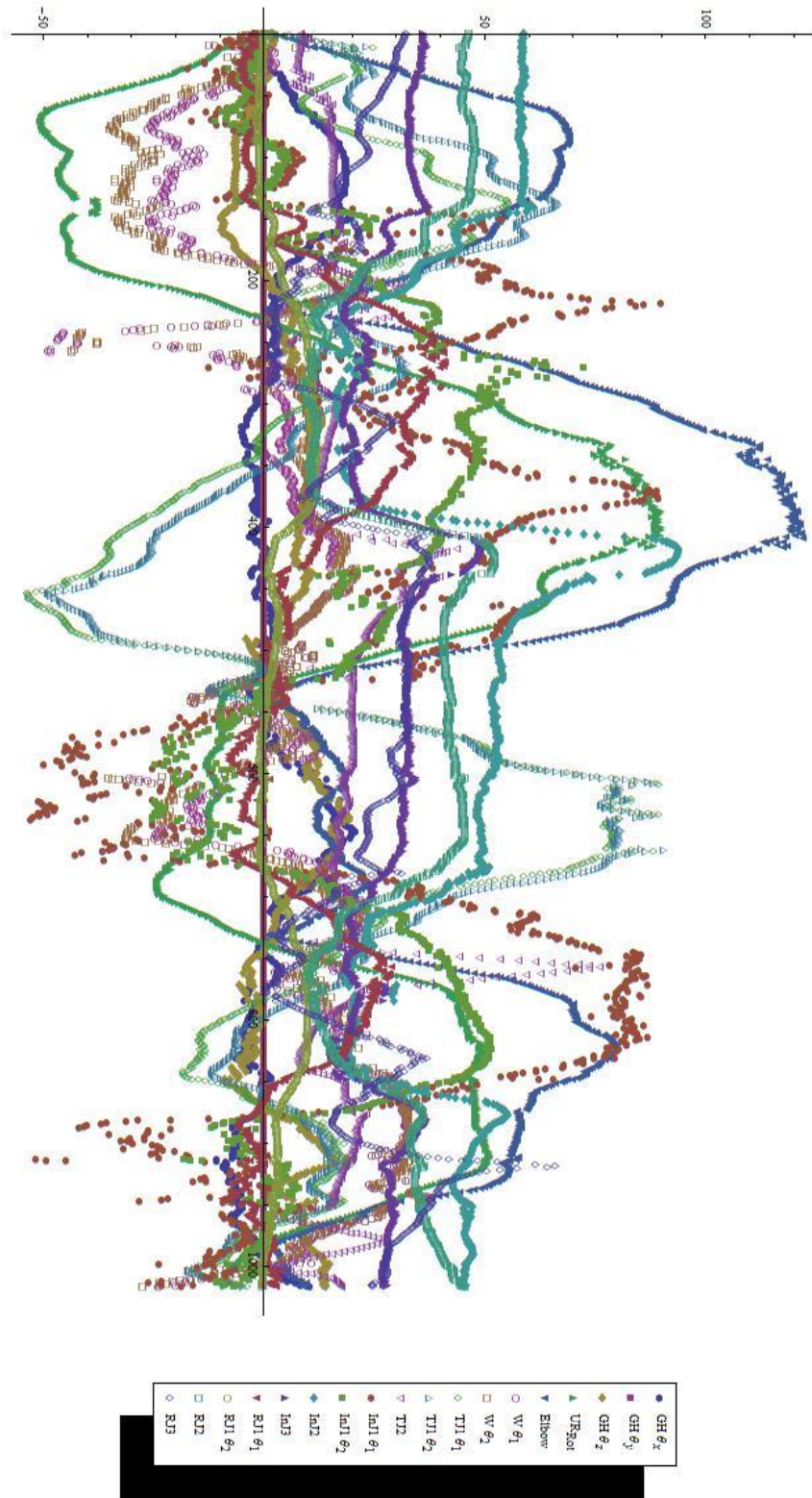


Figure 13: reconstructed angles of SV1 Open Door, in degrees, at each frame.

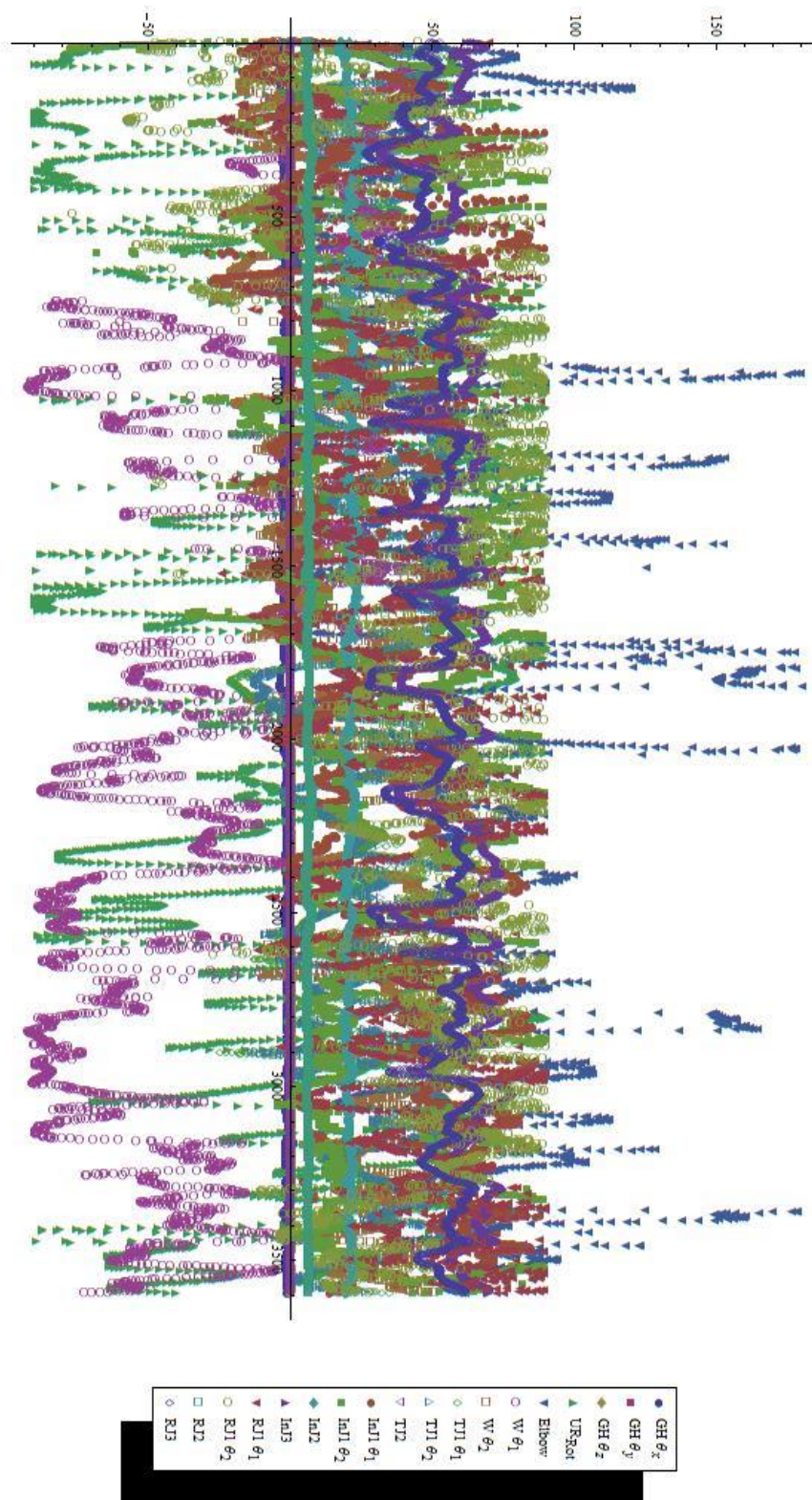


Figure 14: reconstructed angles of SV1 Writing, in degrees, at each frame.

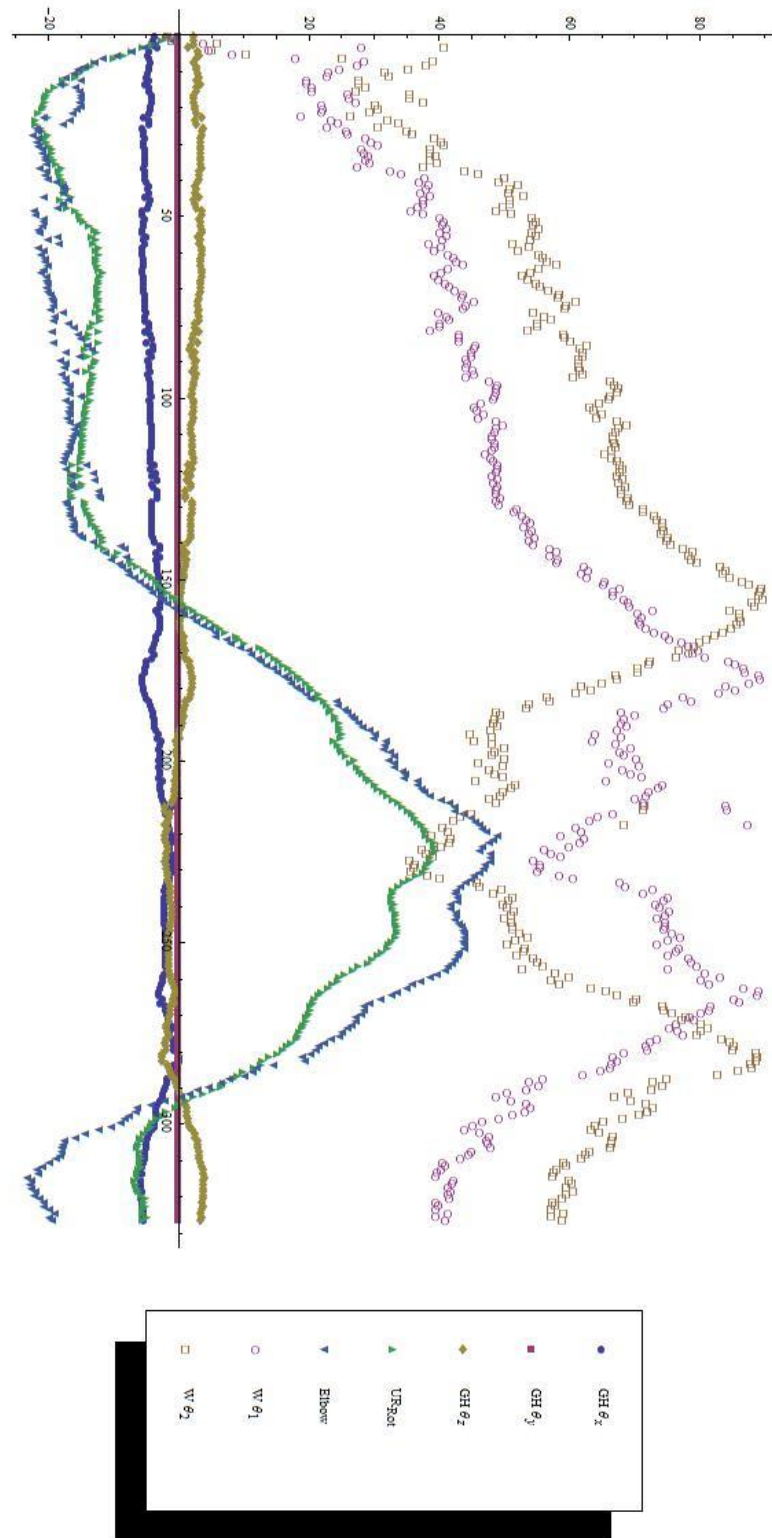


Figure 15: reconstructed angles of SV2 2 Finger Point, in degrees, at each frame.

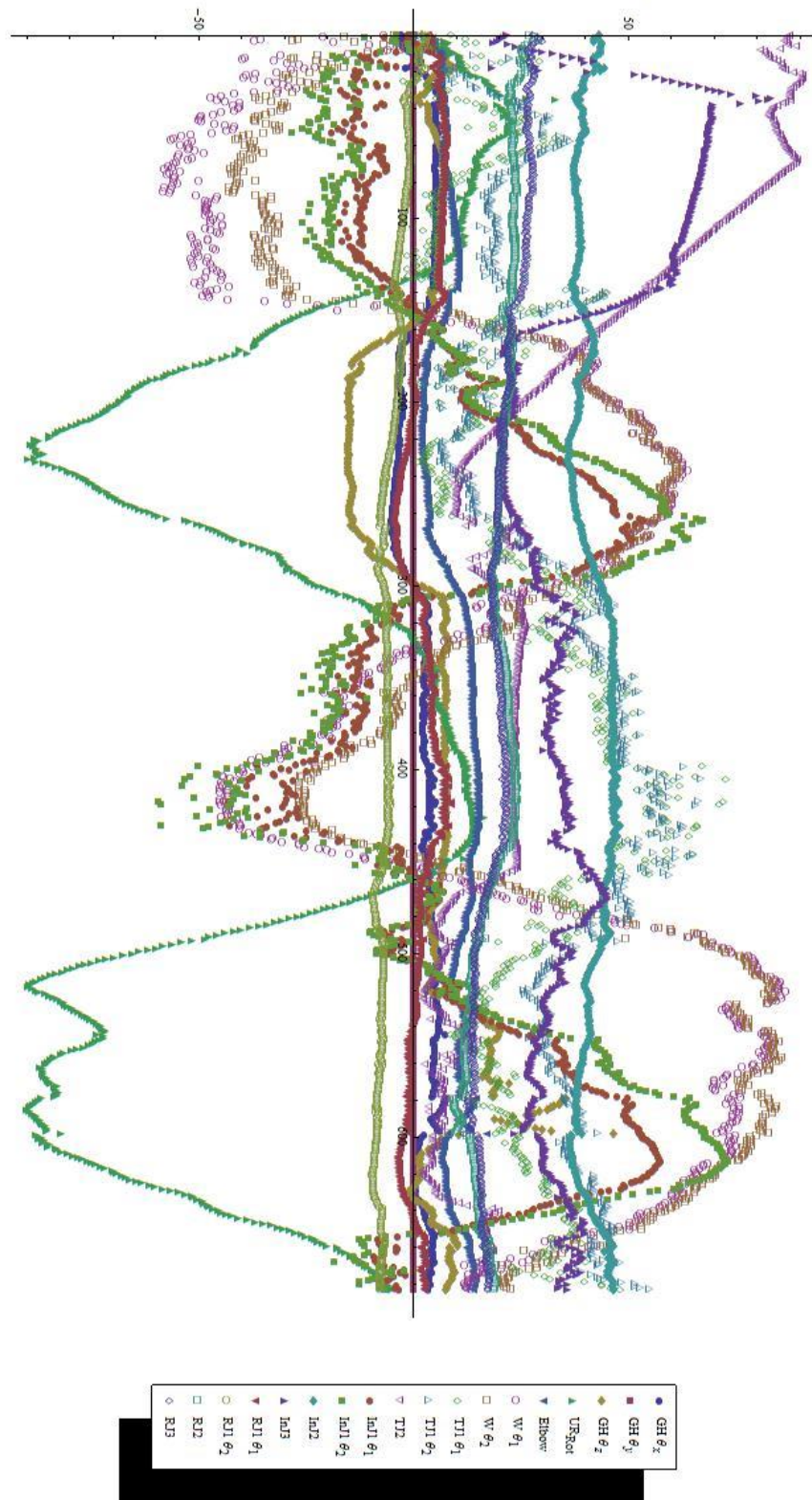


Figure 16: reconstructed angles of SV2 Ball Manipulation, in degrees, at each frame.

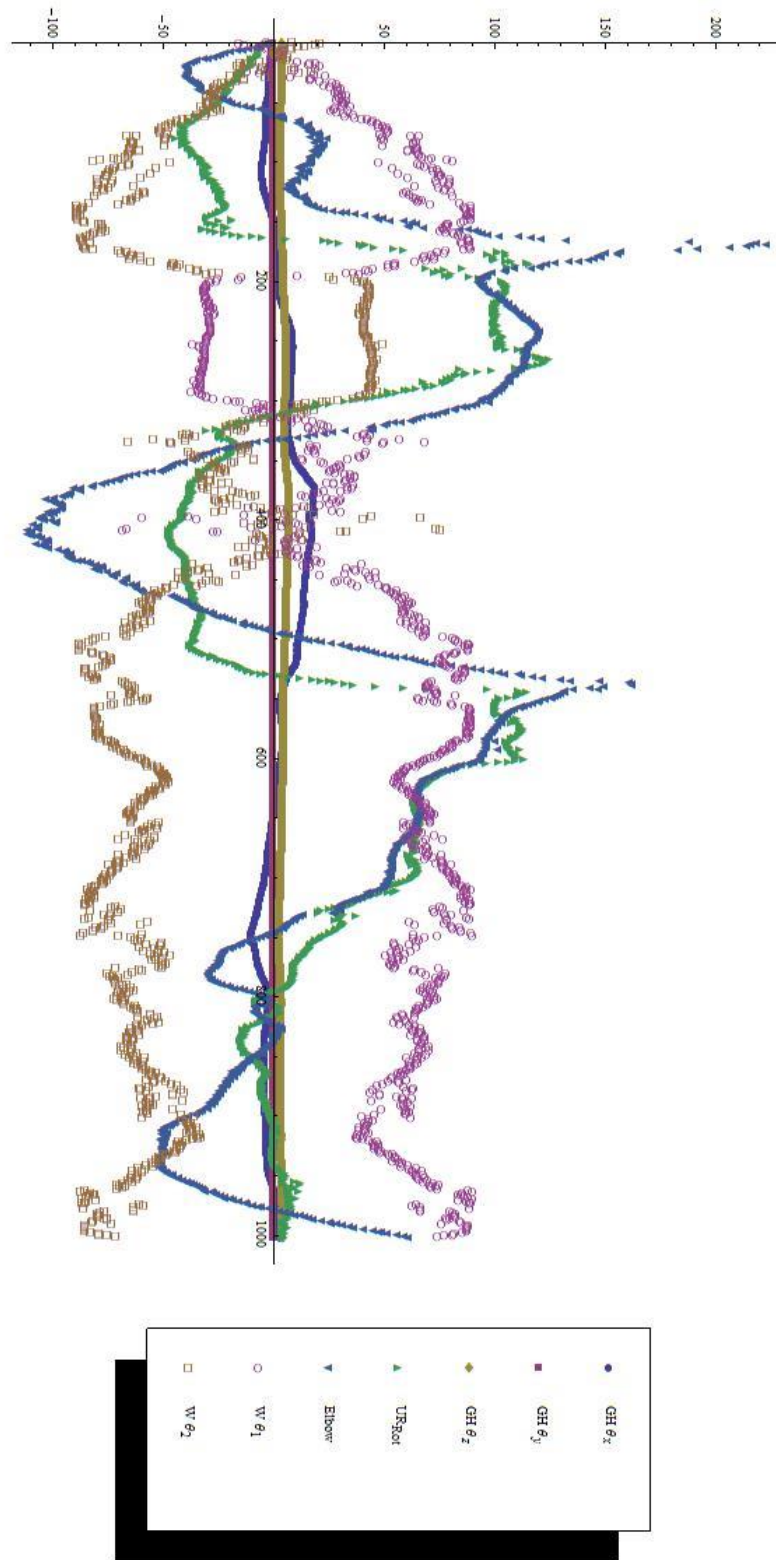


Figure 17: reconstructed angles of SV2 Drinking, in degrees, at each frame.

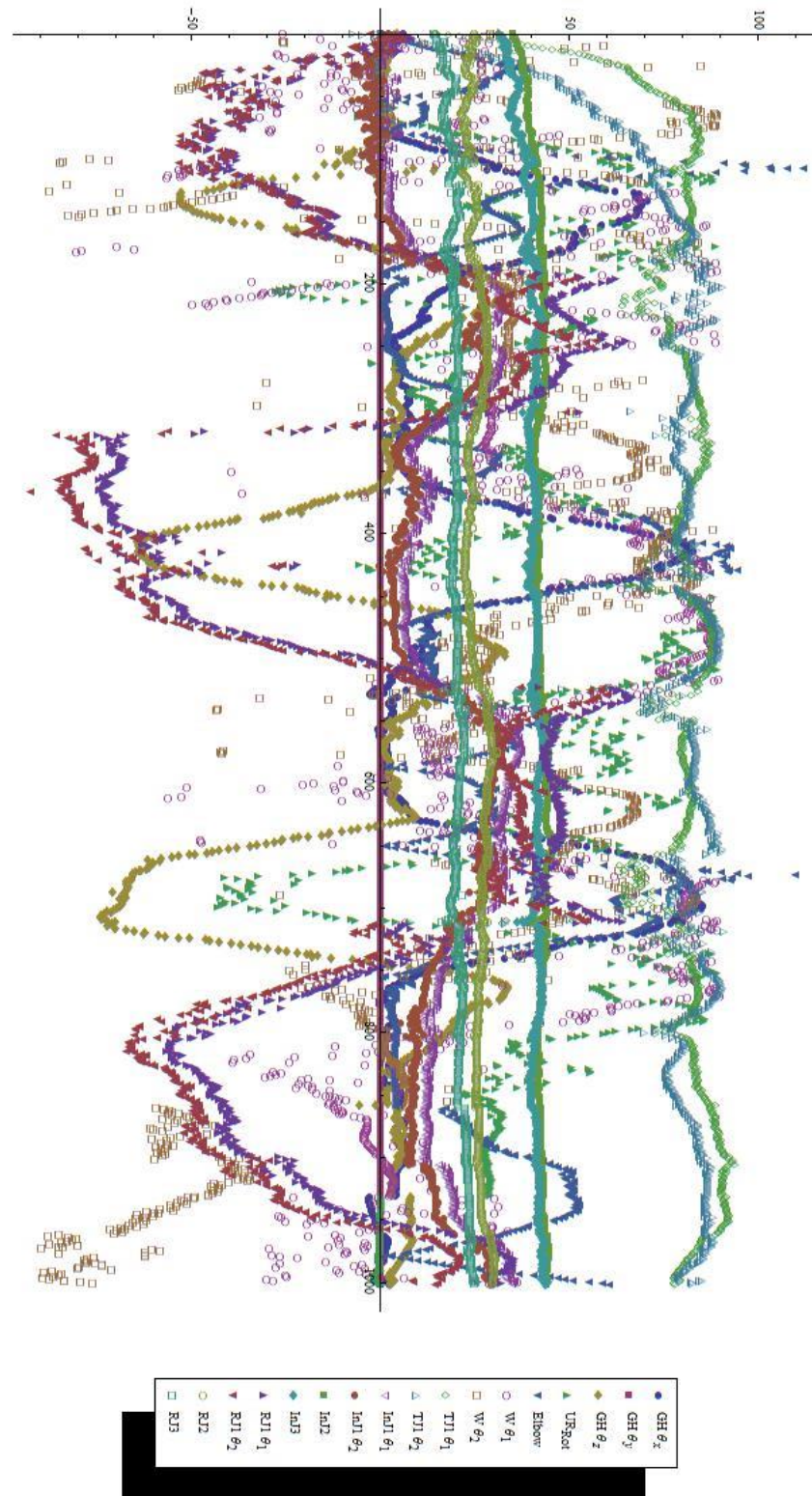


Figure 18: reconstructed angles of SV2 Open Door, in degrees, at each frame.

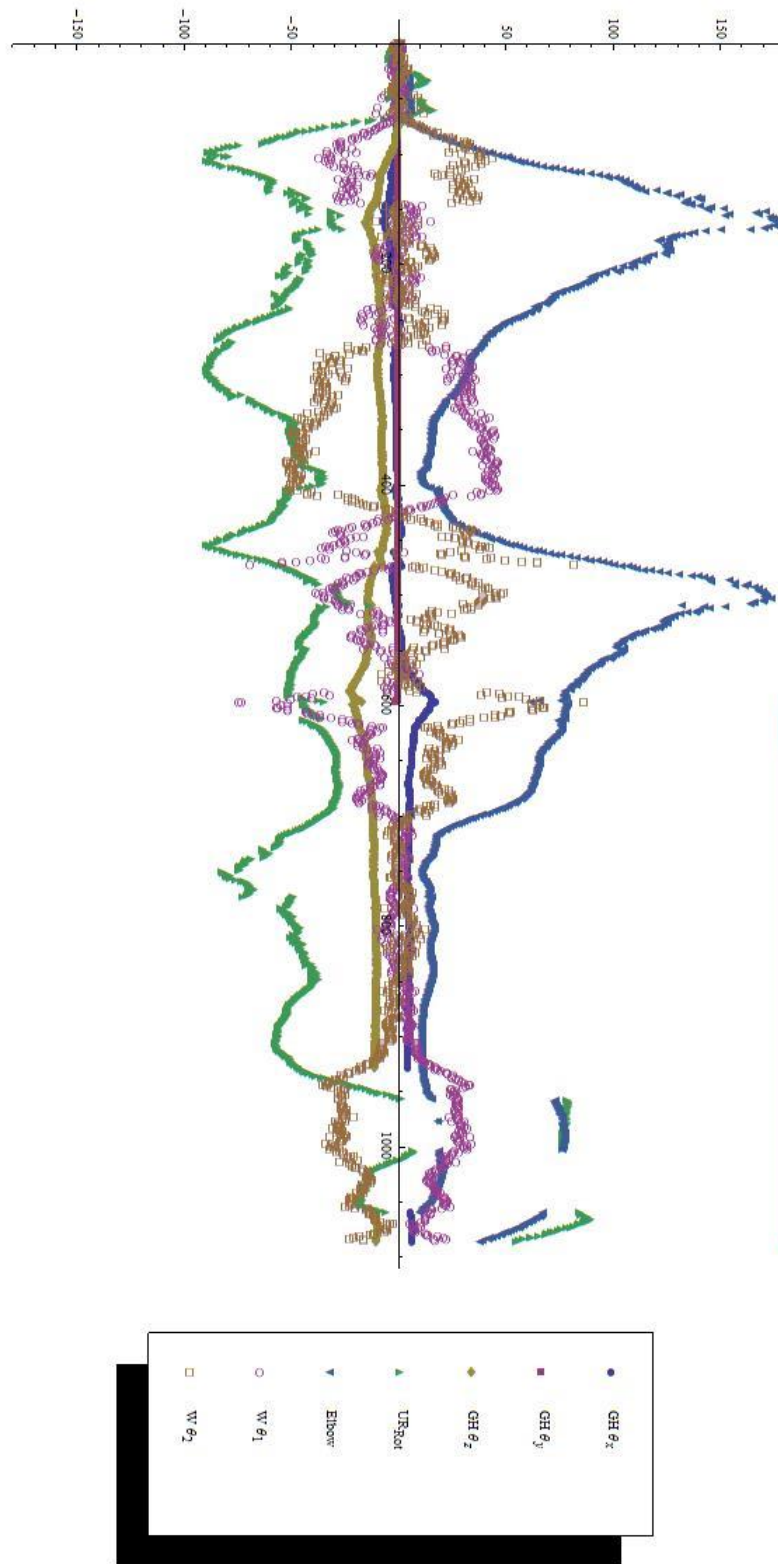


Figure 19: reconstructed angles of SV2 Writing, in degrees, at each frame.

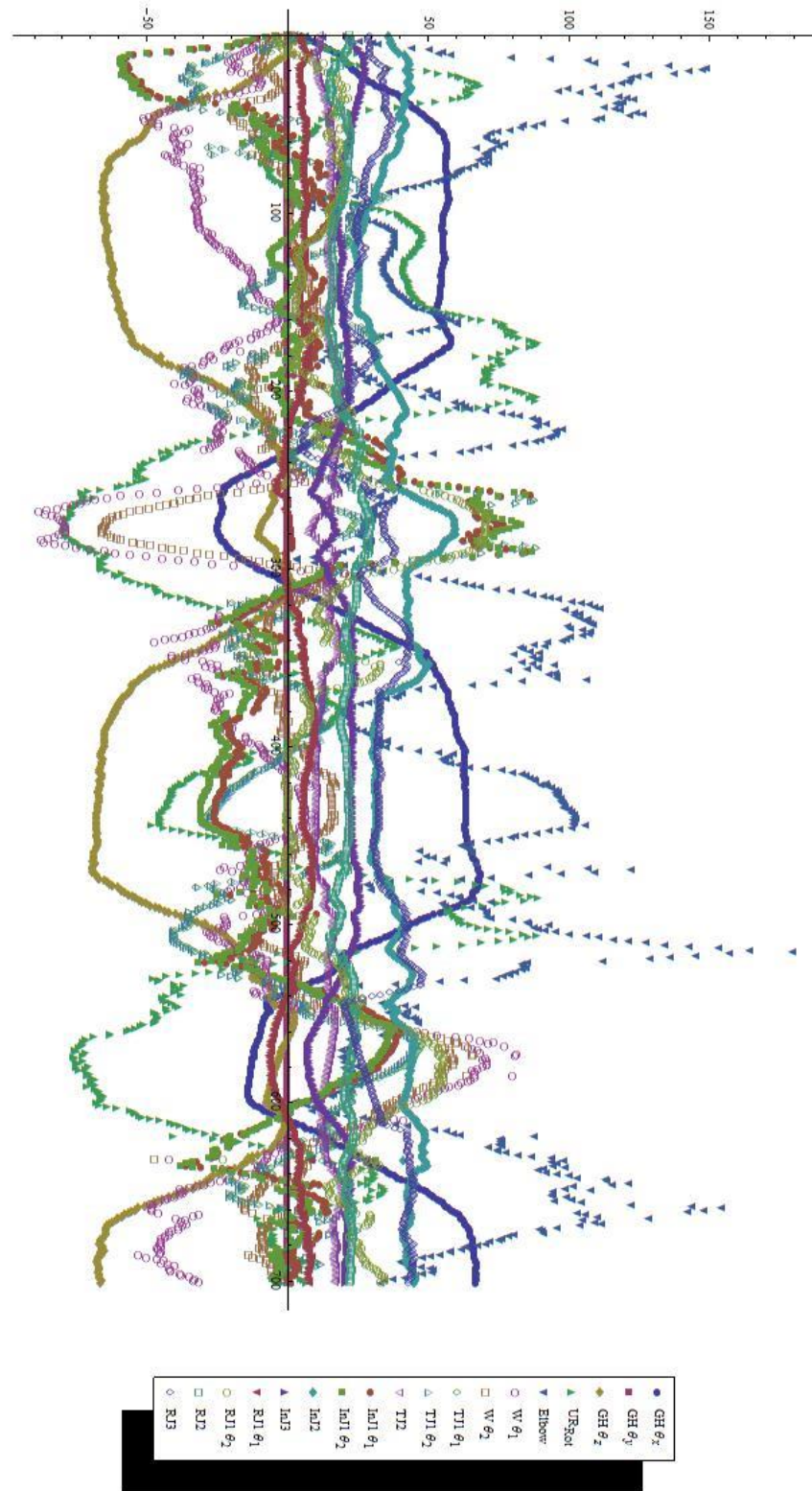


Figure 20: reconstructed angles of NS2 2 Finger Point, in degrees, at each frame.

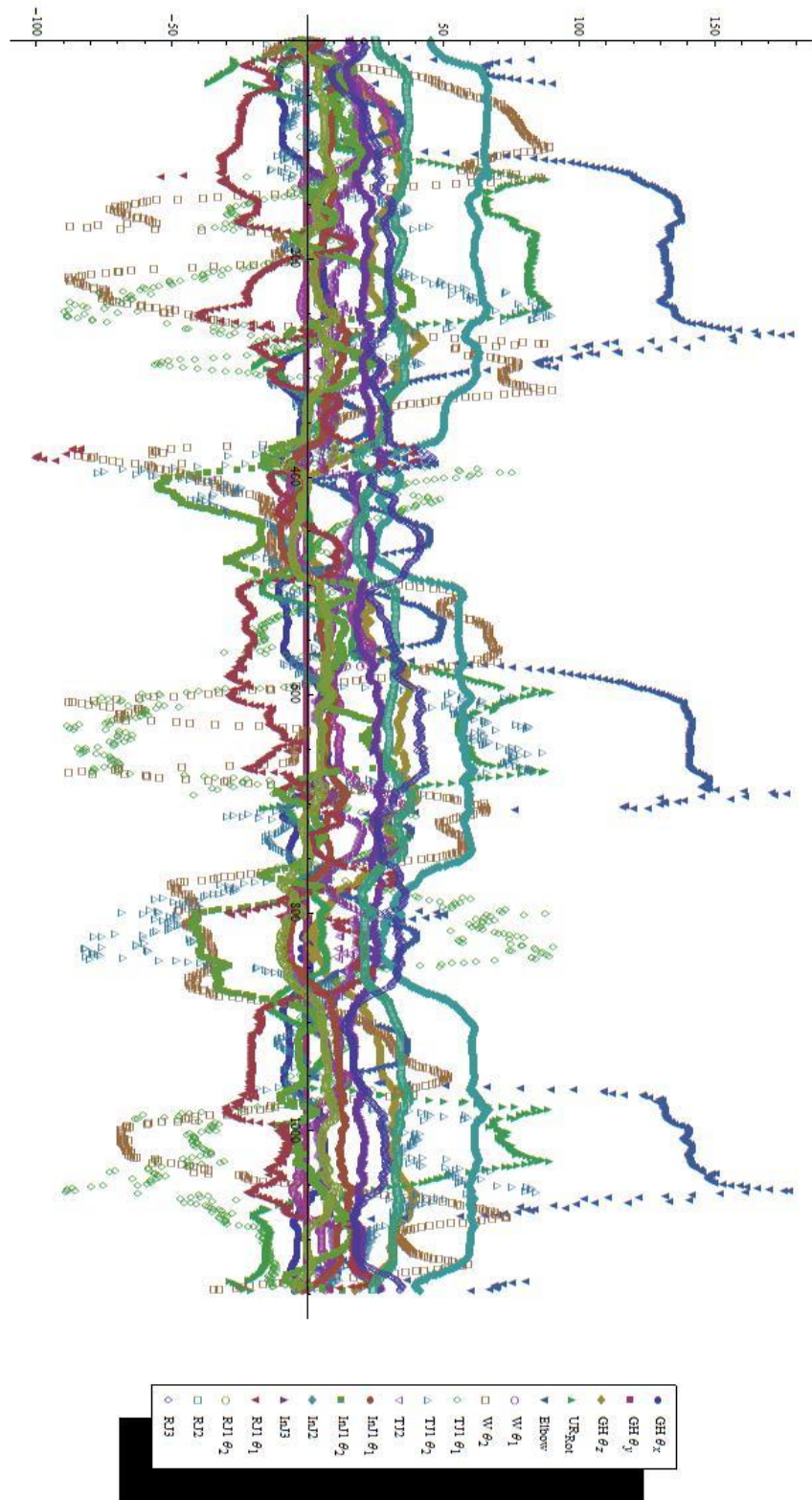


Figure 21: reconstructed angles of NS2 Ball Manipulation, in degrees, at each frame.

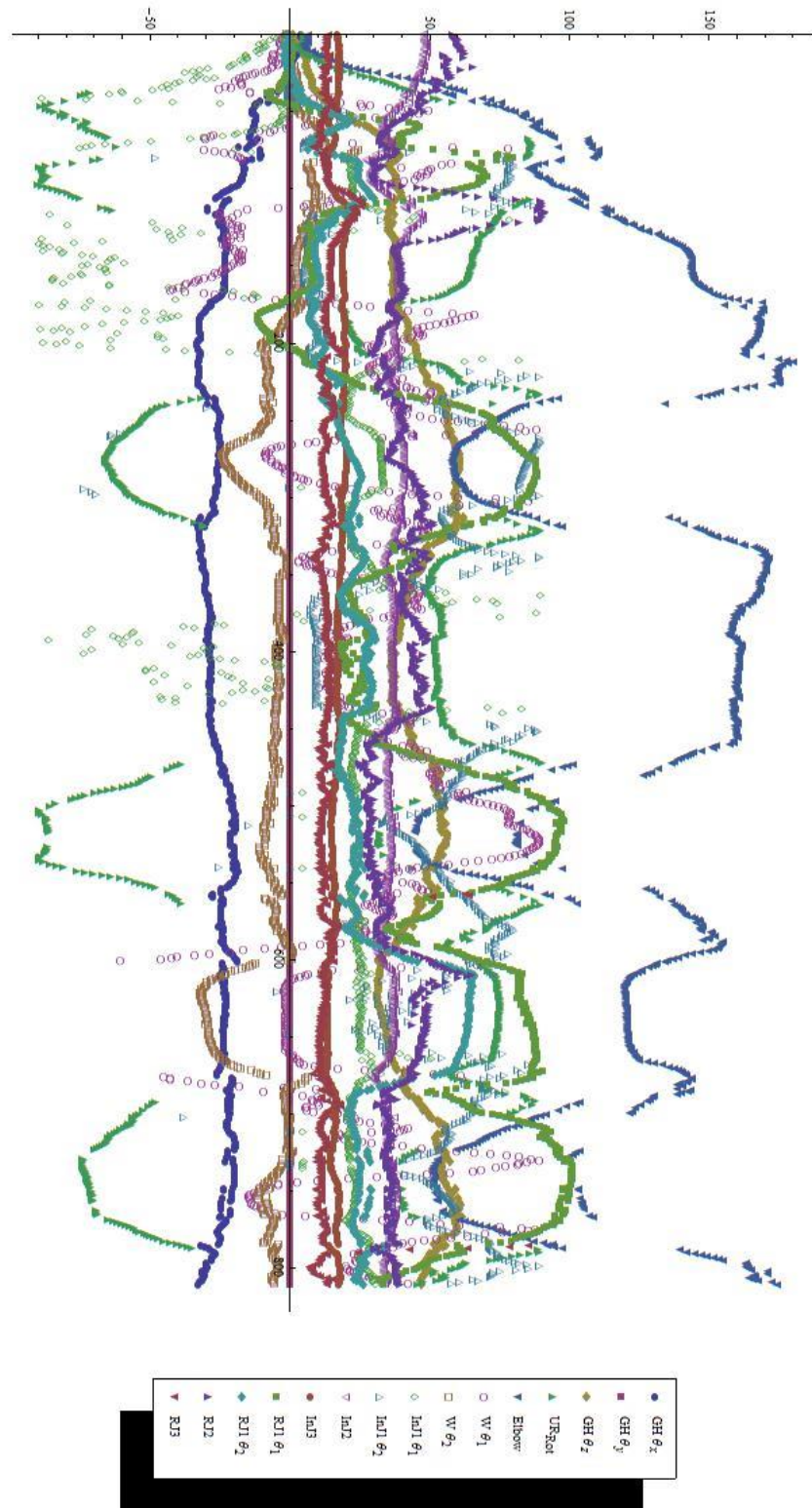


Figure 22: reconstructed angles of NS2 Drinking, in degrees, at each frame.

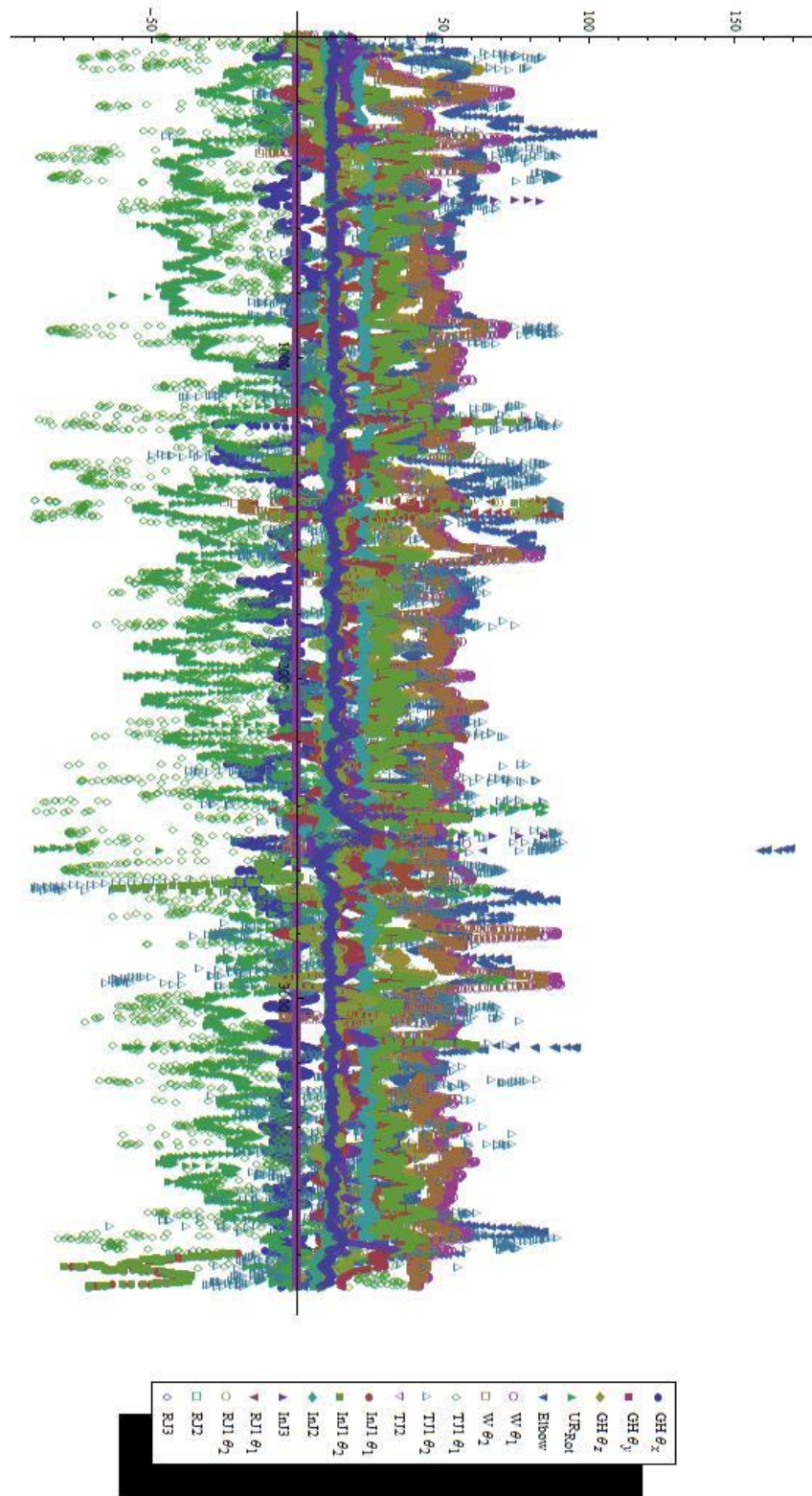


Figure 24: reconstructed angles of NS2 Writing, in degrees, at each frame.

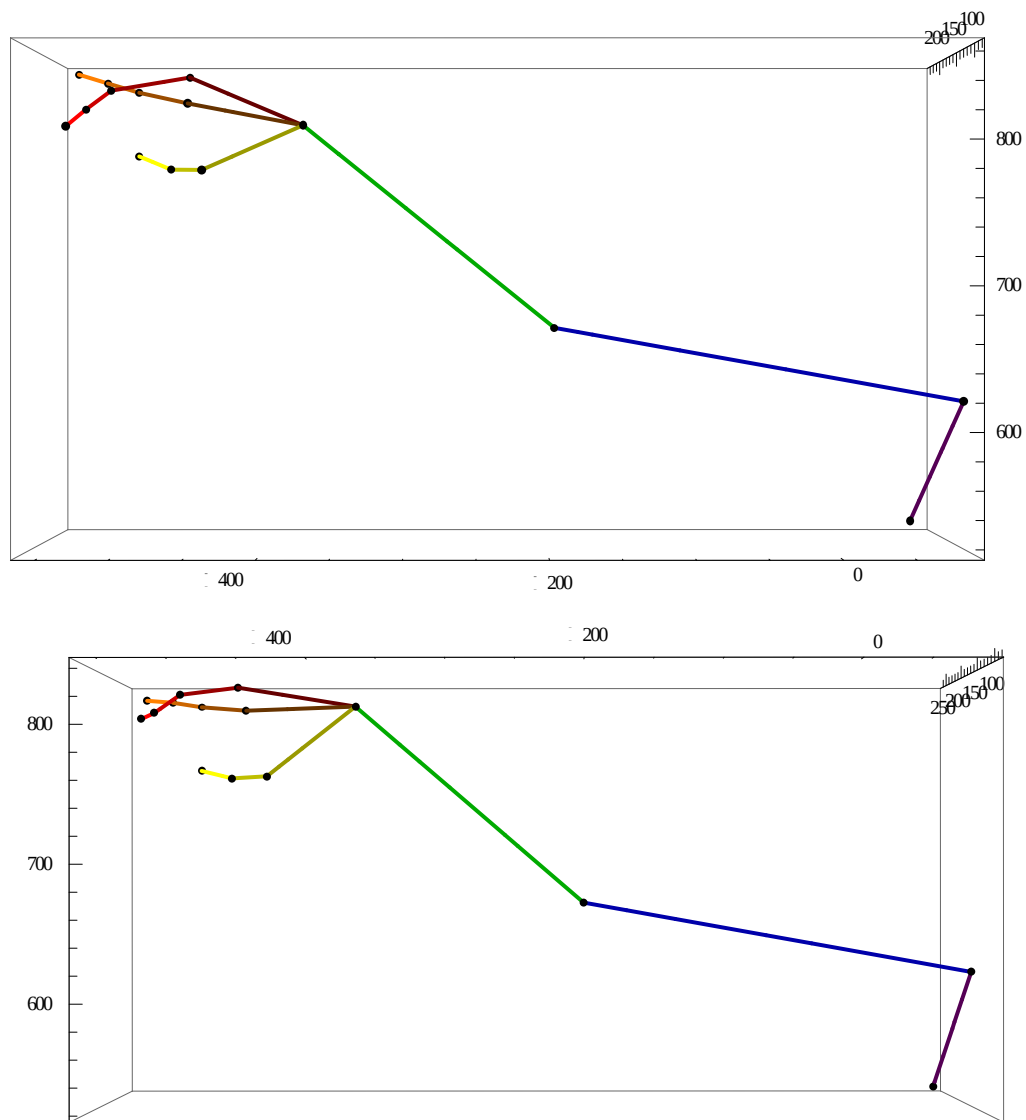


Figure 25: Supposed synergy found from NS2, 2 Finger Point data. Image was taken at frame 277 (top) and 585 (bottom). Precision pose.

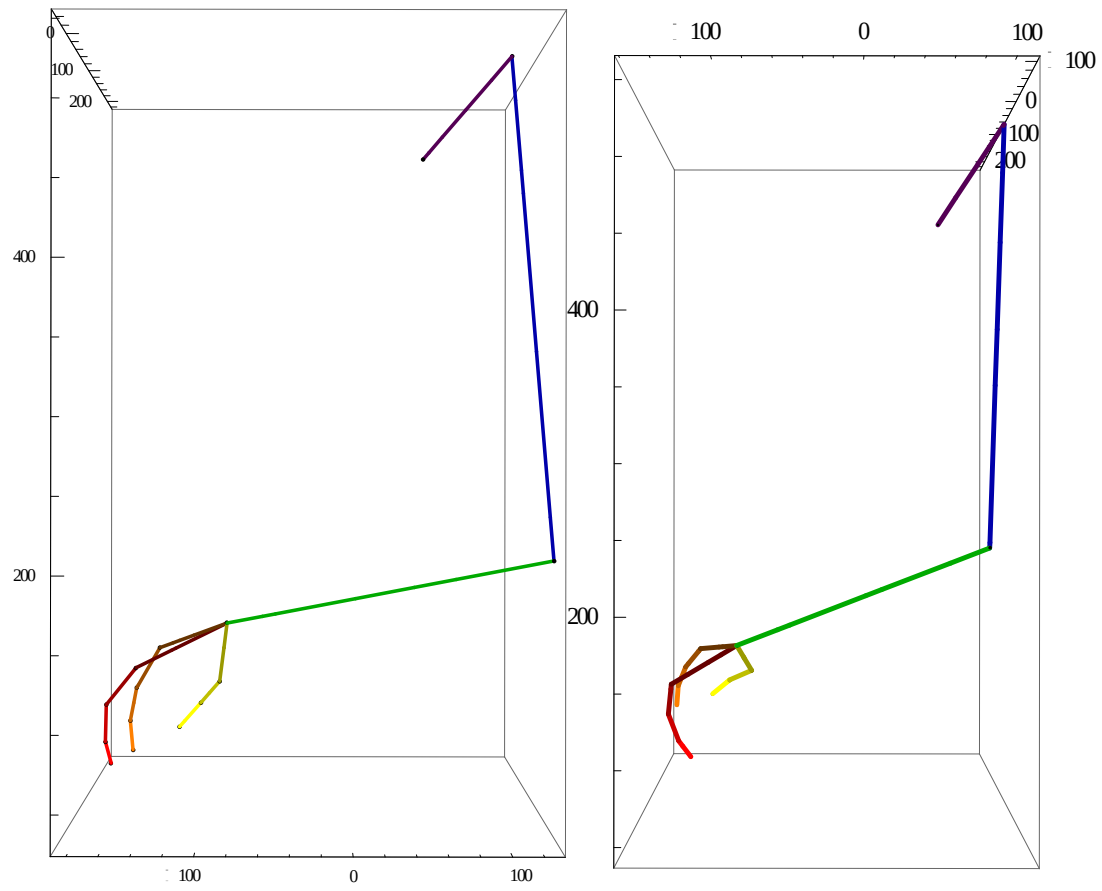


Figure 26: Supposed synergy found from NS2, 2 Finger Point data. Image was taken at frame 110 (right) and 430 (left). Power pose.

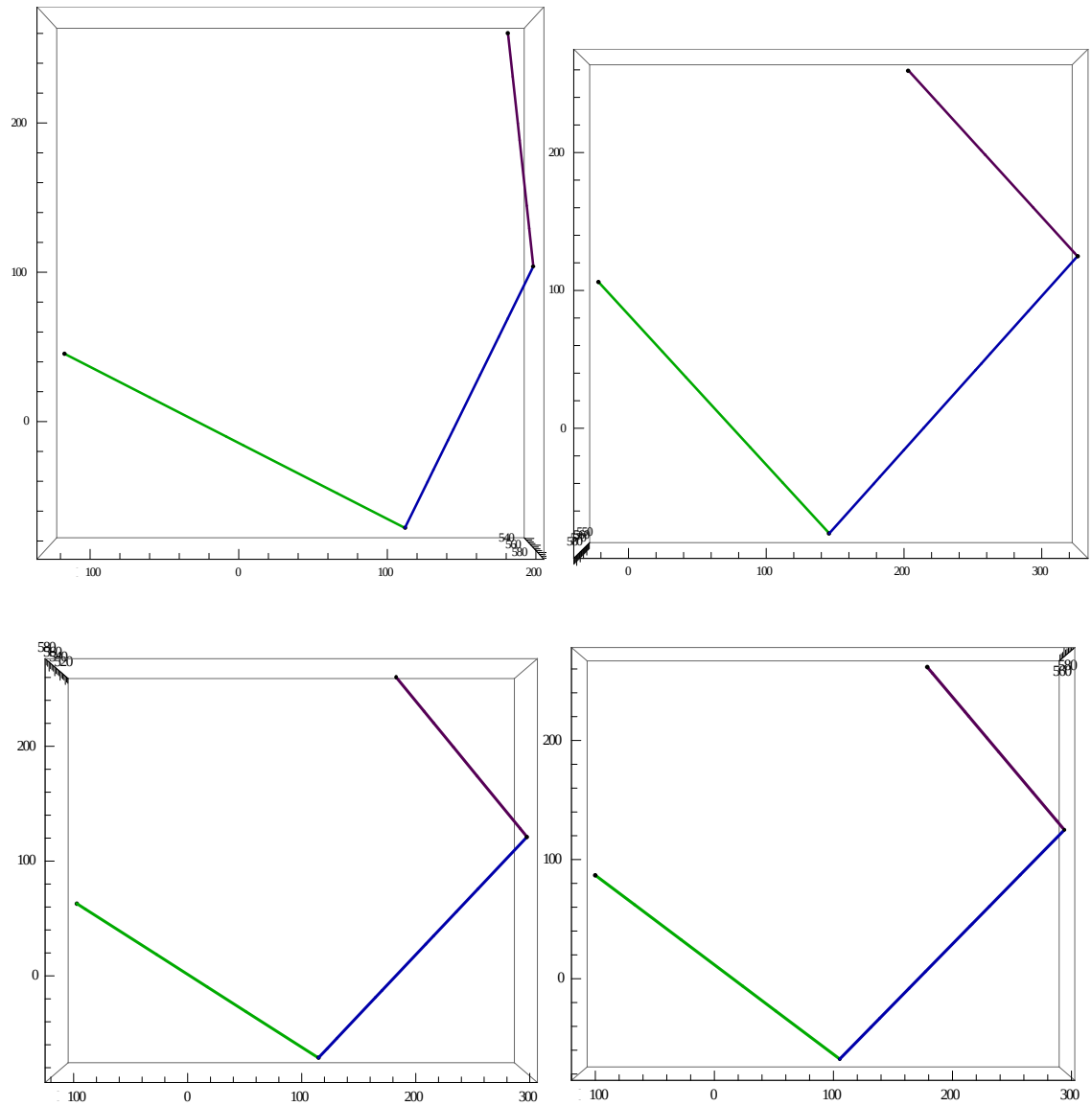


Figure 27: Supposed synergy found from SV2, 2 Finger Point data. Image was taken at frame 73 (top left), 228 (top right), 158 (bottom left) and 295 (bottom right). Power pose.

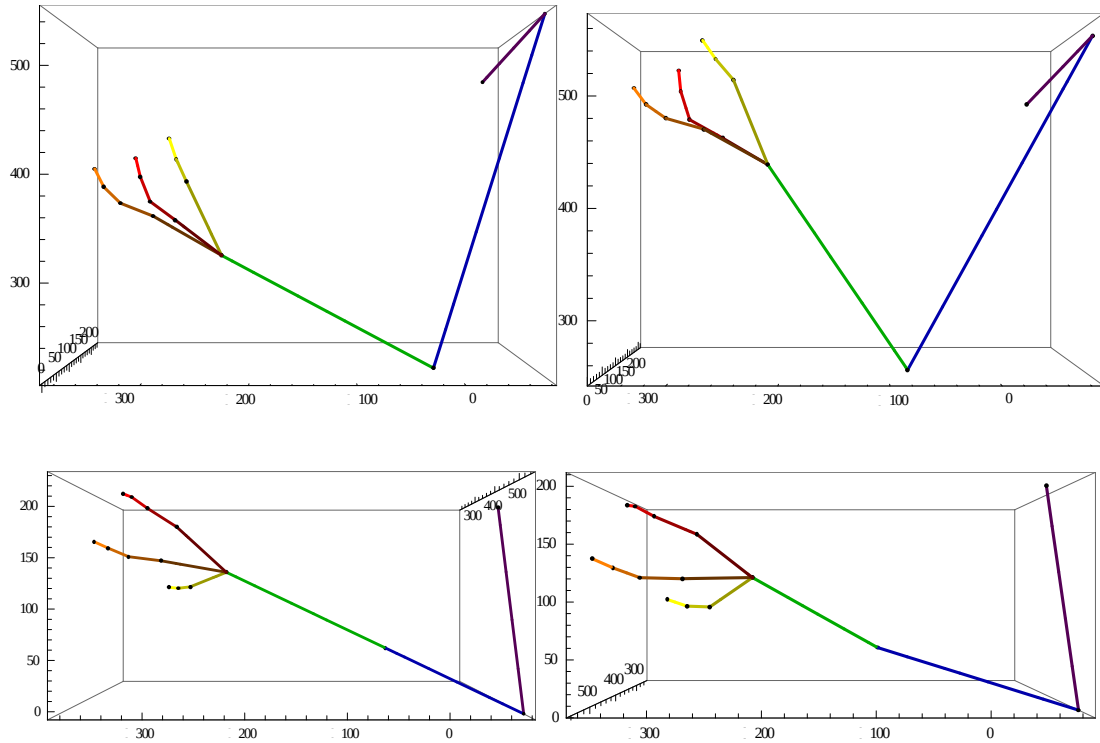


Figure 28: Supposed synergy found from NS2, Ball Manipulation data. Image was taken at frame 190 (left) and 647 (right) with front view on top and top view on bottom. Precision pose.

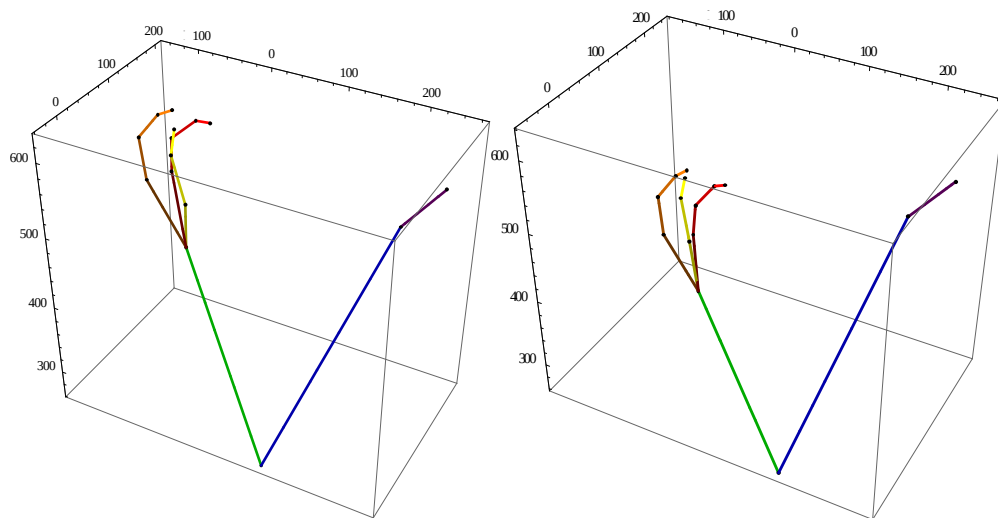


Figure 29: Supposed synergy found from SV1, Ball Manipulation data. Image was taken at frame 900 (left) and 1570 (right) with front view on top and top view on bottom. Precision pose.

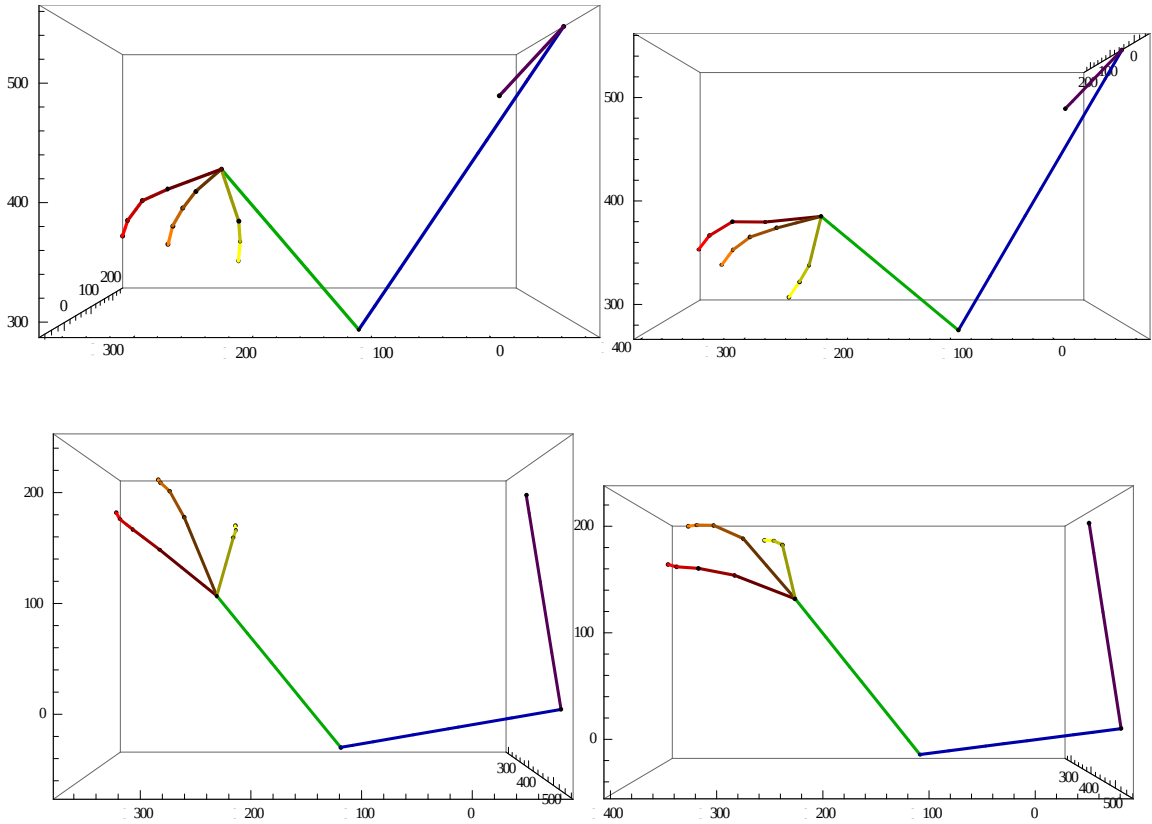


Figure 30: Supposed synergy found from NS2, Ball Manipulation data. Image was taken at frame 320 (left) and 725 (right) with front view on top and top view on bottom. Power pose.

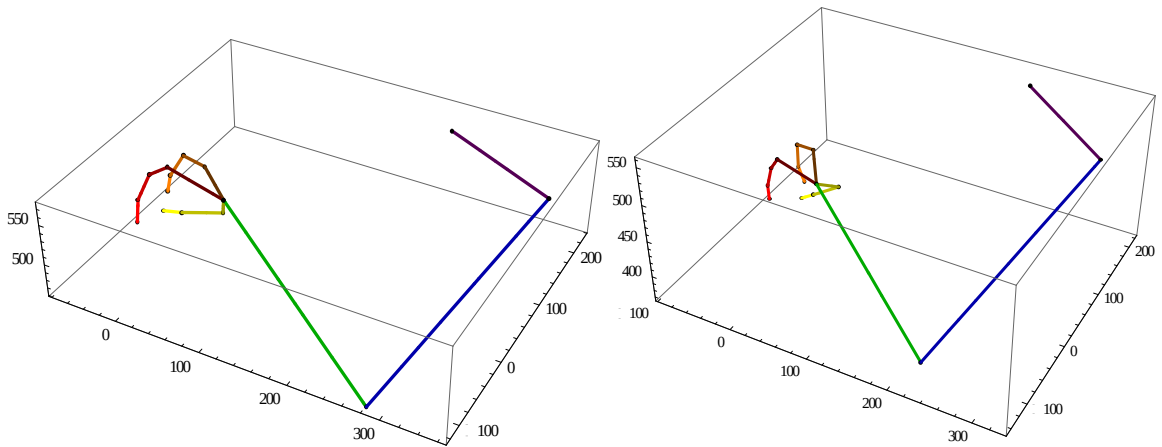


Figure 31: Supposed synergy found from SV2, Ball Manipulation data. Image was taken at frame 265 (left) and 620 (right). Power pose.

Appendix B, Code

Matrix.m

Written by Idaho State University's Department of Mechanical Engineering:

```
Off[General::spell]
DualCross[W_, V_] := {W[[1]]□V[[1]],
  W[[1]]□V[[2]] + W[[2]]□V[[1]]}
DualDot[W_, V_] := {W[[1]].V[[1]], W[[1]].V[[2]] + W[[2]].V[[1]]}
MakeLine[W_] := {W[[1]]/
  Sqrt[W[[1]].W[[1]]], ((W[[1]]□
    W[[2]])/(W[[1]].W[[1]]))□(W[[1]]/Sqrt[W[[1]].W[[1]]])}
MakePoint[W_] := (W[[1]]□W[[2]])/(W[[1]].W[[1]])
(*SinAngle[W_, V_] := ((W[[1]]□V[[1]]).(W[[1]]□V[[1]]))/
  Sqrt[(W[[1]]□V[[1]]).(W[[1]]□V[[1]])*)
SinAngle[W_, V_] := Sqrt[((W[[1]]□V[[1]]).(W[[1]]□V[[1]]))/
  (Sqrt[(W[[1]].W[[1]])]*Sqrt[(V[[1]].V[[1]])])
LineTrans[disp_, line_] := {RotPart[disp].line[[1]],
  TransPart[disp]□(RotPart[disp].line[[1]]) +
  RotPart[disp].line[[2]]}
AxisToSkew[omega_?VectorQ] :=
  {{0, -omega[[3]], omega[[2]]}, {omega[[3]], 0, -omega[[1]]}, {-omega[[2]],
    omega[[1]], 0}}

UnitTwistMatrix[omega_?VectorQ, c_?VectorQ] :=
  Append[Transpose[Append[Transpose[AxisToSkew[omega]], c□omega]], {0,0,0,0}]

UnitTwistMatrix[omega_?VectorQ, c_?VectorQ, h_] :=
  Append[Transpose[Append[Transpose[AxisToSkew[omega]], c□omega +
    h*omega]], {0,0,0,0}]

RPToHomogeneous[R_, p_?VectorQ] := Module[{tmp},
  tmp = Transpose[R];
  AppendTo[tmp, p];
  tmp = Transpose[tmp];
  AppendTo[tmp, {0, 0, 0, 1}];
  Return[tmp];
];
RotationZ[
  theta_] := {{Cos[theta], -Sin[theta], 0}, {Sin[theta], Cos[theta], 0}, {0, 0,
    1}}
RotationX[
  theta_] := {{1, 0, 0}, {0, Cos[theta], -Sin[theta]}, {0, Sin[theta],
    Cos[theta]}}
RotationY[
```

```

theta_] := {{Cos[theta], 0, Sin[theta]}, {0, 1, 0}, {-Sin[theta], 0,
Cos[theta]}}
RotationTranslationX[theta_,
d_] := {{1, 0, 0, d}, {0, Cos[theta], -Sin[theta], 0}, {0, Sin[theta], Cos[theta],
0}, {0, 0, 0, 1}}
RotationTranslationZ[
theta_, d_] := {{Cos[theta], -Sin[theta], 0, 0}, {Sin[theta], Cos[theta], 0, 0}, {0, 0,
1, d}, {0, 0, 0, 1}}
RotationTranslationY[
theta_, d_] := {{Cos[theta], 0, Sin[theta], 0}, {0, 1, 0, d}, {-Sin[theta], 0,
Cos[theta], 0}, {0, 0, 0, 1}}

RotPart[D_?MatrixQ] := Table[D[[i, j]], {i, 3}, {j, 3}];
TransPart[D_?MatrixQ] := Table[D[[i, 4]], {i, 3}];

Ztrans[theta_, d_] := {{Cos[theta], -Sin[theta], 0, 0}, {Sin[theta], Cos[theta], 0, 0}, {0, 0, 1, d}, {0, 0, 0, 1}}
Ytrans[theta_, d_] := {{Cos[theta], 0, Sin[theta], 0}, {0, 1, 0, d}, {-Sin[theta], 0, Cos[theta], 0}, {0, 0, 0, 1}}
Xtrans[alpha_, a_] := {{1, 0, 0, a}, {0, Cos[alpha], -Sin[alpha], 0}, {0, Sin[alpha], Cos[alpha], 0}, {0, 0, 0, 1}}

(*CommonNormal assumes that both inputs are nonparallel.
CommonNormal2 is for parallel screws*)
(*ATT! modification!: this function and the next one take LINES or DUAL VECTORS
as input, not direction and point on the line!!!*)
CommonNormal[{vs1_?VectorQ, vcs1_?VectorQ}, {vs2_?VectorQ, vcs2_?VectorQ}] :=
Module[{s1, s2, cs1, cs2, m1, m2, X, DT, asinAlpha, sinAlpha, cosAlpha, a, Alpha,
Xline, n, n0, p1, p2, t1, t2,
q1, q2, p3},
s1 = vs1/Sqrt[vs1.vs1];
s2 = vs2/Sqrt[vs2.vs2];
cs1 = (vs1 vcs1)/vs1.vs1;
cs2 = (vs2 vcs2)/vs2.vs2;
m1 = cs1 s1;
m2 = cs2 s2;
X = DualCross[{s1, m1}, {s2, m2}];
DT = DualDot[{s1, m1}, {s2, m2}];
asinAlpha = -DT[[2]];
cosAlpha = DT[[1]];
If[Abs[asinAlpha]<0.000001,
sinAlpha = SinAngle[{s1, m1}, {s2, m2}],
sinAlpha = Sign[asinAlpha]*SinAngle[{s1, m1}, {s2, m2}]];
a = asinAlpha/sinAlpha;
Alpha = ArcTan[cosAlpha, sinAlpha];
If[Abs[asinAlpha]<0.000001,
Xline = MakeLine[X],
Xline = Sign[asinAlpha]*MakeLine[X]];
n = Xline[[1]]; (*unit vector of the common normal*)

```

```

    n0 = Xline[[2]];
    p1 = cs1;
    p2 = cs2;
    t1 = n.Cross[(p2 - p1), s2]/(n.Cross[s1, s2]);
    t2 = n.Cross[(p2 - p1), s1]/(n.Cross[s1, s2]);
    (*my new formulas:*)
    p3 = n*n0/(n.n);
    t1 = p3.s1;
    t2 = p3.s2;
    q1 = p1 + t1*s1;(*the intersection point of the common normal and line1*)
    q2 = p2 + t2*s2;(*the intersection point of the common normal and line2*)

    Return[{{Xline, a, Alpha}, {q1, q2}}];
];

(*CommonNormal2 works except for the case in which both lines are the same*)
CommonNormal2[l1_,l2_]:=Module[{p1, p2, t2, i2, n, n0},
    p1=l1[[1]]-l1[[2]]/(l1[[1]].l1[[1]]);
    p2=l2[[1]]-l2[[2]]/(l2[[1]].l2[[1]]);
    t2 = (p2-p1).l2[[1]];
    i2 = p2 - t2*l2[[1]];
    n = (i2-p1)/Sqrt[(i2-p1).(i2-p1)];
    n0 = p1-n;
    Return[{ {n,n0},{p1,i2}}];
];

CommonNormalSym[l1_,l2_]:=Module[{s1, s2, cs1,cs2, X,DT,
asinAlpha,cosAlpha,Xline,n,n0,p3,t1,t2,q1,q2,sinAlpha,a,Alpha,m1,m2},
    s1 = l1[[1]]/Sqrt[l1[[1]].l1[[1]]];
    s2 = l2[[1]]/Sqrt[l2[[1]].l2[[1]]];
    cs1 = (l1[[1]]-l1[[2]]/l1[[1]].l1[[1]]);
    cs2 = (l2[[1]]-l2[[2]]/l2[[1]].l2[[1]]);
    m1=cs1-s1;
    m2=cs2-s2;
    X = DualCross[ {s1, m1}, {s2, m2}];
    DT = DualDot[ {s1, m1}, {s2, m2}];
    asinAlpha = -DT[[2]];
    cosAlpha = DT[[1]];
    Xline = MakeLine[X];
    n = Xline[[1]]; (*unit vector of the common normal*)
    n0 = Xline[[2]];
    sinAlpha = SinAngle[{s1, m1},{s2, m2}];
    a = asinAlpha/sinAlpha;
    Alpha = ArcTan[cosAlpha, sinAlpha];
    p3 = n*n0/(n.n);
    t1 = p3.s1;
    t2 = p3.s2;

```

```

q1 = cs1 + t1*s1;(*the intersection point of the common normal and line1*)
q2 = cs2 + t2*s2;(*the intersection point of the common normal and line2*)
Return[{{ {n,n0},a,Alpha},{q1,q2}}]
];
(*CommonNormal with options for all cases, to be used in robot simulation*)

CommonNormalGen[{vs1_?VectorQ, vcs1_?VectorQ}, {vs2_?VectorQ,
vcs2_?VectorQ}] :=
Module[{s1, s2, cs1, cs2, X, DT, asinAlpha, sinAlpha,cosAlpha, a, Alpha, Xline, n,
n0, p1, p2, t1, t2, i1,i2,
q1, q2, p3},
s1 = vs1/Sqrt[vs1.vs1];
s2 = vs2/Sqrt[vs2.vs2];
cs1 = (vs1-vcs1/vs1.vs1)-s1;
cs2 = (vs2-vcs2/vs2.vs2)-s2;
X = DualCross[ {s1, cs1}, {s2, cs2}];
DT = DualDot[ {s1, cs1}, {s2, cs2}];
asinAlpha = -DT[[2]];
cosAlpha = DT[[1]];
If[Abs[Abs[cosAlpha]-1] < 0.000001, (*Parallel case*)
p1=vs1-vcs1/vs1.vs1;
p2=vs2-vcs2/vs2.vs2;
t2 = (p2-p1).vs2;
i2 = p2 - t2*vs2;
n = (i2-p1)/Sqrt[(i2-p1).(i2-p1)];
n0 = p1-n;
Return[{{ {n,n0},Sqrt[(i2-p1).(i2-p1)],ArcCos[cosAlpha]},{p1,i2}}],
(*Here starts the nonparallel case*)
If[Abs[asinAlpha]<0.000001,
sinAlpha = SinAngle[{s1, cs1},{s2, cs2}],
sinAlpha = Sign[asinAlpha]*SinAngle[{s1, cs1},{s2, cs2}]];
a = asinAlpha/sinAlpha;
Alpha = ArcTan[cosAlpha, sinAlpha];
If[Abs[asinAlpha]<0.000001,
Xline = MakeLine[X],
Xline = Sign[asinAlpha]*MakeLine[X]];
n = Xline[[1]]; (*unit vector of the common normal*)
n0 = Xline[[2]];
p1 = Cross[s1, cs1];
p2 = Cross[s2, cs2];
t1 = n.Cross[(p2 - p1), s2]/(n.Cross[s1, s2]);
t2 = n.Cross[(p2 - p1), s1]/(n.Cross[s1, s2]);
(*my new formulas:*)
p3 = n-n0/(n.n);
t1 = p3.s1;
t2 = p3.s2;

```

```

    q1 = p1 + t1*s1;(*the intersection point of the common normal and line1*)
    q2 = p2 + t2*s2;(*the intersection point of the common normal and line2*)
    Return[{Xline, a, Alpha}, {q1, q2}];
];
];

(*Points on common normal?
pointsCN takes quaternions and pointsCN2 takes lines*)
pointsCN[q1_,q2_]:=
Module[{q1e,q2e,p1,p2,p,k1,k2,k,sols},
  q1e=q1[[1]]+ε q1[[2]];
  q2e=q2[[1]]+ε q2[[2]];
  p=makeLine[quatNormal[q1e,q2e]];
  p1=makeLine[q1e[[1]]□makeLine[q1e[[2]]];
  p2=makeLine[q2e[[1]]□makeLine[q2e[[2]]];
  sols=Solve[p1+k1*makeLine[q1e[[1]]]-
(p2+k2*makeLine[q2e[[1]])==k*p[[1]],{k1,k2,k}];
  {p1+k1*makeLine[q1e[[1]],p2+k2*makeLine[q2e[[1]]]/.sols[[1]]]

pointsCN2[q1_,q2_]:=
Module[{q1e,q2e,n,p1,p2,p,k1,k2,k,sols},
  q1e=q1[[1]]+ε q1[[2]];
  q2e=q2[[1]]+ε q2[[2]];
  n=Collect[q1e□q2e,ε]/.ε^2->0;
  p=makeLine[n];
  p1=makeLine[q1e[[1]]□makeLine[q1e[[2]]];
  p2=makeLine[q2e[[1]]□makeLine[q2e[[2]]];
  sols=Solve[p1+k1*makeLine[q1e[[1]]]-
(p2+k2*makeLine[q2e[[1]])==k*p[[1]],{k1,k2,k}];
  {p1+k1*makeLine[q1e[[1]],p2+k2*makeLine[q2e[[1]]]/.sols[[1]]]
AngleOfTwoVector[v1_?VectorQ, v2_?VectorQ, v12_?VectorQ] :=
Module[{vc12},
  (*v12 is the given normal of v1 and v2
  Now the angle is about the axis v12 from v1 to v2
  *)
  vc12 = Cross[v1, v2];(*vc12 = sin (th)v12*)
  Return[ArcTan[v1.v2, vc12.v12]];
];
NormOfVector[v_?VectorQ] := Sqrt[v.v];
NormalizeVector[v_?VectorQ] := v/NormOfVector[v];
RYPXYZToHomogeneous[roll_, pitch_, yaw_, x_, y_, z_] :=
Module[{rotM},
  rotM = {{Cos[roll], 0, Sin[roll]}, {0, 1, 0}, {-Sin[roll], 0,
Cos[roll]}}.{1, 0, 0}, {0, Cos[pitch],
Sin[pitch]}, {0, -Sin[pitch],
Cos[pitch]}}.{Cos[yaw], -Sin[yaw], 0}, {Sin[yaw], Cos[yaw],

```

```

0}, {0, 0, 1}};
rotM = Transpose[rotM];
AppendTo[rotM, {x, y, z}];
rotM = Transpose[rotM];
AppendTo[rotM, {0, 0, 0, 1}];
Return[rotM];
];

```

(*Quaternion stuff*)

```

quatmult[r_,t_]:=
Collect[{r[[4]]*t[[1]]+t[[4]]*r[[1]]+r[[2]]*t[[3]]-t[[2]]*r[[3]],
r[[4]]*t[[2]]+t[[4]]*r[[2]]+r[[3]]*t[[1]]-t[[3]]*r[[1]],
r[[4]]*t[[3]]+t[[4]]*r[[3]]+r[[1]]*t[[2]]-t[[1]]*r[[2]],
-r[[1]]*t[[1]]-t[[2]]*r[[2]]-r[[3]]*t[[3]]+t[[4]]*r[[4]]},ε]/.ε^2->0

```

```

quatmultSep[r_,t_]:=
Module[{a,b},
a = r[[1]]+ε*r[[2]];
b = t[[1]]+ε*t[[2]];
sepQ2[quatmult[a,b]]]

```

```

quatconj[q_]:=
If[Length[q]==4,{-q[[1]],-q[[2]],-q[[3]],q[[4]]},
{{-q[[1,1]],-q[[1,2]],-q[[1,3]],q[[1,4]]},{-q[[2,1]],-q[[2,2]],-q[[2,3]],q[[2,4]]}}]

```

(*This other conjugation to move points:*)

```

quatconj4[q_]:=
If[Length[q]==4,
{-Coefficient[q,ε,0][[1]],-Coefficient[q,ε,0][[2]],-
Coefficient[q,ε,0][[3]],Coefficient[q,ε,0][[4]]} + ε
{Coefficient[q,ε,1][[1]],Coefficient[q,ε,1][[2]],Coefficient[q,ε,1][[3]],-
Coefficient[q,ε,1][[4]]},
{{-q[[1,1]],-q[[1,2]],-q[[1,3]],q[[1,4]]},{q[[2,1]],q[[2,2]],q[[2,3]],-q[[2,4]]}}]
dualInv[a_]:=1/Coefficient[a,ε,0]-Coefficient[a,ε,1]/(Coefficient[a,ε,0]^2)*ε

```

```

quatInv[q_]:=quatconj[q]*dualInv[quatmult[q,quatconj[q]][[4]]]

```

```

sepQ[q_]:=Flatten[{Coefficient[q,ε,0],Coefficient[q,ε,1]}]

```

```

sepQ2[q_]:={Coefficient[q,ε,0],Coefficient[q,ε,1]}

```

(*makeLine is for the epsilon notation
and makeLine2 for the dual vector notation - for dual vectors and quaternions - *)
makeLine[q_]:=


```

Module[{s,w},

s=Coefficient[q,ε,0][[{1,2,3}]]/Sqrt[Coefficient[q,ε,0][[{1,2,3}]].Coefficient[q,ε,
0][[{1,2,3}]]];

w=s□Coefficient[q,ε,1][[{1,2,3}]]/Sqrt[Coefficient[q,ε,0][[{1,2,3}]].Coefficient[q
,ε,0][[{1,2,3}]]];
{s,w□s}
makeLine2[q_]:=
Module[{s,w},
s=q[[1,{1,2,3}]]/Sqrt[q[[1,{1,2,3}]].q[[1,{1,2,3}]]];
w=s□q[[2,{1,2,3}]]/Sqrt[(q[[1,{1,2,3}]].q[[1,{1,2,3}]]);
{s,w□s}

```

(*Pure dual quaternion perpendicular to any two dual quaternions*)
quatNormal[r_,t_]:=1/2 (quatmult[r,t]-quatmult[t,r])

Random functions

```

makedir[]:=
Module[{v,v1,v2,v3},v1=Random[Real,{-1,1}]; v2=Random[Real,{-1,1}];
v3=Random[Real,{-1,1}];v={v1,v2,v3};v/Sqrt[v.v]]

```

```

makepoint[]:=
Module[{v1,v2,v3},v1=Random[Real,{-2,2}];
v2=Random[Real,{-2,2}]; v3=Random[Real,{-2,2}];{v1,v2,v3}]

```

(*The following function allows to choose the range of the point,*)

```

makepoint[int_]:=
Module[{v1,v2,v3},v1=Random[Real,{-int,int}];
v2=Random[Real,{-int,int}]; v3=Random[Real,{-int,int}];{v1,v2,v3}]

```

```

makedvec[]:=
Module[{dir,point},dir=makedir[]; point=makepoint[];{dir,point□dir}]

```

```

makedvec[int_]:=
Module[{dir,point},dir=makedir[]; point=makepoint[int];{dir,point□dir}]

```

```

makedvecgen[]:=Module[{dir,point,de,ph},dir=makedir[];
point=makepoint[];de=makede[];ph=makeph[];{dir,(point□dir)+(de/ph)*dir}]

```

```

makeph[]:=Random[Real,{-6.29,6.29}]

```

```

makede[]:=Random[Real,{-10,10}]
makede[num_]:=Random[Real,{-num,num}]

```

```

(*Attention! New makequat functions!!*)
(*NOT CORRECT
makequatN[dvec_,ph_,de_]:= Module[{quat},
    If[ph == 0,
        quat = {{0,0,0,1},Append[dvec[[2]],0]*1/2 de},
        quat = {Flatten[{Sin[ph/2]*dvec[[1]],Cos[ph/2]}],
        Flatten[{Sin[ph/2]*dvec[[2]]+de/2*Cos[ph/2]*dvec[[1]],-
de/2*Sin[ph/2]}}];
    ];
    quat
]
*)
makequat[dvec_,ph_,de_]:= Module[{v,q},
    If[Length[dvec]==2,
        v = dvec,
        v = {Coefficient[dvec, $\epsilon$ ,0],Coefficient[dvec, $\epsilon$ ,1]}
    ];
    {Flatten[{Sin[ph/2]*v[[1]],Cos[ph/2]}],
    Flatten[{Sin[ph/2]*v[[2]]+de/2*Cos[ph/2]*v[[1]],-de/2*Sin[ph/2]}}}
]
(* WRONG AND NO LONGER NEEDED
makequate[dvec_,ph_,de_]:= Module[{quat},
    If[ph == 0,
        quat = {0,0,0,1}+ $\epsilon$  Append[dvec[[2]],0]*de,
        quat = Flatten[{Sin[ph/2]*dvec[[1]],Cos[ph/2]}]+ $\epsilon$ *
        Flatten[{Sin[ph/2]*dvec[[2]]+de/2*Cos[ph/2]*dvec[[1]],-de/2*Sin[ph/2]}}
    ];
    quat
]
*)

```

Dual quaternion to absolute homogeneous matrix and viceversa assuming
reference configuration is the identity

```
Id={{1,0,0},{0,1,0},{0,0,1}}
```

```
makeB[b_,psi_]:=Tan[psi/2]*{{0,-b[[3]],b[[2]]},{b[[3]],0,-b[[1]]},{-b[[2]],b[[1]],0}}
```

```
makeA[Id_,B_]:= (Inverse[Id-B]).(Id+B)
```

(*OLD FUNCTIONS

```

dq2HM[q_]:=Module[{q2,b12,point12,s12,psi12,t12,B12,A12,A2,
d1,dsol,eq12,sols,d12,d2,Pos2,dx,dy,dz, Pos1,A1,Id},
Pos1={{1,0,0,0},{0,1,0,0},{0,0,1,0},{0,0,0,1}};
A1=Pos1[[{1,2,3},{1,2,3}]];

```

```

Id={{1,0,0},{0,1,0},{0,0,1}};
q2={Coefficient[q,ε,0],Coefficient[q,ε,1]};
b12={q2[[1,1]],q2[[1,2]],q2[[1,3]]};
point12=(b12□{q2[[2,1]],q2[[2,2]],q2[[2,3]]})/(b12.b12);
s12={b12/Sqrt[b12.b12],point12□b12/Sqrt[b12.b12]};
psi12=2 ArcTan[q2[[1,4]],Sqrt[b12.b12]];
t12=-2 q2[[2,4]]/Sqrt[b12.b12];
B12=N[makeB[s12[[1]],psi12]];
A12=makeA[Id,B12];
A2=A12.A1;
d1=Pos1[{{1,2,3},4}];
dsol={dx,dy,dz};
eq12=(Id-A12).s12[[2]]-dsol□s12[[1]];
sols=Solve[{eq12[[1]]==0,eq12[[2]]==0,dsol.s12[[1]]==0},{dx,dy,dz}];
d12=(dsol/.{sols[[1,1]],sols[[1,2]],sols[[1,3]]})+t12*s12[[1]];
d2=d12+A12.d1;
Pos2=Append[Transpose[Append[Transpose[A2],d2]],{0,0,0,1}];
Pos2]

```

(*New one, including translation-only elements!!*)

```

dq2HMN[q_]:=Module[{q2,b12,point12,s12,psi12,t12,B12,A12,A2,
d1,dsol,eq12,sols,d12,d2,Pos2,dx,dy,dz, Pos1,A1,Id},
Pos1={{1,0,0,0},{0,1,0,0},{0,0,1,0},{0,0,0,1}};
A1=Pos1[{{1,2,3},{1,2,3}}];
Id={{1,0,0},{0,1,0},{0,0,1}};
If[Length[q]==4,
q2={Coefficient[q,ε,0],Coefficient[q,ε,1]},
q2=q;
];
b12={q2[[1,1]],q2[[1,2]],q2[[1,3]]};
If[b12.b12<0.0000001,
A12 = IdentityMatrix[3];
t12=2*Sqrt[q2[[2,{1,2,3}]].q2[[2,{1,2,3}]]];
s12={0,0,0},q2[[2,{1,2,3}]]/(t12/2));
A2=A12.A1;
d12=t12*s12[[2]],
point12=(b12□{q2[[2,1]],q2[[2,2]],q2[[2,3]]})/(b12.b12);
s12={b12/Sqrt[b12.b12],point12□b12/Sqrt[b12.b12]};
psi12=2 ArcTan[q2[[1,4]],Sqrt[b12.b12]];
t12=-2 q2[[2,4]]/Sqrt[b12.b12];
If[(Abs[psi12]-3.14159265)<0.000001,
A2={2 s12[[1,1]]^2-1,2 s12[[1,1]]s12[[1,2]],2 s12[[1,1]]s12[[1,3]],{2
s12[[1,1]]s12[[1,2]],2 s12[[1,2]]^2-1,2 s12[[1,2]]s12[[1,3]]},
{2 s12[[1,1]]s12[[1,3]],2 s12[[1,2]]s12[[1,3]],2 s12[[1,3]]^2-1}};
A12=A2,

```

```

B12=N[makeB[s12[[1]],psi12]];
A12=makeA[Id,B12];
A2=A12.A1;
];
(*FIX CALCULATION OF d*)
dsol={dx,dy,dz};
eq12=(Id-A12).s12[[2]]-
dsol.s12[[1]];sols=Solve[{eq12[[1]]==0,eq12[[2]]==0,dsol.s12[[1]]==0},{dx,dy,dz}];
d12=(dsol/.{sols[[1,1]],sols[[1,2]],sols[[1,3]]})+t12*s12[[1]];
];
d1=Pos1[{1,2,3},4];
d2=d12+A12.d1;
Pos2=Append[Transpose[Append[Transpose[A2],d2]],{0,0,0,1}];
Pos2]
*)

```

(*THIS IS A FULLY NEW DQ2HM FUNCITON, HOPEFULLY ABLE TO TAKE CARE OF EEEVVEERYTHING IT IS VALID FOR A SINGLE POSITION, NOT A RELATIVE ONE, AND FOR EPSILON AND NON-EPSILON FORMULATION*)

```

dq2HM[q_]:=Module[{q2,b,pointc,s,psi,t,B12,Rot,
d,Pos,A1,Id},
(*UNIFY INPUT DATA*)
If[Length[q]==4,
q2={Coefficient[q,ε,0],Coefficient[q,ε,1]},
q2=q;
];
b={q2[[1,1]],q2[[1,2]],q2[[1,3]]};
(*CALCULATE ROTATION AXIS AND ANGLE. ONLY SPECIAL CASE IS
WHEN ANGLE=0*)
psi=2 ArcTan[q2[[1,4]],Sqrt[b.b]];
If[Abs[psi]<0.0000001,
(*TRANSLATION dq*)
Rot= IdentityMatrix[3];
t = 2*Sqrt[q2[[2,{1,2,3}]].q2[[2,{1,2,3}]]];
d = 2*q2[[2,{1,2,3}]],
(*NOT TRANSLATION dq*)
s = b/Sin[psi/2];
Rot = {{s[[1]]^2(1-Cos[psi])+Cos[psi],s[[1]]s[[2]](1-Cos[psi])-s[[3]]Sin[psi],
s[[1]]s[[3]](1-Cos[psi])+s[[2]]Sin[psi]},{s[[1]]s[[2]](1-Cos[psi])+s[[3]]Sin[psi],
s[[2]]^2(1-Cos[psi])+Cos[psi],s[[2]]s[[3]](1-Cos[psi])-s[[1]]Sin[psi]},{
s[[1]]s[[3]](1-Cos[psi])-s[[2]]Sin[psi],s[[2]]s[[3]](1-Cos[psi])+s[[1]]Sin[psi],
s[[3]]^2(1-Cos[psi])+Cos[psi]}};
(*Translation part: point on the line:*)
pointc = s.q2[[2,{1,2,3}]]/Sin[psi/2];

```

```

t = -2*q2[[2,4]]/Sin[psi/2];
d = (IdentityMatrix[3]-Rot).pointc+t*s;
];
Pos=Append[Transpose[Append[Transpose[Rot],d]],{0,0,0,1}]
]

(*THIS IS THE "THEORETICAL VERSION" TO LOOK NICE*)
dq2HMGGen[q_]:=Module[{q2,b12,point12,s12,psi12,t12,B12,A12,A2,
  dsol,eq12,sols,d12,d2,Pos2,dx,dy,dz,Id},
  Id={{1,0,0},{0,1,0},{0,0,1}};
  q2={Coefficient[q,ε,0],Coefficient[q,ε,1]};
  b12={q2[[1,1]],q2[[1,2]],q2[[1,3]]};
  point12=(b12□{q2[[2,1]],q2[[2,2]],q2[[2,3]]})/(b12.b12);
  s12={b12/Sqrt[b12.b12],point12□b12/Sqrt[b12.b12]};
  psi12=2 ArcTan[q2[[1,4]],Sqrt[b12.b12]];
  t12=-2 q2[[2,4]]/Sqrt[b12.b12];
  B12=N[makeB[s12[[1]],psi12]];
  A12=makeA[Id,B12];
  dsol={dx,dy,dz};
  eq12=(Id-A12).s12[[2]]-dsol□s12[[1]];
  sols=Solve[{eq12[[1]]==0,eq12[[2]]==0,dsol.s12[[1]]==0},{dx,dy,dz}];
  d12=(dsol/.{sols[[1,1]],sols[[1,2]],sols[[1,3]]})+t12*s12[[1]];
  Pos2=Append[Transpose[Append[Transpose[A12],d12]],{0,0,0,1}];
  Pos2]

This one for first configuration not being the identity
(*Case when we are computing an absolute from a relative, knowing the reference
configuration Pos1*)
dq2HMPos1[Pos1_,q_]:=Module[{q2,b,pointc,s,psi,t,B12,Rot,
  d,Pos,A1,d1,Id,Pos2,Rot2,d2},
  A1=Pos1[{{1,2,3},{1,2,3}}];
  d1=Pos1[{{1,2,3},4}];
  (*UNIFY INPUT DATA*)
  If[Length[q]==4,
    q2={Coefficient[q,ε,0],Coefficient[q,ε,1]},
    q2=q;
  ];
  b={q2[[1,1]],q2[[1,2]],q2[[1,3]]};
  (*CALCULATE ROTATION AXIS AND ANGLE. ONLY SPECIAL CASE IS
WHEN ANGLE=0*)
  psi=2 ArcTan[q2[[1,4]],Sqrt[b.b]];
  If[Abs[psi]<0.0000001,
    (*TRANSLATION dq*)
    Rot= IdentityMatrix[3];
    t = 2*Sqrt[q2[[2,{1,2,3}]].q2[[2,{1,2,3}]]];
    d = 2*q2[[2,{1,2,3}]],
    (*NOT TRANSLATION dq*)

```

```

s = b/Sin[psi/2];
Rot = {{s[[1]]^2(1-Cos[psi])+Cos[psi],s[[1]]s[[2]](1-Cos[psi])-s[[3]]Sin[psi],
s[[1]]s[[3]](1-Cos[psi])+s[[2]]Sin[psi]},{s[[1]]s[[2]](1-Cos[psi])+s[[3]]Sin[psi],
s[[2]]^2(1-Cos[psi])+Cos[psi],s[[2]]s[[3]](1-Cos[psi])-s[[1]]Sin[psi]},{
s[[1]]s[[3]](1-Cos[psi])-s[[2]]Sin[psi],s[[2]]s[[3]](1-Cos[psi])+s[[1]]Sin[psi],
s[[3]]^2(1-Cos[psi])+Cos[psi]}};
(*Translation part: point on the line:*)
pointc = s^2 q2[[2,{1,2,3}]]/Sin[psi/2];
t = -2*q2[[2,4]]/Sin[psi/2];
d = (IdentityMatrix[3]-Rot).pointc+t*s;
];
Rot2=Rot.A1;
d2=d+Rot.d1;
Pos2=Append[Transpose[Append[Transpose[Rot2],d2]],{0,0,0,1}];
Pos2]
HM2dq[M_]:=Module[{checkA,checkB,checkS,checkb,checks,checkd,checkv,Id,dq,axis
,phi,d},
Id={{1,0,0},{0,1,0},{0,0,1}};
checkA=M[{{1,2,3},{1,2,3}}];
If[Abs[1/2(checkA[[1,1]]+checkA[[2,2]]+checkA[[3,3]]-1)+1]<0.00001,
checkS=1/2(checkA-Id);
(*checks={Sign[checkS[[1,3]]]*Sqrt[1/2(checkS[[1,1]]-checkS[[2,2]]-checkS[[3,3]]),
Sign[checkS[[2,3]]]*Sqrt[1/2(-checkS[[1,1]]+checkS[[2,2]]-checkS[[3,3]]),
Sqrt[1/2(-checkS[[1,1]]-checkS[[2,2]]+checkS[[3,3]])]};*)
checks={Sqrt[1/2(checkS[[1,1]]-checkS[[2,2]]-checkS[[3,3]])],
Sqrt[1/2(-checkS[[1,1]]+checkS[[2,2]]-checkS[[3,3]])],
Sqrt[1/2(-checkS[[1,1]]-checkS[[2,2]]+checkS[[3,3]])]};
checkb=checks;
phi=Pi,
checkB=(checkA-Id).Inverse[checkA+Id];
checkb={checkB[[3,2]],checkB[[1,3]],checkB[[2,1]]};

If[Sqrt[checkb.checkb]<0.000001,checks={0,0,0},checks=checkb/Sqrt[checkb.checkb]];
phi=2*ArcTan[Sqrt[checkb.checkb]];
checkd=Flatten[M[{{1,2,3},{4}}]];
If[Sqrt[checkb.checkb]<0.000001,
checks=checkd/Sqrt[checkd.checkd];
checkv={0,0,0};
d=Sqrt[checkd.checkd],
checkv=N[-1/(2*Tan[phi/2]^3)*checkB.checkB.(Id-checkB).checkd];
d=checkd.checks;
];
axis={checks,checkv};
dq=makequat[axis,phi,d]
]

```

```

HM2dqN[M_]:=Module[{checkA,checkB,checkb,checks,checkd,checkv,Id,dq,axis,phi,d},
  Id={{1,0,0},{0,1,0},{0,0,1}};
  checkA=M[{{1,2,3},{1,2,3}}];
  checkB=(checkA-Id).Inverse[checkA+Id];
  checkb={checkB[[3,2]],checkB[[1,3]],checkB[[2,1]]};

  If[Sqrt[checkb.checkb]<0.000001,checks={0,0,0},checks=checkb/Sqrt[checkb.checkb]];
  phi=2*ArcTan[Sqrt[checkb.checkb]];
  checkd=Flatten[M[{{1,2,3},{4}}]];
  If[Sqrt[checkb.checkb]<0.000001,
    checkv=checkd/Sqrt[checkd.checkd];
    d=Sqrt[checkd.checkd];
    checkv=N[-1/(2*Tan[phi/2]^3)*checkB.checkB.(Id-checkB).checkd];
    d=checkd.checks;
  ];
  axis={checks,checkv};
  dq=makequat[axis,phi,d]
]
(*Function to compute screw, unit and with finite pitch,
of displacement from homogeneous matrix*)
HM2sc[M_]:=Module[{checkA,checkB,checkb,checks,checkd,checkv,Id,dq,axis,phi,d,pi},
  Id={{1,0,0},{0,1,0},{0,0,1}};
  checkA=M[{{1,2,3},{1,2,3}}];
  checkB=(checkA-Id).Inverse[checkA+Id];
  checkb={checkB[[3,2]],checkB[[1,3]],checkB[[2,1]]};
  checks=checkb/Sqrt[checkb.checkb];
  phi=2*ArcTan[Sqrt[checkb.checkb]];
  checkd=Flatten[M[{{1,2,3},{4}}]];
  checkv=N[-1/(2*Tan[phi/2]^3)*checkB.checkB.(Id-checkB).checkd];
  axis={checks,checkv};
  d=checkd.checks;
  pitch=(d/2)/(Tan[phi/2]);
  screw={checks,checkv+pitch*checks}
]
(*FUNCTIONS FOR SCREWS*)

GetScrew[q_]:=Module[{q1},
  GetScrew::usage="GetScrew takes a dual quaternion and returns the screw";
  q1=q[{{1,2},{1,2,3}}]
]

MakeScrew[dvec_,p_]:=Module[{sc,s},
  MakeScrew::usage="MakeScrew takes a line and a pitch and returns the screw";
  s=makedir[];

```

```
sc={s,makepoint[]□s+p*s};
]
```

(*Functions to plot screws (S is a list of screws), from the common normal line between consecutive ones,
First one for arbitrary screws, second one for parallel screws.*)

```
plotLine[S_,len_]:=Module[{Sn,c1,lS},
  Sn=S/Sqrt[S[[1]].S[[1]]];
  c1=(S[[1]]□S[[2]])/(S[[1]].S[[1]]);
  lS=Line[{c1-len*Sn[[1]],c1+len*Sn[[1]]},VertexColors->{Blue,Green}];
  graT=Graphics3D[{Blue,Thick,Opacity[.8],lS},AspectRatio->1,Axes->True]
]
```

```
plotLines[S_,len_]:=Module[{Sn,l,c1,c2,ctot,n,pit,lS},
  l=Length[S];
  Sn=Table[S[[i]]/Sqrt[S[[i,1]].S[[i,1]]],{i,1,l}];
  n=Table[CommonNormal[S[[i]],S[[i+1]]][[1,1]],{i,1,l-1}];
  c1=Table[CommonNormal[S[[i]],S[[i+1]]][[2,1]],{i,1,l-1}];
  c2=Table[CommonNormal[S[[i]],S[[i+1]]][[2,2]],{i,1,l-1}];
  ctot=Append[c1,c2[[1-1]]];
  pit=Table[S[[i,1]].S[[i,2]]/(S[[i,1]].S[[i,1]]),{i,1,l}];
  lS=Table[Line[{ctot[[i]]-len*Sn[[i,1]],ctot[[i]]+len*Sn[[i,1]]},VertexColors->{Blue,Green}],{i,1,l}];
  graT=Graphics3D[{Blue,Thick,Opacity[.8],lS},AspectRatio->1,Axes->True]
]
```

```
plotLines2[S_,len_]:=Module[{Sn,l,c1,c2,ctot,n,pit,lS},
  l=Length[S];
  Sn=Table[S[[i]]/Sqrt[S[[i,1]].S[[i,1]]],{i,1,l}];
  n=Table[CommonNormal2[S[[i]],S[[i+1]]][[1,1]],{i,1,l-1}];
  c1=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,1]],{i,1,l-1}];
  c2=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,2]],{i,1,l-1}];
  ctot=Append[c1,c2[[1-1]]];
  pit=Table[S[[i,1]].S[[i,2]]/(S[[i,1]].S[[i,1]]),{i,1,l}];
  lS=Table[Line[{ctot[[i]],ctot[[i]]+len*Sn[[i,1]]},VertexColors->{Blue,Green}],{i,1,l}];
  graT=Graphics3D[{Blue,Thick,Opacity[.8],lS},AspectRatio->1,Axes->True,PlotRange->{{-25,25},{-25,25},{-25,25}}]
]
```

```
plotScrew[S_]:=Module[{Sn,l,c1,c2,ctot,n,pit,lS},
  l=Length[S];
  Sn=Table[S[[i]]/Sqrt[S[[i,1]].S[[i,1]]],{i,1,l}];
  n=Table[CommonNormal[S[[i]],S[[i+1]]][[1,1]],{i,1,l-1}];
  c1=Table[CommonNormal[S[[i]],S[[i+1]]][[2,1]],{i,1,l-1}];
  c2=Table[CommonNormal[S[[i]],S[[i+1]]][[2,2]],{i,1,l-1}];
```



```

ctot=Append[c1,c2[[1-1]]];
pit=Table[S[[i,1]].S[[i,2]]/(S[[i,1]].S[[i,1]]),{i,1,l}];
lS=Table[Line[{ctot[[i]]-pit[[i]]*Sn[[i,1]],ctot[[i]]+pit[[i]]*Sn[[i,1]]},VertexColors-
>{Blue,Green}],{i,1,l}];
graT=Graphics3D[{Blue,Thick,Opacity[.8],lS},AspectRatio->1,Axes->True]
]

plotScrew2[S_]:=Module[{Sn,l,c1,c2,ctot,n,pit,lS},
l=Length[S];
Sn=Table[S[[i]]/Sqrt[S[[i,1]].S[[i,1]]],{i,1,l}];
n=Table[CommonNormal2[S[[i]],S[[i+1]]][[1,1]],{i,1,l-1}];
c1=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,1]],{i,1,l-1}];
c2=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,2]],{i,1,l-1}];
ctot=Append[c1,c2[[1-1]]];
pit=Table[S[[i,1]].S[[i,2]]/(S[[i,1]].S[[i,1]]),{i,1,l}];
lS=Table[Line[{ctot[[i]],ctot[[i]]+pit[[i]]*Sn[[i,1]]},VertexColors-
>{Blue,Green}],{i,1,l}];
graT=Graphics3D[{Blue,Thick,Opacity[.8],lS},AspectRatio->1,Axes->True,PlotRange-
>{{-25,25},{-25,25},{-25,25}}]
]

plotScrew2col[S_,colors_]:=Module[{Sn,l,c1,c2,ctot,n,pit,lS},
l=Length[S];
Sn=Table[S[[i]]/Sqrt[S[[i,1]].S[[i,1]]],{i,1,l}];
n=Table[CommonNormal2[S[[i]],S[[i+1]]][[1,1]],{i,1,l-1}];
c1=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,1]],{i,1,l-1}];
c2=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,2]],{i,1,l-1}];
ctot=Append[c1,c2[[1-1]]];
pit=Table[S[[i,1]].S[[i,2]]/(S[[i,1]].S[[i,1]]),{i,1,l}];
lS=Table[Line[{ctot[[i]],ctot[[i]]+pit[[i]]*Sn[[i,1]]},VertexColors->colors],{i,1,l}];
graT=Graphics3D[{Blue,Thick,Opacity[.8],lS},AspectRatio->1,Axes->True,PlotRange-
>{{-25,25},{-25,25},{-25,25}}]
]

plotScrew2Arrow[S_,arrowHead_,tubeWidth_]:=Module[{Sn,l,c1,c2,ctot,n,pit,lS},
l=Length[S];
Sn=Table[S[[i]]/Sqrt[S[[i,1]].S[[i,1]]],{i,1,l}];
n=Table[CommonNormal2[S[[i]],S[[i+1]]][[1,1]],{i,1,l-1}];
c1=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,1]],{i,1,l-1}];
c2=Table[CommonNormal2[S[[i]],S[[i+1]]][[2,2]],{i,1,l-1}];
ctot=Append[c1,c2[[1-1]]];
pit=Table[S[[i,1]].S[[i,2]]/(S[[i,1]].S[[i,1]]),{i,1,l}];

lS=Table[{Hue[1/6],Arrowheads[arrowHead*Sign[pit[[i]]]*pit[[i]]/10],Arrow[Tube[{cto
t[[i]],ctot[[i]]+pit[[i]]*Sn[[i,1]]},tubeWidth]]}],{i,1,l}];
Show[Graphics3D[{Thick,Opacity[.8],lS},AspectRatio->1,Axes->True,Boxed->True,

```

```

ViewPoint->{1,1,1},ImageSize->600,PlotRange->{{-25,25},{-25,25},{-
25,25}},DisplayFunction->(PopupWindow[Button["Click here"],#,
Window Size->{650,650}]&)]
]

```

```

generatePlot[X_,col_,x_] :=
Module[{matr, axisX, axisY, axisZ, axisP, Xplot, Yplot, Zplot, plotme},
  matr = Table[dq2HM[makeEpsilon[X[[i]]], {i, 1, Length[X]}];
  axisX = Table[matr[[i, {1, 2, 3}, 1]], {i, 1, Length[X]}];
  axisY = Table[matr[[i, {1, 2, 3}, 2]], {i, 1, Length[X]}];
  axisZ = Table[matr[[i, {1, 2, 3}, 3]], {i, 1, Length[X]}];
  axisP = Table[matr[[i, {1, 2, 3}, 4]], {i, 1, Length[X]}];
  Xplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisX[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  Yplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisY[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  Zplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisZ[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  plotme = Table[
    Graphics3D[{Xplot[[j]], Yplot[[j]], Zplot[[j]]},
    AspectRatio -> Automatic], {j, 1, Length[X]}]

```

```

generatePlotM[X_,col_,x_] :=
Module[{matr, axisX, axisY, axisZ, axisP, Xplot, Yplot, Zplot, plot},
  matr = X;
  axisX = Table[matr[[i, {1, 2, 3}, 1]], {i, 1, Length[X]}];
  axisY = Table[matr[[i, {1, 2, 3}, 2]], {i, 1, Length[X]}];
  axisZ = Table[matr[[i, {1, 2, 3}, 3]], {i, 1, Length[X]}];
  axisP = Table[matr[[i, {1, 2, 3}, 4]], {i, 1, Length[X]}];
  Xplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisX[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  Yplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisY[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  Zplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisZ[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  plot = Table[
    Graphics3D[{Xplot[[j]], Yplot[[j]], Zplot[[j]]},
    AspectRatio -> Automatic], {j, 1, Length[X]}]

```

```

generatePlotM2D[X_,col_,x_] :=

```

```

Module[{matr, axisX, axisY, axisZ, axisP, Xplot, Yplot, Zplot, plot},
  matr = X;
  axisX = Table[matr[[i, {1, 2}, 1]], {i, 1, Length[X]}];
  axisY = Table[matr[[i, {1, 2}, 2]], {i, 1, Length[X]}];
  axisP = Table[matr[[i, {1, 2}, 3]], {i, 1, Length[X]}];
  Xplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisX[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  Yplot = Table[
    Line[{axisP[[i]], axisP[[i]] + x*axisY[[i]]},
    VertexColors -> {col}], {i, 1, Length[X]}];
  plot = Table[
    Graphics[{Thick, Xplot[[j]], Yplot[[j]]},
    AspectRatio -> Automatic], {j, 1, Length[X]}];
  Show[plot, Axes -> True]
]

```

```

makeEpsilon[q_]:=Module[{g},g=Flatten[q];Table[g[[i]],{i,1,4}]+ε*Table[g[[i]],{i,5,8}]
]

```

```

generatePlotwithArrow[X_, len_, col_, ArrowHead_, TubeWidth_] :=
Module[{matr, axisX, axisY, axisZ, axisP, Xplot, Yplot, Zplot, plot},
  matr = Table[dq2HM[makeEpsilon[X[[i]]]], {i, 1, Length[X]}];
  axisX = Table[matr[[i, {1, 2, 3}, 1]], {i, 1, Length[X]}];
  axisY = Table[matr[[i, {1, 2, 3}, 2]], {i, 1, Length[X]}];
  axisZ = Table[matr[[i, {1, 2, 3}, 3]], {i, 1, Length[X]}];
  axisP = Table[matr[[i, {1, 2, 3}, 4]], {i, 1, Length[X]}];
  Xplot =
    Table[{Arrowheads[ArrowHead],
    Arrow[Tube[{axisP[[i]], axisP[[i]] + len*axisX[[i]]},
    TubeWidth]]}, {i, 1, Length[X]}];
  Yplot =
    Table[{Arrowheads[ArrowHead],
    Arrow[Tube[{axisP[[i]], axisP[[i]] + len*axisY[[i]]},
    TubeWidth]]}, {i, 1, Length[X]}];
  Zplot =
    Table[{Arrowheads[ArrowHead],
    Arrow[Tube[{axisP[[i]], axisP[[i]] + len*axisZ[[i]]},
    TubeWidth]]}, {i, 1, Length[X]}];
  plot = Table[
    Graphics3D[{col, Thick,
    Opacity[.8], {Xplot[[j]], Yplot[[j]], Zplot[[j]]}},
    AspectRatio -> Automatic], {j, 1, Length[X]}]

```

This new section is for the automated analysis of solutions. Given a solution robot, expressed as joint axes in a reference configuration, and given the set of task

positions, including the reference configuration, find the DH parameters and check whether the chain really hits the given positions (absolute).

```
GetH[i_,data_]:=data[[i,{1,2,3}]]
GetG[i_,data_]:= {data[[i,{4,5,6}]],data[[i,{7,8,9}]]}
GetW[i_,data_]:= {data[[i,{10,11,12}]],data[[i,{13,14,15}]]}
GetF[i_,data_]:= {data[[i,{16,17,18}]],data[[i,{19,20,21}]]}
```

```
MakeH[H_,G_]:=Module[{hdir,nhg,pointG,Hline},
  hdir=H/Sqrt[H.H];
  nhg=G[[1]]□hdir;
  pointG=G[[2]]□G[[1]];
  Hline={hdir,(pointG+2 nhg)□hdir}]
```

```
MakeUnit[S_]:= {S[[1]]/Sqrt[S[[1]].S[[1]]],S[[2]]/Sqrt[S[[1]].S[[1]]]}
```

```
MakeNormal[G_,W_]:=Module[{Ngw,pointNgw,Ngwline,p1,p2,t2,i2},
  If[G[[1]]==W[[1]],
    p1=G[[1]]□G[[2]]/(G[[1]].G[[1]]);
    p2=W[[1]]□W[[2]]/(W[[1]].W[[1]]);
    t2 = (p2-p1).W[[1]];
    i2 = p2 - t2*W[[1]];
    Ngw = (i2-p1)/Sqrt[(i2-p1).(i2-p1)];
    pointNgw = p1□Ngw;
    Ngwline={Ngw,pointNgw},
```

```
    Ngw=({G[[1]]□W[[1]],G[[1]]□W[[2]]+G[[2]]□W[[1]]})/Sqrt[(G[[1]]□W[[1]]).(G
[[1]]□W[[1]]);
    pointNgw=Ngw[[1]]□Ngw[[2]];
    Ngwline={Ngw[[1]],pointNgw□Ngw[[1]]};
  Return[Ngwline];
]
```

```
MakeGtransP[H_,G_]:=Module[{pointH,Hl,xdir,ydir,Rgtrans,Ghom,Gtot},
  Hl=MakeH[H,G];
  pointH=Hl[[1]]□Hl[[2]];
  xdir=pointH/Sqrt[pointH.pointH];
  ydir=Hl[[1]]□xdir;
  Rgtrans={xdir,ydir,Hl[[1]],pointH};
  Ghom=Transpose[Rgtrans];
  Gtot=Append[Ghom,{0,0,0,1}]]
```

```
MakeDH1[Gtot_,H_,
G_]:=Module[{Nhglie,Hl,pointH,xdir,cosangle,minusasinangle,angle1,lenght1,dhpars},
  Hl=MakeH[H,G];
  pointH=Hl[[1]]□Hl[[2]];
  xdir=pointH/Sqrt[pointH.pointH];
```

```

Nhgline=MakeNormal[Hl,G];
cosangle=xdir.Nhgline[[1]];
minusasinangle=xdir.Nhgline[[2]]+{0,0,0}.Nhgline[[1]];
angle1=ArcCos[cosangle];
length1=-minusasinangle/Sqrt[1-cosangle^2];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[angle1,length1]).{1,0,0,0})),{4})).Nhgline[[1]]==1,Null,angle1=-angle1;length1=-length1];
dhpars={angle1,length1}

MakeDH2[Gtot_,H_,G_,DH1_]:=Module[{Hl,cosangle,minusasinangle,angle2,length2,dhpars},
Hl=MakeH[H,G];
cosangle=Hl[[1]].G[[1]];
minusasinangle=Hl[[1]].G[[2]]+Hl[[2]].G[[1]];
angle2=ArcCos[cosangle];
length2=-minusasinangle/Sqrt[1-cosangle^2];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[DH1[[1]],DH1[[2]]).Xtrans[angle2,length2]).{0,0,1,0})),{4})).G[[1]]==1,Null,angle2=-angle2;length2=-length2];
dhpars={angle2,length2}

MakeDH3[Gtot_,H_,G_,W_,DH1_,DH2_]:=Module[{Nhg,Ngw,Hl,cosangle,minusasinangle,angle,length,dhpars},
Hl=MakeH[H,G];
Nhg=MakeNormal[Hl,G];
Ngw=MakeNormal[G,W];
cosangle=Nhg[[1]].Ngw[[1]];
minusasinangle=Nhg[[1]].Ngw[[2]]+Nhg[[2]].Ngw[[1]];
angle=ArcCos[cosangle];
length=-minusasinangle/Sqrt[1-cosangle^2];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[DH1[[1]],DH1[[2]]).Xtrans[DH2[[1]],DH2[[2]]).Ztrans[angle,length]).{1,0,0,0})),{4})).Ngw[[1]]==1,Null,angle=-angle;length=-length];
dhpars={angle,length}

MakeDH4[Gtot_,G_,W_,DH1_,DH2_,DH3_]:=Module[{cosangle,minusasinangle,angle,length,dhpars},
cosangle=G[[1]].W[[1]];
minusasinangle=G[[1]].W[[2]]+G[[2]].W[[1]];
angle=ArcCos[cosangle];
length=-minusasinangle/Sqrt[1-cosangle^2];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[DH1[[1]],DH1[[2]]).Xtrans[DH2[[1]],DH2[[2]]).Ztrans[DH3[[1]],DH3[[2]]].Xtrans[angle,length]).{0,0,1,0})),{4})).W[[1]]==1,Null,angle=-angle;length=-length];

```

```

dhpars={angle,length}}

MakeDH5[Gtot_,G_,W_,F_,DH1_,DH2_,DH3_,DH4_]:=Module[{Ngw,Nwf,cosangle,minusasinangle,angle,length,dhpars},
  Ngw=MakeNormal[G,W];
  Nwf=MakeNormal[W,F];
  cosangle=Ngw[[1]].Nwf[[1]];
  minusasinangle=Ngw[[1]].Nwf[[2]]+Ngw[[2]].Nwf[[1]];
  angle=ArcCos[cosangle];
  length=-minusasinangle/Sqrt[1-cosangle^2];

  If[(Drop[({Gtot.Ztrans[0,0].Ztrans[DH1[[1]],DH1[[2]]].Xtrans[DH2[[1]],DH2[[2]]].Ztrans[DH3[[1]],DH3[[2]]].Xtrans[DH4[[1]],DH4[[2]]].Ztrans[angle,length]).{1,0,0,0}),{4}]).Nwf[[1]]==1,Null,angle=-angle;length=-length];
  dhpars={angle,length}}

MakeDH6[Gtot_,W_,F_,DH1_,DH2_,DH3_,DH4_,DH5_]:=Module[{cosangle,minusasinangle,angle,length,dhpars},
  cosangle=W[[1]].F[[1]];
  minusasinangle=W[[1]].F[[2]]+W[[2]].F[[1]];
  angle=ArcCos[cosangle];
  length=-minusasinangle/Sqrt[1-cosangle^2];

  If[(Drop[({Gtot.Ztrans[0,0].Ztrans[DH1[[1]],DH1[[2]]].Xtrans[DH2[[1]],DH2[[2]]].Ztrans[DH3[[1]],DH3[[2]]].Xtrans[DH4[[1]],DH4[[2]]].Ztrans[DH5[[1]],DH5[[2]]].Xtrans[angle,length]).{0,0,1,0}),{4}]).F[[1]]==1,Null,angle=-angle;length=-length];
  dhpars={angle,length}}

MakeHtransP[Gtot_,DH1_,DH2_,DH3_,DH4_,DH5_,DH6_,Pos1_]:=Module[{theta10,phi10,psi10,d10,KE10},
  theta10=0;
  phi10=0;
  psi10=0;
  d10=0;

  KE10=Gtot.Ztrans[d10,0].Ztrans[DH1[[1]],DH1[[2]]].Xtrans[DH2[[1]],DH2[[2]]].Ztrans[theta10,0].Ztrans[DH3[[1]],DH3[[2]]].Xtrans[DH4[[1]],DH4[[2]]].Ztrans[phi10,0].Ztrans[DH5[[1]],DH5[[2]]].Xtrans[DH6[[1]],DH6[[2]]].Ztrans[psi10,0];
  Htot=Inverse[KE10].Pos1

  CheckPos[pos_,i_,data_,angles_,Gtot_,DH1_,DH2_,DH3_,DH4_,DH5_,DH6_,Htot_]:=
Module[{hnorm,solsAngles,thetas,phis,psis,ds,KE1pos},
  solsAngles=data[[i,Range[22,49]]];
  hnorm=Sqrt[data[[i,{1,2,3}]].data[[i,{1,2,3}]]];
  ds=Table[solsAngles[[4j-3]]*hnorm,{j,1,Length[solsAngles]/4}];
  thetas=Table[solsAngles[[4j-2]],{j,1,Length[solsAngles]/4}];

```

```

phis=Table[solsAngles[[4 j-1]],{j,1,Length[solsAngles]/4}];
psis=Table[solsAngles[[4 j]],{j,1,Length[solsAngles]/4}];
KE1pos=Gtot.Ztrans[0,ds[[pos-
1]]].Ztrans[DH1[[1]],DH1[[2]]].Xtrans[DH2[[1]],DH2[[2]]].Ztrans[thetas[[pos-
1]],0].Ztrans[DH3[[1]],DH3[[2]]].Xtrans[DH4[[1]],DH4[[2]]].Ztrans[phis[[pos-
1]],0].Ztrans[DH5[[1]],DH5[[2]]].Xtrans[DH6[[1]],DH6[[2]]].Ztrans[psis[[pos-
1]],0].Htot]
(* FROM HERE ON, NEW FUNCTIONS *)

(*MakeGtrans is different from MakeGtransP because here all joints are R.
The transformation goes up to the common normal with the first axis *)

MakeGtrans[axes_]:=Module[{pointH,xdir,ydir,Rgtrans,Ghom,Gtot},
pointH=axes[[1,1]]□axes[[1,2]];

If[Sqrt[pointH.pointH]<0.0001,xdir=axes[[1,1]]□{1,0,0},xdir=pointH/Sqrt[pointH.point
H]];
ydir=axes[[1,1]]□xdir;
Rgtrans={xdir,ydir,axes[[1,1]],pointH};
Ghom=Transpose[Rgtrans];
Gtot=Append[Ghom,{0,0,0,1}]

(*DHs contains all sets,
Pos1 is the reference configuration as absolute displacement
MakeHtrans fixes all angles equal to zero to reach the first position
(fixes in fact the reference configuration) *)

MakeHtrans[Gtot_,DHs_,Pos1_]:=Module[{angles,KE10},
angles=ConstantArray[0,{8,2}];
KE10=Gtot;
Table[KE10=KE10.Ztrans[angles[[i,1]],angles[[i,2]]].Ztrans[DHs[[2 i-1,1]],DHs[[2 i-
1,2]]].Xtrans[DHs[[2 i,1]],DHs[[2 i,2]]],{i,1,7}];
KE10=KE10.Ztrans[angles[[8,1]],angles[[8,2]]];
Htot=Inverse[KE10].Pos1]
(* This is the general DH function. Creates always 14 sets, last ones may be zeroed.
Gtot is the transformation from base to first axis
axes contain all the axes {{axis1dir,axis1mom},{axis2dir,axis2mom},...} (only one
solution!!!)
*)
CommonNormal2[l1_,l2_]:=Module[{p1,p2,t2,i2,n,n0},
p1=l1[[1]]□l1[[2]]/(l1[[1]].l1[[1]]);
p2=l2[[1]]□l2[[2]]/(l2[[1]].l2[[1]]);
t2=(p2-p1).l2[[1]];
i2=p2-t2*l2[[1]];
n=(i2-p1)/Sqrt[(i2-p1).(i2-p1)];
n0=p1□n;

```

```

Return[{ {n,n0},{p1,i2}}];
];

```

```

MakeDH[Gtot_,axes2_]:=Module[{naxes,axes,addedAxes,nline,nline2,xdir,cosangle,min
usasinangle,angle,length,dhpars,p1,p2},
  naxes=Length[axes2];
  (*If we have less than 8 axes, we complement them adding one zero sets*)
  addedAxes=ConstantArray[0,{2,3}];
  axes=axes2;
  Table[axes=Append[axes,addedAxes],{i,1,8-naxes}];
  (*Start creating DH sets. The first two have no check.*)
  nline=MakeNormal[axes[[1]],axes[[2]]];
  xdir=Gtot[{ {1,2,3},1}];
  cosangle=xdir.nline[[1]];
  minusasinangle=xdir.nline[[2]]+{0,0,0}.nline[[1]];
  angle=ArcCos[cosangle];
  If[-0.00001<(Abs[cosangle]-1)<0.000001,
    p1=xdir[[1]] $\square$ xdir[[2]]/(xdir[[1]].xdir[[1]]);
    p2=nline[[1]] $\square$ nline[[2]]/(nline[[1]].nline[[1]]);
    If[Abs[(p2-p1).(p2-p1)]<0.000001,
      length=0,
      p1=CommonNormal2[xdir,nline][[2]];
      p2=p1[[2]]-p1[[1]];
      length=Sqrt[p2.p2],
      length=-minusasinangle/Sqrt[1-cosangle^2]
    ];
  If[(Drop[({Gtot.Ztrans[0,0].Ztrans[angle,length]).{1,0,0,0}},{4}]).nline[[1]]==1,Null,angle=-angle;length=-length];
  dhpars={ {angle,length} };
  (*second DH set*)
  cosangle=axes[[1,1]].axes[[2,1]];
  minusasinangle=axes[[1,1]].axes[[2,2]]+axes[[1,2]].axes[[2,1]];
  angle=ArcCos[cosangle];
  If[-0.00001<(Abs[cosangle]-1)<0.000001,
    p1=axes[[1,1]] $\square$ axes[[1,2]]/(axes[[1,1]].axes[[1,1]]);
    p2=axes[[2,1]] $\square$ axes[[2,2]]/(axes[[2,1]].axes[[2,1]]);
    If[Abs[(p2-p1).(p2-p1)]<0.000001,
      length=0,
      p1=CommonNormal2[axes[[1]],axes[[2]]][[2]];
      p2=p1[[2]]-p1[[1]];
      length=Sqrt[p2.p2],
      length=-minusasinangle/Sqrt[1-cosangle^2]
    ];
];

```



```

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpars[[1,1]],dhpars[[1,2]]).Xtrans[angle,length]).{0,0,1,0}),{4})).axes[[2,1]]==1,Null,angle=-angle;length=-length];
dhpars=Append[dhpars,{angle,length}];
(*From here on we check that it is not zero*)
If[axes[[3]]=={{0,0,0},{0,0,0}}, dhpars=Append[dhpars,{0,0}],
nline=MakeNormal[axes[[1]],axes[[2]]];
nline2=MakeNormal[axes[[2]],axes[[3]]];
cosangle=nline[[1]].nline2[[1]];
minusasineangle=nline[[1]].nline2[[2]]+nline[[2]].nline2[[1]];
angle=ArcCos[cosangle];
If[-0.00001<(Abs[cosangle]-1)<0.000001,
p1=nline[[1]] $\times$ nline[[2]]/(nline[[1]].nline[[1]]);
p2=nline2[[1]] $\times$ nline2[[2]]/(nline2[[1]].nline2[[1]]);
If[Abs[(p2-p1).(p2-p1)]<0.000001,
length=0,
p1=CommonNormal2[nline,nline2][[2]];
p2=p1[[2]]-p1[[1]];
length=Sqrt[p2.p2],
length=-minusasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpars[[1,1]],dhpars[[1,2]]).Xtrans[dhpars[[2,1]],dhpars[[2,2]]].Ztrans[angle,length]).{1,0,0,0}),{4})).nline2[[1]]==1,
Null,angle=-angle;length=-length];
dhpars=Append[dhpars,{angle,length}];
If[axes[[3]]=={{0,0,0},{0,0,0}}, dhpars=Append[dhpars,{0,0}],
cosangle=axes[[2,1]].axes[[3,1]];
minusasineangle=axes[[2,1]].axes[[3,2]]+axes[[2,2]].axes[[3,1]];
angle=ArcCos[cosangle];
If[-0.00001<(Abs[cosangle]-1)<0.000001,
p1=axes[[2,1]] $\times$ axes[[2,2]]/(axes[[2,1]].axes[[2,1]]);
p2=axes[[3,1]] $\times$ axes[[3,2]]/(axes[[3,1]].axes[[3,1]]);
If[Abs[(p2-p1).(p2-p1)]<0.000001,
length=0,
p1=CommonNormal2[axes[[2]],axes[[3]]][[2]];
p2=p1[[2]]-p1[[1]];
length=Sqrt[p2.p2],
length=-minusasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpars[[1,1]],dhpars[[1,2]]).Xtrans[dhpars[[2,1]],dhpars[[2,2]]].Ztrans[dhpars[[3,1]],dhpars[[3,2]]].Xtrans[angle,length]).{0,0,1,0}),{4})).axes[[3,1]]==1,
Null,angle=-angle;length=-length];
dhpars=Append[dhpars,{angle,length}];

```

```

If[axes[[4]]=={{0,0,0},{0,0,0}}, dhpars=Append[dhpars,{0,0}],
  nline=MakeNormal[axes[[2]],axes[[3]]];
  nline2=MakeNormal[axes[[3]],axes[[4]]];
  cosangle=nline[[1]].nline2[[1]];
  minusasinangle=nline[[1]].nline2[[2]]+nline[[2]].nline2[[1]];
  angle=ArcCos[cosangle];
  If[-0.00001<(Abs[cosangle]-1)<0.000001,
    p1=nline[[1]] $\square$ nline[[2]]/(nline[[1]].nline[[1]]);
    p2=nline2[[1]] $\square$ nline2[[2]]/(nline2[[1]].nline2[[1]]);
    If[Abs[(p2-p1).(p2-p1)]<0.000001,
      length=0,
      p1=CommonNormal2[nline,nline2][[2]];
      p2=p1[[2]]-p1[[1]];
      length=Sqrt[p2.p2]],
    length=-minusasinangle/Sqrt[1-cosangle^2]
  ];

  If[(Drop[({Gtot.Ztrans[0,0].Ztrans[dhpars[[1,1]],dhpars[[1,2]]].Xtrans[dhpars[[2,1]]],dhpars[[2,2]]].Ztrans[dhpars[[3,1]],dhpars[[3,2]]].Xtrans[dhpars[[4,1]],dhpars[[4,2]]].Ztrans[angle,length]).{1,0,0,0},{4}).nline2[[1]]==1,
    Null,angle=-angle;length=-length];
  dhpars=Append[dhpars,{angle,length}];
  If[axes[[4]]=={{0,0,0},{0,0,0}}, dhpars=Append[dhpars,{0,0}],
    cosangle=axes[[3,1]].axes[[4,1]];
    minusasinangle=axes[[3,1]].axes[[4,2]]+axes[[3,2]].axes[[4,1]];
    angle=ArcCos[cosangle];
    If[-0.00001<(Abs[cosangle]-1)<0.000001,
      p1=axes[[3,1]] $\square$ axes[[3,2]]/(axes[[3,1]].axes[[3,1]]);
      p2=axes[[4,1]] $\square$ axes[[4,2]]/(axes[[4,1]].axes[[4,1]]);
      If[Abs[(p2-p1).(p2-p1)]<0.000001,
        length=0,
        p1=CommonNormal2[axes[[3]],axes[[4]]][[2]];
        p2=p1[[2]]-p1[[1]];
        length=Sqrt[p2.p2]],
      length=-minusasinangle/Sqrt[1-cosangle^2]
    ];

  If[(Drop[({Gtot.Ztrans[0,0].Ztrans[dhpars[[1,1]],dhpars[[1,2]]].Xtrans[dhpars[[2,1]]],dhpars[[2,2]]].Ztrans[dhpars[[3,1]],dhpars[[3,2]]].Xtrans[dhpars[[4,1]],dhpars[[4,2]]].Ztrans[dhpars[[5,1]],dhpars[[5,2]]].Xtrans[angle,length]).{0,0,1,0},{4}).axes[[4,1]]==1,
    Null,angle=-angle;length=-length];
  dhpars=Append[dhpars,{angle,length}];
  If[axes[[5]]=={{0,0,0},{0,0,0}}, dhpars=Append[dhpars,{0,0}],
    nline=MakeNormal[axes[[3]],axes[[4]]];
    nline2=MakeNormal[axes[[4]],axes[[5]]];
    cosangle=nline[[1]].nline2[[1]];

```

```

minusasinangle=nline[[1]].nline2[[2]]+nline[[2]].nline2[[1]];
angle=ArcCos[cosangle];
If[-0.00001<(Abs[cosangle]-1)<0.000001,
  p1=nline[[1]]□nline[[2]]/(nline[[1]].nline[[1]]);
  p2=nline2[[1]]□nline2[[2]]/(nline2[[1]].nline2[[1]]);
  If[Abs[(p2-p1).(p2-p1)]<0.000001,
    length=0,
    p1=CommonNormal2[nline,nline2][[2]];
    p2=p1[[2]]-p1[[1]];
    length=Sqrt[p2.p2]],
  length=-minusasinangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[angle,length]).{1,0,0,0}),{4})).nline2[[1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
If[axes[[5]]=={{0,0,0},{0,0,0}}, dhpar=Append[dhpar,{0,0}],
  cosangle=axes[[4,1]].axes[[5,1]];
  minusasinangle=axes[[4,1]].axes[[5,2]]+axes[[4,2]].axes[[5,1]];
  angle=ArcCos[cosangle];
  If[-0.00001<(Abs[cosangle]-1)<0.000001,
    p1=axes[[4,1]]□axes[[4,2]]/(axes[[4,1]].axes[[4,1]]);
    p2=axes[[5,1]]□axes[[5,2]]/(axes[[5,1]].axes[[5,1]]);
    If[Abs[(p2-p1).(p2-p1)]<0.000001,
      length=0,
      p1=CommonNormal2[axes[[4]],axes[[5]]][[2]];
      p2=p1[[2]]-p1[[1]];
      length=Sqrt[p2.p2]],
    length=-minusasinangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[dhpar[[7,1]],dhpar[[7,2]]].Xtrans[angle,length]).{0,0,1,0}),{4})).axes[[5,1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
If[axes[[6]]=={{0,0,0},{0,0,0}}, dhpar=Append[dhpar,{0,0}],
  nline=MakeNormal[axes[[4]],axes[[5]]];
  nline2=MakeNormal[axes[[5]],axes[[6]]];
  cosangle=nline[[1]].nline2[[1]];
  minusasinangle=nline[[1]].nline2[[2]]+nline[[2]].nline2[[1]];
  angle=ArcCos[cosangle];

```

```

If[-0.00001<(Abs[cosangle]-1)<0.000001,
  p1=nline[[1]]□nline[[2]]/(nline[[1]].nline[[1]]);
  p2=nline2[[1]]□nline2[[2]]/(nline2[[1]].nline2[[1]]);
  If[Abs[(p2-p1).(p2-p1)]<0.000001,
    length=0,
    p1=CommonNormal2[nline,nline2][[2]];
    p2=p1[[2]]-p1[[1]];
    length=Sqrt[p2.p2]],
  length=-minusasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[dhpar[[7,1]],dhpar[[7,2]]].Xtrans[dhpar[[8,1]],dhpar[[8,2]]].Ztrans[angle,length]).{1,0,0,0}),{4})).nline2[[1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
If[axes[[6]]=={{0,0,0},{0,0,0}}, dhpar=Append[dhpar,{0,0}],
  cosangle=axes[[5,1]].axes[[6,1]];
  minusasinangle=axes[[5,1]].axes[[6,2]]+axes[[5,2]].axes[[6,1]];
  angle=ArcCos[cosangle];
  If[-0.00001<(Abs[cosangle]-1)<0.000001,
    p1=axes[[5,1]]□axes[[5,2]]/(axes[[5,1]].axes[[5,1]]);
    p2=axes[[6,1]]□axes[[6,2]]/(axes[[6,1]].axes[[6,1]]);
    If[Abs[(p2-p1).(p2-p1)]<0.000001,
      length=0,
      p1=CommonNormal2[axes[[5]],axes[[6]]][[2]];
      p2=p1[[2]]-p1[[1]];
      length=Sqrt[p2.p2]],
    length=-minusasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[dhpar[[7,1]],dhpar[[7,2]]].Xtrans[dhpar[[8,1]],dhpar[[8,2]]].Ztrans[dhpar[[9,1]],dhpar[[9,2]]].Xtrans[angle,length]).{0,0,1,0}),{4})).axes[[6,1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
If[axes[[7]]=={{0,0,0},{0,0,0}}, dhpar=Append[dhpar,{0,0}],
  nline=MakeNormal[axes[[5]],axes[[6]]];
  nline2=MakeNormal[axes[[6]],axes[[7]]];
  cosangle=nline[[1]].nline2[[1]];
  minusasinangle=nline[[1]].nline2[[2]]+nline[[2]].nline2[[1]];
  angle=ArcCos[cosangle];

```

```

If[-0.00001<(Abs[cosangle]-1)<0.000001,
  p1=nline[[1]]□nline[[2]]/(nline[[1]].nline[[1]]);
  p2=nline2[[1]]□nline2[[2]]/(nline2[[1]].nline2[[1]]);
  If[Abs[(p2-p1).(p2-p1)]<0.000001,
    length=0,
    p1=CommonNormal2[nline,nline2][[2]];
    p2=p1[[2]]-p1[[1]];
    length=Sqrt[p2.p2]],
  length=-minusasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[dhpar[[7,1]],dhpar[[7,2]]].Xtrans[dhpar[[8,1]],dhpar[[8,2]]].Ztrans[dhpar[[9,1]],dhpar[[9,2]]].Xtrans[dhpar[[10,1]],dhpar[[10,2]]].Ztrans[angle,length]).{1,0,0,0},{4}}).nline2[[1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
If[axes[[7]]=={{0,0,0},{0,0,0}}, dhpar=Append[dhpar,{0,0}],
cosangle=axes[[6,1]].axes[[7,1]];
minusasineangle=axes[[6,1]].axes[[7,2]]+axes[[6,2]].axes[[7,1]];
angle=ArcCos[cosangle];
If[-0.00001<(Abs[cosangle]-1)<0.000001,
  p1=axes[[6,1]]□axes[[6,2]]/(axes[[6,1]].axes[[6,1]]);
  p2=axes[[7,1]]□axes[[7,2]]/(axes[[7,1]].axes[[7,1]]);
  If[Abs[(p2-p1).(p2-p1)]<0.000001,
    length=0,
    p1=CommonNormal2[axes[[6]],axes[[7]]][[2]];
    p2=p1[[2]]-p1[[1]];
    length=Sqrt[p2.p2]],
  length=-minusasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[dhpar[[7,1]],dhpar[[7,2]]].Xtrans[dhpar[[8,1]],dhpar[[8,2]]].Ztrans[dhpar[[9,1]],dhpar[[9,2]]].Xtrans[dhpar[[10,1]],dhpar[[10,2]]].Ztrans[dhpar[[11,1]],dhpar[[11,2]]].Xtrans[angle,length]).{0,0,1,0},{4}}).axes[[7,1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
If[axes[[8]]=={{0,0,0},{0,0,0}}, dhpar=Append[dhpar,{0,0}],
nline=MakeNormal[axes[[6]],axes[[7]]];
nline2=MakeNormal[axes[[7]],axes[[8]]];
cosangle=nline[[1]].nline2[[1]];
minusasineangle=nline[[1]].nline2[[2]]+nline[[2]].nline2[[1]];

```

```

angle=ArcCos[cosangle];
If[-0.00001<(Abs[cosangle]-1)<0.000001,
  p1=nline[[1]]□nline[[2]]/(nline[[1]].nline[[1]]);
  p2=nline2[[1]]□nline2[[2]]/(nline2[[1]].nline2[[1]]);
  If[Abs[(p2-p1).(p2-p1)]<0.000001,
    length=0,
    p1=CommonNormal2[nline,nline2][[2]];
    p2=p1[[2]]-p1[[1]];
    length=Sqrt[p2.p2]],
  length=-minuasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[dhpar[[7,1]],dhpar[[7,2]]].Xtrans[dhpar[[8,1]],dhpar[[8,2]]].Ztrans[dhpar[[9,1]],dhpar[[9,2]]].Xtrans[dhpar[[10,1]],dhpar[[10,2]]].Ztrans[dhpar[[11,1]],dhpar[[11,2]]].Xtrans[dhpar[[12,1]],dhpar[[12,2]]].Ztrans[angle,length]).{1,0,0,0},{4})].nline2[[1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
If[axes[[8]]=={{0,0,0},{0,0,0}}, dhpar=Append[dhpar,{0,0}],
cosangle=axes[[7,1]].axes[[8,1]];
minuasineangle=axes[[7,1]].axes[[8,2]]+axes[[7,2]].axes[[8,1]];
angle=ArcCos[cosangle];
If[-0.00001<(Abs[cosangle]-1)<0.000001,
  p1=axes[[7,1]]□axes[[7,2]]/(axes[[7,1]].axes[[7,1]]);
  p2=axes[[8,1]]□axes[[8,2]]/(axes[[8,1]].axes[[8,1]]);
  If[Abs[(p2-p1).(p2-p1)]<0.000001,
    length=0,
    p1=CommonNormal2[axes[[7]],axes[[8]]][[2]];
    p2=p1[[2]]-p1[[1]];
    length=Sqrt[p2.p2]],
  length=-minuasineangle/Sqrt[1-cosangle^2]
];

If[(Drop[(((Gtot.Ztrans[0,0].Ztrans[dhpar[[1,1]],dhpar[[1,2]]).Xtrans[dhpar[[2,1]],dhpar[[2,2]]).Ztrans[dhpar[[3,1]],dhpar[[3,2]]].Xtrans[dhpar[[4,1]],dhpar[[4,2]]].Ztrans[dhpar[[5,1]],dhpar[[5,2]]].Xtrans[dhpar[[6,1]],dhpar[[6,2]]].Ztrans[dhpar[[7,1]],dhpar[[7,2]]].Xtrans[dhpar[[8,1]],dhpar[[8,2]]].Ztrans[dhpar[[9,1]],dhpar[[9,2]]].Xtrans[dhpar[[10,1]],dhpar[[10,2]]].Ztrans[dhpar[[11,1]],dhpar[[11,2]]].Xtrans[dhpar[[12,1]],dhpar[[12,2]]].Ztrans[dhpar[[13,1]],dhpar[[13,2]]].Xtrans[angle,length]).{0,0,1,0},{4})].axes[[8,1]]==1,
  Null,angle=-angle;length=-length];
dhpar=Append[dhpar,{angle,length}];
Return[dhpar]

```

(* CheckPosRev is for general chains with only revolute joints
data contains all the axes to analyze, organized
as follows: {{axis1dir,axis1mom},{axis2dir,axis2mom},...} (only 1 solution at a time)
angles contains the joint variables as
{{angle1,...,anglex}for first rel pos,{...}second rel pos,....}
*)

(* Consider the general case, with 8 joints. If there are less, we fill up the rest so that the matrix is the identity *)

(* We have eliminated the variable i and assume that we only have one solution here.*)

(* DHs has the structure {DH1,DH2,...,DHn}

In total, there are $2(n_{axes}-2)+2 = 2*n_{axes}-2$ DH sets. When we add nadded axes, we need to add $2*nadded$ DH sets.*)

```
CheckPosRev[pos_,angles_,Gtot_,DHs_,Htot_]:=Module[{nang,addedzeros,addedDHs,
myangles,myangles2,DHs2,KE1pos},
  nang=Length[angles[[1]]];
  myangles=angles[[pos]];
  If[nang < 8, addedzeros=ConstantArray[0,8-nang];
    myangles2=Flatten[Append[myangles,addedzeros]],
    myangles2=myangles];
```

```
KE1pos=Gtot.Ztrans[myangles2[[1]],0].Ztrans[DHs[[1,1]],DHs[[1,2]]].Xtrans[DHs[[2,1]],
DHs[[2,2]]].Ztrans[myangles2[[2]],0]
.Ztrans[DHs[[3,1]],DHs[[3,2]]].Xtrans[DHs[[4,1]],DHs[[4,2]]].Ztrans[myangles2[[3]],
0].
```

```
Ztrans[DHs[[5,1]],DHs[[5,2]]].Xtrans[DHs[[6,1]],DHs[[6,2]]].Ztrans[myangles2[[4]],0].
```

```
Ztrans[DHs[[7,1]],DHs[[7,2]]].Xtrans[DHs[[8,1]],DHs[[8,2]]].Ztrans[myangles2[[5]],0]
.Ztrans[DHs[[9,1]],DHs[[9,2]]].Xtrans[DHs[[10,1]],DHs[[10,2]]].Ztrans[myangles2[[6]],0]
.Ztrans[DHs[[11,1]],DHs[[11,2]]].Xtrans[DHs[[12,1]],DHs[[12,2]]].Ztrans[myangles2[[7]],0]
.Ztrans[DHs[[13,1]],DHs[[13,2]]].Xtrans[DHs[[14,1]],DHs[[14,2]]].Ztrans[myangles2[[8]],0].Htot]
```

(*In Analyze, we assume that
axes contains all the axes to analyze, organized
as follows: {{axis1dir,axis1mom},{axis2dir,axis2mom},...} for one solution only.
angles contains the joint variables for all positions, as
{{angle1,...,anglex}for first rel pos,{...}second rel pos,....},
ref is the reference configuration as an absolute transformation.
If ref is unknown, set it to the identity *)

```
Analyze[axes_,angles_,ref_]:=Module[{n,Gtot,DHs,Htot,KEs},
```

```

n=Length[angles];
Gtot=MakeGtrans[axes];
DHs=MakeDH[Gtot,axes];
Htot=MakeHtrans[Gtot,DHs,ref];
KEs=Table[CheckPosRev[i,angles,Gtot,DHs,Htot],{i,1,n}]
]

```

(*CheckPosRevC is similar to CheckPosRev, but containing variables for cylindrical joints. Joint variables are organized as {{angle1,slide1},...,{anglex,slidex}} for first rel pos,{},{...} second rel pos,....).

AnalyzeC is similar to Analyze, but with cylindrical joints for all variables, so that they can be converted to either prismatic or revolute just by making one of the variables equal to zero *)

```

CheckPosRevC[pos_,angles_,Gtot_,DHs_,Htot_]:=Module[{nang,addedzeros,addedDHs,
myangles,myangles2,DHs2,KE1pos},
  nang=Length[angles[[1]]];
  myangles=angles[[pos]];
  If[nang < 8, addedzeros=ConstantArray[0,{8-nang,2}];
    myangles2=Flatten[Append[{myangles},addedzeros],1], myangles2=myangles];

```

```

KE1pos=Gtot.Ztrans[myangles2[[1,1]],myangles2[[1,2]]].Ztrans[DHs[[1,1]],DHs[[1,2]]].
Xtrans[DHs[[2,1]],DHs[[2,2]]].Ztrans[myangles2[[2,1]],myangles2[[2,2]]].Ztrans[DHs[[
3,1]],DHs[[3,2]]].Xtrans[DHs[[4,1]],DHs[[4,2]]].Ztrans[myangles2[[3,1]],myangles2[[3,
2]]].

```

```

Ztrans[DHs[[5,1]],DHs[[5,2]]].Xtrans[DHs[[6,1]],DHs[[6,2]]].Ztrans[myangles2[[4,1]],
myangles2[[4,2]]].

```

```

Ztrans[DHs[[7,1]],DHs[[7,2]]].Xtrans[DHs[[8,1]],DHs[[8,2]]].Ztrans[myangles2[[5,1]],
myangles2[[5,2]]]
  .Ztrans[DHs[[9,1]],DHs[[9,2]]].Xtrans[DHs[[10,1]],DHs[[10,2]]].Ztrans[myangle
s2[[6,1]],myangles2[[6,2]]]
  .Ztrans[DHs[[11,1]],DHs[[11,2]]].Xtrans[DHs[[12,1]],DHs[[12,2]]].Ztrans[myan
gles2[[7,1]],myangles2[[7,2]]]
  .Ztrans[DHs[[13,1]],DHs[[13,2]]].Xtrans[DHs[[14,1]],DHs[[14,2]]].Ztrans[myan
gles2[[8,1]],myangles2[[8,2]]].Htot]

```

```

AnalyzeC[axes_,angles_,ref_]:=Module[{n,Gtot,DHs,Htot,KEs},
  n=Length[angles];
  Gtot=MakeGtrans[axes];
  DHs=MakeDH[Gtot,axes];
  Htot=MakeHtrans[Gtot,DHs,ref];

```



```

    KEs=Table[CheckPosRevC[i,angles,Gtot,DHs,Htot},{i,1,n}]
]

```

To Matlab.m

Written by Harri Ojanen:

(***** ToMatlab.mth -- Mathematica expressions into Matlab form *****)

(*

ToMatlab[expr]

Converts the expression expr into matlab syntax and returns it as a String.

ToMatlab[expr, name]

Returns an assignment of expr into name as a String. name can be also a more complicated string, e.g.,

```
ToMatlab[If[t,a,b],"function y=iffun(t,a,b)\ny"].
```

The special symbol Colon can be used to denote the matlab colon operator :, and Colon[a,b] for a:b, Colon[a,b,c] for a:b:c.

WriteMatlab[expr, file]

WriteMatlab[expr, file, name]

Writes the expr in matlab form into the given file. The second form makes this an assignment into the variable name. Example:

```

f = OpenWrite["file.m"];
WriteMatlab[Cos[x]-x, f, y];
Close[f];

```

The file argument can also be a String that gives the name of the file:

```
WriteMatlab[Cos[x]-x, "file.m", y];
```

achieves the same result as the previous example (but this limits one expression per file).

PrintMatlab[expr]

`PrintMatlab[expr, name]`

is like `ToMatlab` but instead of returning the String, it is printed on the screen.

`RulesToMatlab[rules]`

Where `rules` is the result from `Solve` or `NSolve`: converts the rules into individual assignment statements.

*)

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<http://www.iki.fi/~harri.ojanen/> *)

(* Last modified April 2 1999 *)

`BeginPackage["MatlabUtils`ToMatlab`"]`

`ToMatlab::usage =`

"`ToMatlab[expr]` converts the expression `expr` into matlab syntax and returns it as a String.
"`ToMatlab[expr, name]` returns an assignment of `expr` into `name` as a String.
`name` can be also a more complicated string, e.g., `ToMatlab[If[t,a,b],\"function y=iffun(t,a,b)\ny\"]`.
The special symbol Colon can be used to denote the matlab colon operator `:`, and `Colon[a,b]` for `a:b`, `Colon[a,b,c]` for `a:b:c`.
See also `WriteMatlab` and `PrintMatlab`.
All functions accept an optional last argument that is the maximum line width."

`WriteMatlab::usage =`

"`WriteMatlab[expr, file]` or `WriteMatlab[expr, file, name]` Writes the `expr` in matlab form into the given file. The second form makes this an assignment into the variable `name`.
Example: `f = OpenWrite["file.m"]; WriteMatlab[Cos[x]-x, f, y]; Close[f];`
The file argument can also be a String that gives the name of the file: `WriteMatlab[Cos[x]-x, \"file.m\", y];` achieves the same result as the previous example (but this limits one expression per file).
See also `ToMatlab` and `PrintMatlab`."

`PrintMatlab::usage =`

"`PrintMatlab[expr]` or `PrintMatlab[expr, name]` is like `ToMatlab` but instead of returning the String, it is printed on the screen. See also `ToMatlab` and `WriteMatlab`."

`RulesToMatlab::usage =`

"`RulesToMatlab[rules]` where `rules` is from `Solve` or `NSolve` converts the rules into individual assignment statements."

```

(*SetMargin::usage = "SetMargin[margin]"
RestoreMargin::usage = "RestoreMargin[]"*)

Begin["`Private`"]

WriteMatlab[e_, file_OutputStream] :=
  (WriteString[file, ToMatlab[e,72]];)

WriteMatlab[e_, file_OutputStream, name_] :=
  (WriteString[file, ToMatlab[e,name,72]];)  /; (!NumberQ[name])

WriteMatlab[e_, file_String] :=
  (Block[{f = OpenWrite[file]},
    WriteString[f, ToMatlab[e,72]];
    Close[f];];)

WriteMatlab[e_, file_String, name_] :=
  (Block[{f = OpenWrite[file]},
    WriteString[f, ToMatlab[e,name,72]];
    Close[f];];)  /; (!NumberQ[name])

WriteMatlab[e_, file_OutputStream, margin_Integer] :=
  (WriteString[file, ToMatlab[e,margin]];)

WriteMatlab[e_, file_OutputStream, name_, margin_Integer] :=
  (WriteString[file, ToMatlab[e,name,margin]];)

WriteMatlab[e_, file_String, margin_Integer] :=
  (Block[{f = OpenWrite[file]},
    WriteString[f, ToMatlab[e,margin]];
    Close[f];];)

WriteMatlab[e_, file_String, name_, margin_Integer] :=
  (Block[{f = OpenWrite[file]},
    WriteString[f, ToMatlab[e,name,margin]];
    Close[f];];)

PrintMatlab[e_] :=
  (Print[ToMatlab[e, 60]];)

PrintMatlab[e_, name_] :=
  (Print[ToMatlab[e, name, 60]];)  /; (!NumberQ[name])

PrintMatlab[e_, margin_Integer] :=

```

```

(Print[ToMatlab[e, margin]]);

PrintMatlab[e_, name_, margin_Integer] :=
(Print[ToMatlab[e, name, margin]]);

ToMatlab[e_] := foldlines[ToMatlabaux[e] <> ";\\n"]

ToMatlab[e_, name_] :=
  ToMatlabaux[name] <> "=" <> foldlines[ToMatlabaux[e] <> ";\\n"] /;
  (!NumberQ[name])

ToMatlab[e_, margin_Integer] :=
  Block[{s},
    SetMargin[margin];
    s = foldlines[ToMatlabaux[e] <> ";\\n"];
    RestoreMargin[];
    s]

ToMatlab[e_, name_, margin_Integer] :=
  Block[{s},
    SetMargin[margin];
    s = ToMatlabaux[name] <> "=" <> foldlines[ToMatlabaux[e] <> ";\\n"];
    RestoreMargin[];
    s]

RulesToMatlab[l_List] :=
  If[Length[l] === 0,
    "",
    Block[{s = RulesToMatlab[l[[1]]]},
      Do[s = s <> RulesToMatlab[l[[i]]], {i, 2, Length[l]}];
      s]]

RulesToMatlab[Rule[x_, a_]] :=
  ToMatlab[a, ToMatlab[x] // StringDrop[#, -2]&]

(***) Numbers and strings
*****

ToMatlabaux[s_String] := s

ToMatlabaux[n_Integer] :=
  If[n >= 0, ToString[n], "(" <> ToString[n] <> ")"]

(*ToMatlabaux[r_Rational] :=

```

```

    "(" <> ToMatlabaux[Numerator[r]] <> "/" <>
      ToMatlabaux[Denominator[r]] <> ")"*)

ToMatlabaux[r_Rational] :=
  "(" <> ToString[Numerator[r]] <> "/" <>
    ToString[Denominator[r]] <> ")"

ToMatlabaux[r_Real] :=
  Block[{a = MantissaExponent[r]},
    If[r >= 0,
      ToString[N[a[[1]],18]] <> "E" <> ToString[a[[2]]],
      "(" <> ToString[N[a[[1]],18]] <> "E" <> ToString[a[[2]]] <> ")"]

ToMatlabaux[I] := "sqrt(-1)";

ToMatlabaux[c_Complex] :=
  "(" <>
    If[Re[c] === 0,
      "",
      ToMatlabaux[Re[c]] <> "+" <>
    If[Im[c] === 1,
      "sqrt(-1)",
      "sqrt(-1)*" <> ToMatlabaux[Im[c]] ] <> ")"

(***) Lists, vectors and matrices *****)

numberMatrixQ[m_] := MatrixQ[m] && (And @@ Map[numberListQ,m])

numberListQ[l_] := ListQ[l] && (And @@ Map[NumberQ,l])

numbermatrixToMatlab[m_] :=
  Block[{i, s=""},
    For[i=1, i<=Length[m], i++,
      s = s <> numbermatrixrow[m[[i]]];
      If[i < Length[m], s = s <> ","];
    s]

numbermatrixrow[l_] :=
  Block[{i, s=""},
    For[i=1, i<=Length[l], i++,
      s = s <> ToMatlabaux[l[[i]]];
      If[i < Length[l], s = s <> ","];
    s]

ToMatlabaux[l_List /; MatrixQ[l]] :=

```

```

If[numberMatrixQ[l],
  "[" <> numbermatrixToMatlab[l] <> "]",
  "[" <> matrixToMatlab[l] <> "]" ]

matrixToMatlab[m_] :=
  If[Length[m] === 1,
    ToMatlabargs[m[[1]]],
    ToMatlabargs[m[[1]]] <> ";" <>
      matrixToMatlab[ argslistdrop[m] ] ]

ToMatlabaux[l_List] := "[" <> ToMatlabargs[l] <> "]"

(*** Symbols
*****

ToMatlabaux[Colon] = ":"
ToMatlabaux[Abs] = "abs"
ToMatlabaux[Min] = "min"
ToMatlabaux[Max] = "max"
ToMatlabaux[Sin] = "sin"
ToMatlabaux[Cos] = "cos"
ToMatlabaux[Tan] = "tan"
ToMatlabaux[Cot] = "cot"
ToMatlabaux[Csc] = "csc"
ToMatlabaux[Sec] = "sec"
ToMatlabaux[ArcSin] = "asin"
ToMatlabaux[ArcCos] = "acos"
ToMatlabaux[ArcTan] = "atan"
ToMatlabaux[ArcCot] = "acot"
ToMatlabaux[ArcCsc] = "acsc"
ToMatlabaux[ArcSec] = "asec"
ToMatlabaux[Sinh] := "sinh"
ToMatlabaux[Cosh] := "cosh"
ToMatlabaux[Tanh] := "tanh"
ToMatlabaux[Coth] := "coth"
ToMatlabaux[Csch] := "csch"
ToMatlabaux[Sech] := "sech"
ToMatlabaux[ArcSinh] := "asinh"
ToMatlabaux[ArcCosh] := "acosh"
ToMatlabaux[ArcTanh] := "atanh"
ToMatlabaux[ArcCoth] := "acoth"
ToMatlabaux[ArcCsch] := "acsch"
ToMatlabaux[ArcSech] := "asech"
ToMatlabaux[Log] := "log"
ToMatlabaux[Exp] := "exp"

```

```

ToMatlabaux[MatrixExp] := "expm"
ToMatlabaux[Pi] := "pi"
ToMatlabaux[E] := "exp(1)"
ToMatlabaux[True] := "1"
ToMatlabaux[False] := "0"

```

```

ToMatlabaux[e_Symbol] := ToString[e]

```

```

(*** Relational operators
*****

```

```

ToMatlabaux[e_ /; Head[e] === Equal] :=
  ToMatlabrelop[ arglist[e], "==" ]
ToMatlabaux[e_ /; Head[e] === Unequal] :=
  ToMatlabrelop[ arglist[e], "~=" ]
ToMatlabaux[e_ /; Head[e] === Less] :=
  ToMatlabrelop[ arglist[e], "<" ]
ToMatlabaux[e_ /; Head[e] === Greater] :=
  ToMatlabrelop[ arglist[e], ">" ]
ToMatlabaux[e_ /; Head[e] === LessEqual] :=
  ToMatlabrelop[ arglist[e], "<=" ]
ToMatlabaux[e_ /; Head[e] === GreaterEqual] :=
  ToMatlabrelop[ arglist[e], ">=" ]
ToMatlabaux[e_ /; Head[e] === And] :=
  ToMatlabrelop[ arglist[e], "&" ]
ToMatlabaux[e_ /; Head[e] === Or] :=
  ToMatlabrelop[ arglist[e], "|" ]
ToMatlabaux[e_ /; Head[e] === Not] :=
  "~(" <> ToMatlabaux[e[[1]]] <> ")"

```

```

ToMatlabrelop[e_, o_] :=
  If[Length[e] === 1,
    "(" <> ToMatlabaux[e[[1]]] <> ")",
    "(" <> ToMatlabaux[e[[1]]] <> ")" <> o <>
    ToMatlabrelop[ arglistdrop[e], o ]

```

```

relopQ[e_] := MemberQ[{Equal, Unequal, Less, Greater, LessEqual,
  GreaterEqual, And, Or, Not}, Head[e]]

```

```

(*** Addition, multiplication and powers *****

```

```

ToMatlabaux[e_ /; Head[e] === Plus] :=
  If[relopQ[e[[1]]],
    "(" <> ToMatlabaux[e[[1]]] <> ")",

```

```

    ToMatlabaux[e[[1]]] ] <>
" + " <>
    If[Length[e] === 2,
      If[relopQ[e[[2]]],
        "(" <> ToMatlabaux[e[[2]]] <> ")",
        ToMatlabaux[e[[2]]] ],
      ToMatlabaux[ dropfirst[e] ]]

ToMatlabaux[e_ /; Head[e] === Times] :=
  If[Head[e[[1]]] === Plus,
    "(" <> ToMatlabaux[e[[1]]] <> ")",
    ToMatlabaux[e[[1]]] ] <>
  ".*" <>
  If[Length[e] === 2,
    If[Head[e[[2]]] === Plus,
      "(" <> ToMatlabaux[e[[2]]] <> ")",
      ToMatlabaux[e[[2]]] ],
    ToMatlabaux[ dropfirst[e] ]]

ToMatlabaux[e_ /; Head[e] === Power] :=
  If[Head[e[[1]]] === Plus || Head[e[[1]]] === Times || Head[e[[1]]] === Power,
    "(" <> ToMatlabaux[e[[1]]] <> ")",
    ToMatlabaux[e[[1]]] ] <>
  ".^" <>
  If[Length[e] === 2,
    If[Head[e[[2]]] === Plus || Head[e[[2]]] === Times || Head[e[[2]]] === Power,
      "(" <> ToMatlabaux[e[[2]]] <> ")",
      ToMatlabaux[e[[2]]] ],
    ToMatlabaux[ dropfirst[e] ]]

(*** Special cases of functions
*****)

ToMatlabaux[Rule[_ ,r_]] := ToMatlabaux[r]

ToMatlabaux[Log[10, z_]] := "log10(" <> ToMatlabaux[z] <> ")"
ToMatlabaux[Log[b_, z_]] :=
  "log(" <> ToMatlabaux[z] <> ")./log(" <> ToMatlabaux[b] <> ")"

ToMatlabaux[Power[e_, 1/2]] := "sqrt(" <> ToMatlabaux[e] <> ")"
ToMatlabaux[Power[E, z_]] := "exp(" <> ToMatlabaux[z] <> ")"

ToMatlabaux[If[test_, t_, f_]] :=
  Block[{teststr = ToMatlabaux[test]},
    "(" <> teststr <> ").*((" <> ToMatlabaux[t] <> ") + (~("

```



```

    <> teststr <> ")).*(" <> ToMatlabaux[f] <> ")")"]

ToMatlabaux[e_] /; (Head[e] === Max || Head[e] == Min) :=
  ToMatlabaux[Head[e]] <> "(" <>
    If[ Length[e] === 2,
      ToMatlabargs[e] <> ")",
      ToMatlabaux[e[[1]]] <> ", " <> ToMatlabaux[dropfirst[e]] <> ")"]

ToMatlabaux[Colon[a_,b_]] :=
  "(" <> ToMatlabaux[a] <> "):(" <> ToMatlabaux[b] <> ")"
ToMatlabaux[Colon[a_,b_,c_]] :=
  "(" <> ToMatlabaux[a] <> "):(" <> ToMatlabaux[b] <>
  "):(" <> ToMatlabaux[c] <> ")"

(***) General functions
*****

ToMatlabaux[e_] :=
  ToMatlabaux[Head[e]] <> "(" <>
  ToMatlabargs[ argslist[e] ] <> ")"

ToMatlabargs[e_] :=
  If[Length[e] === 1,
    ToMatlabaux[e[[1]]],
    ToMatlabaux[e[[1]]] <> ", " <>
    ToMatlabargs[ argslistdrop[e] ] ]

(***) Argument lists
*****

(***) argslist returns a List of the arguments ***)
argslist[e_] :=
  Block[{ARGSLISTINDEX}, Table[ e[[ARGSLISTINDEX]],
    {ARGSLISTINDEX, 1, Length[e]}]]

(***) argslistdrop returns a List of all arguments except the first one ***)
argslistdrop[e_] :=
  Block[{ARGSLISTINDEX}, Table[ e[[ARGSLISTINDEX]],
    {ARGSLISTINDEX, 2, Length[e]}]]

(***) dropfirst is like argslistdrop but retains the original Head ***)
dropfirst[e_] :=
  e[[ Block[{i}, Table[i, {i,2,Length[e]}]] ] ]

```

```

(***) Folding long lines
*****)

MARGIN = 66
MARGINS = {}

SetMargin[m_] := (MARGINS = Prepend[MARGINS, MARGIN]; MARGIN = m;
MARGINS)

RestoreMargin[] :=
  If[Length[MARGINS] > 0,
    MARGIN = MARGINS[[1]];
    MARGINS = Drop[MARGINS, 1]]

foldlines[s_String] :=
  Block[{cut, sin=s, sout=""},
    While[StringLength[sin] >= MARGIN,
      cut = findcut[sin];
      If[cut > 0,
        sout = sout <> StringTake[sin, cut] <> "...\n ";
        sin = StringDrop[sin, cut],
        (* else *)
        sout = sout <> StringTake[sin, MARGIN];
        sin = StringDrop[sin, MARGIN]]];
    sout <> sin]

findcut[s_String] :=
  Block[{i=MARGIN},
    While[i > 0 &&
      !MemberQ[{",", "(", ")", "+", "*", " "}, StringTake[s, {i}]],
      i--];
    i]

End[]

EndPackage[]

```

Mathematica Code

Written by John Roylance:

Set UP

Grabbing Data

```
<<"\\Users\\John\\Documents\\Mechanical_Engineering\\Robotics and
Automation\\Matrix.m"
```

```
<<"\\Users\\John\\Documents\\Mechanical_Engineering\\Robotics and
Automation\\ToMatlab.m"
```

```

Data=Import["C:\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\SV1\\SV1\\SV1\\2182016\\Drinking.csv"];
The minus five comes in as there are five rows till the actual data begins.
n=Length[Data]-5
1011
0's
Theta1=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering
\\Thesis", "Th1"}], PageWidth→400];
Table[WriteString[Theta1, "th1", i, " ", {i, 7500}];
Close[Theta1];
Theta2=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering
\\Thesis", "Th2"}], PageWidth→400];
Table[WriteString[Theta2, "th2", i, " ", {i, 7500}];
Close[Theta2];
Theta3=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering
\\Thesis", "Th3"}], PageWidth→400];
Table[WriteString[Theta3, "th3", i, " ", {i, 7500}];
Close[Theta3];
TH1={th11,th12,th13,th14,th15,th16,th17,th18,th19,th110,th111,th112,th113,th114,th11
5,th116,th117,th118,th119,th120,th121,th122,th123,th124,th125,th126,th127,th128,th129
,th130,th131,th132,th133,th134,th135,th136,th137,th138,th139,th140,th141,th142,th143,t
h144,th145,th146,th147,th148,th149,th150,th151,th152,th153,th154,th155,th156,th157,th
158,th159,th160,th161,th162,th163,th164,th165,th166,th167,th168,th169,th170,th171,th1
72,th173,th174,th175,th176,th177,th178,th179,th180,th181,th182,th183,th184,th185,th18
6,th187,th188,th189,th190,th191,th192,th193,th194,th195,th196,th197,th198,th199,th110
0,th1101,th1102,th1103,th1104,th1105,th1106,th1107,th1108,th1109,th1110,th1111,th11
12,th1113,th1114,th1115,th1116,th1117,th1118,th1119,th1120,th1121,th1122,th1123,th1
124,th1125,th1126,th1127,th1128,th1129,th1130,th1131,th1132,th1133,th1134,th1135,th
1136,th1137,th1138,th1139,th1140,th1141,th1142,th1143,th1144,th1145,th1146,th1147,t
h1148,th1149,th1150,th1151,th1152,th1153,th1154,th1155,th1156,th1157,th1158,th1159
,th1160,th1161,th1162,th1163,th1164,th1165,th1166,th1167,th1168,th1169,th1170,th117
1,th1172,th1173,th1174,th1175,th1176,th1177,th1178,th1179,th1180,th1181,th1182,th11
83,th1184,th1185,th1186,th1187,th1188,th1189,th1190,th1191,th1192,th1193,th1194,th1
195,th1196,th1197,th1198,th1199,th1200,th1201,th1202,th1203,th1204,th1205,th1206,th
1207,th1208,th1209,th1210,th1211,th1212,th1213,th1214,th1215,th1216,th1217,th1218,t
h1219,th1220,th1221,th1222,th1223,th1224,th1225,th1226,th1227,th1228,th1229,th1230
,th1231,th1232,th1233,th1234,th1235,th1236,th1237,th1238,th1239,th1240,th1241,th124
2,th1243,th1244,th1245,th1246,th1247,th1248,th1249,th1250,th1251,th1252,th1253,th12
54,th1255,th1256,th1257,th1258,th1259,th1260,th1261,th1262,th1263,th1264,th1265,th1
266,th1267,th1268,th1269,th1270,th1271,th1272,th1273,th1274,th1275,th1276,th1277,th
1278,th1279,th1280,th1281,th1282,th1283,th1284,th1285,th1286,th1287,th1288,th1289,t
h1290,th1291,th1292,th1293,th1294,th1295,th1296,th1297,th1298,th1299,th1300,th1301
,th1302,th1303,th1304,th1305,th1306,th1307,th1308,th1309,th1310,th1311,th1312,th131
3,th1314,th1315,th1316,th1317,th1318,th1319,th1320,th1321,th1322,th1323,th1324,th13
25,th1326,th1327,th1328,th1329,th1330,th1331,th1332,th1333,th1334,th1335,th1336,th1

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Sternum

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DataSternM1=Table[Data[[5+i,{3,4,5}]],{i,1,n}];
DataSternM2=Table[Data[[5+i,{6,7,8}]],{i,1,n}];
DataSternM3=Table[Data[[5+i,{9,10,11}]],{i,1,n}];
XStern=Table[(DataSternM1[[i]]-DataSternM2[[i]])/Sqrt[(DataSternM1[[i]]-DataSternM2[[i]])*(DataSternM1[[i]]-DataSternM2[[i]])],{i,1,Length[DataSternM1]}];
YStern=Table[(DataSternM3[[i]]-DataSternM2[[i]])/Sqrt[(DataSternM3[[i]]-DataSternM2[[i]])*(DataSternM3[[i]]-DataSternM2[[i]])],{i,1,Length[DataSternM1]}];
ZStern=Table[XStern[[i]]*YStern[[i]],{i,1,Length[DataSternM1]}];
OTStern=Table[Orthogonalize[{XStern[[i]],YStern[[i]],ZStern[[i]]}],{i,1,Length[XStern]}];
DStern=Table[Append[Transpose[{OTStern[[i,1]],OTStern[[i,2]],OTStern[[i,3]],DataSternM2[[i]]}],{0,0,0,1}],{i,1,Length[XStern]}];
MatrixForm[DStern[[1]]];
Table[Det[DStern[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DStern]}];
dqEStern=Table[HM2dq[DStern[[i]]],{i,1,Length[DStern]}];
plotSternAbs=generatePlotM[DStern,Black,5];
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Scapula

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DataScapM1=Table[Data[[5+i,{18,19,20}]],{i,1,n}];
DataScapM2=Table[Data[[5+i,{21,22,23}]],{i,1,n}];
DataScapM3=Table[Data[[5+i,{24,25,26}]],{i,1,n}];
XScap=Table[(DataScapM1[[i]]-DataScapM2[[i]])/Sqrt[(DataScapM1[[i]]-
DataScapM2[[i]]).(DataScapM1[[i]]-DataScapM2[[i]])],{i,1,Length[DataScapM1]}];
YScap=Table[(DataScapM3[[i]]-DataScapM2[[i]])/Sqrt[(DataScapM3[[i]]-
DataScapM2[[i]]).(DataScapM3[[i]]-DataScapM2[[i]])],{i,1,Length[DataScapM1]}];
ZScap=Table[XScap[[i]]YScap[[i]],{i,1,Length[DataScapM1]}];
OTScap=Table[Orthogonalize[{XScap[[i]],YScap[[i]],ZScap[[i]]}],{i,1,Length[XScap]}];
;
DScap=Table[Append[Transpose[{OTScap[[i,1]],OTScap[[i,2]],OTScap[[i,3]],DataScap
M2[[i]]}],{0,0,0,1}],{i,1,Length[XScap]}];
MatrixForm[DScap[[1]]];
Table[Det[DScap[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DScap]}];
dqEScap=Table[HM2dq[DScap[[i]]],{i,1,Length[DScap]}];
plotScapAbs=generatePlotM[DScap,Purple,1];
Upper Arm
DataUpArmM1=Table[Data[[5+i,{30,31,32}]],{i,1,n}];
DataUpArmM2=Table[Data[[5+i,{33,34,35}]],{i,1,n}];
DataUpArmM3=Table[Data[[5+i,{36,37,38}]],{i,1,n}];
XUpArm=Table[(DataUpArmM1[[i]]-DataUpArmM2[[i]])/Sqrt[(DataUpArmM1[[i]]-
DataUpArmM2[[i]]).(DataUpArmM1[[i]]-
DataUpArmM2[[i]])],{i,1,Length[DataUpArmM1]}];
YUpArm=Table[(DataUpArmM3[[i]]-DataUpArmM2[[i]])/Sqrt[(DataUpArmM3[[i]]-
DataUpArmM2[[i]]).(DataUpArmM3[[i]]-
DataUpArmM2[[i]])],{i,1,Length[DataUpArmM1]}];
ZUpArm=Table[XUpArm[[i]]YUpArm[[i]],{i,1,Length[DataUpArmM1]}];
OTUpArm=Table[Orthogonalize[{XUpArm[[i]],YUpArm[[i]],ZUpArm[[i]]}],{i,1,Lengt
h[XUpArm]}];
DUpArm=Table[Append[Transpose[{OTUpArm[[i,1]],OTUpArm[[i,2]],OTUpArm[[i,3]
],DataUpArmM2[[i]]}],{0,0,0,1}],{i,1,Length[XUpArm]}];
MatrixForm[DUpArm[[1]]];
Table[Det[DUpArm[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DUpArm]}];
dqEUpArm=Table[HM2dq[DUpArm[[i]]],{i,1,Length[DUpArm]}];
plotUpArmAbs=generatePlotM[DUpArm,Blue,5];
ForeArm
DataForeArmM1=Table[Data[[5+i,{45,46,47}]],{i,1,n}];
DataForeArmM2=Table[Data[[5+i,{48,49,50}]],{i,1,n}];
DataForeArmM3=Table[Data[[5+i,{51,52,53}]],{i,1,n}];
XForeArm=Table[(DataForeArmM1[[i]]-
DataForeArmM2[[i]])/Sqrt[(DataForeArmM1[[i]]-
DataForeArmM2[[i]]).(DataForeArmM1[[i]]-
DataForeArmM2[[i]])],{i,1,Length[DataForeArmM1]}];
YForeArm=Table[(DataForeArmM3[[i]]-
DataForeArmM2[[i]])/Sqrt[(DataForeArmM3[[i]]-
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DataForeArmM2[[i]]).(DataForeArmM3[[i]]-
DataForeArmM2[[i]]),{i,1,Length[DataForeArmM1]}}];
ZForeArm=Table[XForeArm[[i]]□YForeArm[[i]],{i,1,Length[DataForeArmM1]}}];
OTForeArm=Table[Orthogonalize[{XForeArm[[i]],YForeArm[[i]],ZForeArm[[i]]}],{i,1,
Length[XForeArm]}}];
DForeArm=Table[Append[Transpose[{OTForeArm[[i,1]],OTForeArm[[i,2]],OTForeAr
m[[i,3]],DataForeArmM2[[i]]}],{0,0,0,1}],{i,1,Length[XForeArm]}}];
MatrixForm[DForeArm[[1]]];
Table[Det[DForeArm[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DForeArm]}}];
dqEForeArm=Table[HM2dq[DForeArm[[i]]],{i,1,Length[DForeArm]}}];
plotForeArmAbs=generatePlotM[DForeArm,Green,5];
MetaCarple
DataMetaCarpleM1=Table[Data[[5+i,{54,55,56}]],{i,1,n}];
DataMetaCarpleM2=Table[Data[[5+i,{57,58,59}]],{i,1,n}];
DataMetaCarpleM3=Table[Data[[5+i,{60,61,62}]],{i,1,n}];
XMetaCarp=Table[(DataMetaCarpleM1[[i]]-
DataMetaCarpleM2[[i]])/Sqrt[(DataMetaCarpleM1[[i]]-
DataMetaCarpleM2[[i]]).(DataMetaCarpleM1[[i]]-
DataMetaCarpleM2[[i]])],{i,1,Length[DataMetaCarpleM1]}}];
YMetaCarp=Table[(DataMetaCarpleM3[[i]]-
DataMetaCarpleM2[[i]])/Sqrt[(DataMetaCarpleM3[[i]]-
DataMetaCarpleM2[[i]]).(DataMetaCarpleM3[[i]]-
DataMetaCarpleM2[[i]])],{i,1,Length[DataMetaCarpleM1]}}];
ZMetaCarp=Table[XMetaCarp[[i]]□YMetaCarp[[i]],{i,1,Length[DataMetaCarpleM1]}}];
OTMetaCarp=Table[Orthogonalize[{XMetaCarp[[i]],YMetaCarp[[i]],ZMetaCarp[[i]]}],{
i,1,Length[XMetaCarp]}}];
DMetaCarp=Table[Append[Transpose[{OTMetaCarp[[i,1]],OTMetaCarp[[i,2]],OTMeta
Carp[[i,3]],DataMetaCarpleM2[[i]]}],{0,0,0,1}],{i,1,Length[XMetaCarp]}}];
MatrixForm[DMetaCarp[[1]]];
Table[Det[DMetaCarp[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DMetaCarp]}}];
dqEMetaCarp=Table[HM2dq[DMetaCarp[[i]]],{i,1,Length[DMetaCarp]}}];
plotMetaCarpAbs=generatePlotM[DMetaCarp,Pink,5];
ThumbS1 (needs L frame)-S3
DataThumbS1M1=Table[Data[[5+i,{69,70,71}]],{i,1,n}];(*Point Proximal on Thumb*);
DataThumbS1M2=Table[Data[[5+i,{63,64,65}]],{i,1,n}];(*Point Distal on Thumb*);
DataThumbS1M3=Table[Data[[5+i,{66,67,68}]],{i,1,n}];(*Point "below" Thumb*);
DataThumbS2M1=Table[Data[[5+i,{72,73,74}]],{i,1,n}];
DataThumbS3M1=Table[Data[[5+i,{75,76,77}]],{i,1,n}];
DataThumbS4M1=Table[Data[[5+i,{78,79,80}]],{i,1,n}];
XThumb=Table[(DataThumbS1M1[[i]]-
DataThumbS1M2[[i]])/Sqrt[(DataThumbS1M1[[i]]-
DataThumbS1M2[[i]]).(DataThumbS1M1[[i]]-
DataThumbS1M2[[i]])],{i,1,Length[DataThumbS1M1]}}];
YThumb=Table[(DataThumbS1M3[[i]]-
DataThumbS1M2[[i]])/Sqrt[(DataThumbS1M3[[i]]-

```

```

DataThumbS1M2[[i]]).(DataThumbS1M3[[i]]-
DataThumbS1M2[[i]]),{i,1,Length[DataThumbS1M1]}}];
ZThumb=Table[XThumb[[i]]YThumb[[i]],{i,1,Length[DataThumbS1M1]}}];
OTThumb=Table[Orthogonalize[{XThumb[[i]],YThumb[[i]],ZThumb[[i]]},{i,1,Length[
XThumb]}}];
DThumb=Table[Append[Transpose[{OTThumb[[i,1]],OTThumb[[i,2]],OTThumb[[i,3]],
DataThumbS1M2[[i]]},{0,0,0,1}],{i,1,Length[XThumb]}}];
MatrixForm[DThumb[[1]]];
Table[Det[DThumb[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DThumb]}}];
dqEThumb=Table[HM2dq[DThumb[[i]]],{i,1,Length[DThumb]}}];
plotThumbAbs=generatePlotM[DThumb,Pink,5];
IndexS1-S4
DataIndexS1M1=Table[Data[[5+i,{60,61,62}]],{i,1,n}];(*Point Proximal on Finger*);
DataIndexS1M2=Table[Data[[5+i,{81,82,83}]],{i,1,n}];(*Point Distal on Finger*);
DataIndexS2M1=Table[Data[[5+i,{84,85,86}]],{i,1,n}];
DataIndexS3M1=Table[Data[[5+i,{87,88,89}]],{i,1,n}];
DataIndexS4M1=Table[Data[[5+i,{90,91,92}]],{i,1,n}];
DataIndexS1M3=Table[Cross[DataIndexS1M1[[i]]-
DataIndexS1M2[[i]],DataIndexS1M2[[i]]-DataIndexS2M1[[i]]],{i,1,n}];
XIndex=Table[(DataIndexS1M1[[i]]-DataIndexS1M2[[i]])/Sqrt[(DataIndexS1M1[[i]]-
DataIndexS1M2[[i]]).(DataIndexS1M1[[i]]-
DataIndexS1M2[[i]])],{i,1,Length[DataIndexS1M1]}}];
YIndex=Table[(DataIndexS1M3[[i]]-DataIndexS1M2[[i]])/Sqrt[(DataIndexS1M3[[i]]-
DataIndexS1M2[[i]]).(DataIndexS1M3[[i]]-
DataIndexS1M2[[i]])],{i,1,Length[DataIndexS1M1]}}];
ZIndex=Table[XIndex[[i]]YIndex[[i]],{i,1,Length[DataIndexS1M1]}}];
OTIndex=Table[Orthogonalize[{XIndex[[i]],YIndex[[i]],ZIndex[[i]]},{i,1,Length[XInd
ex]}}];
DIndex=Table[Append[Transpose[{OTIndex[[i,1]],OTIndex[[i,2]],OTIndex[[i,3]],DataIn
dexS1M2[[i]]},{0,0,0,1}],{i,1,Length[XIndex]}}];
MatrixForm[DIndex[[1]]];
Table[Det[DIndex[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DIndex]}}];
dqEIndex=Table[HM2dq[DIndex[[i]]],{i,1,Length[DIndex]}}];
plotIndexAbs=generatePlotM[DIndex,Pink,5];
RingS1
DataRingS1M1=Table[Data[[5+i,{57,58,59}]],{i,1,n}];(*Point Proximal on Finger*);
DataRingS1M2=Table[Data[[5+i,{93,94,95}]],{i,1,n}];(*Point Distal on Finger*);
DataRingS2M1=Table[Data[[5+i,{96,97,98}]],{i,1,n}];
DataRingS3M1=Table[Data[[5+i,{99,100,101}]],{i,1,n}];
DataRingS4M1=Table[Data[[5+i,{102,103,104}]],{i,1,n}];
DataRingS1M3=Table[Cross[DataRingS1M1[[i]]-
DataRingS1M2[[i]],DataRingS1M2[[i]]-DataRingS2M1[[i]]],{i,1,n}];
XRing=Table[(DataRingS1M1[[i]]-DataRingS1M2[[i]])/Sqrt[(DataRingS1M1[[i]]-
DataRingS1M2[[i]]).(DataRingS1M1[[i]]-
DataRingS1M2[[i]])],{i,1,Length[DataRingS1M1]}}];

```

```

YRing=Table[(DataRingS1M3[[i]]-DataRingS1M2[[i]])/Sqrt[(DataRingS1M3[[i]]-
DataRingS1M2[[i]])*(DataRingS1M3[[i]]-
DataRingS1M2[[i]])],{i,1,Length[DataRingS1M1]}}];
ZRing=Table[XRing[[i]]□YRing[[i]],{i,1,Length[DataRingS1M1]}}];
OTRing=Table[Orthogonalize[{XRing[[i]],YRing[[i]],ZRing[[i]]}],{i,1,Length[XRing]
}];
DRing=Table[Append[Transpose[{OTRing[[i,1]],OTRing[[i,2]],OTRing[[i,3]],DataRing
S1M2[[i]]}],{0,0,0,1}],{i,1,Length[XRing]}}];
MatrixForm[DRing[[1]]];
Table[Det[DRing[[i,{1,2,3},{1,2,3}]]],{i,1,Length[DRing]}}];
dqERing=Table[HM2dq[DRing[[i]]],{i,1,Length[DRing]}}];
plotRingAbs=generatePlotM[DRing,Pink,5];
Plots
Show[plotSternAbs];
Show[plotScapAbs];
Show[plotUpArmAbs];
Show[plotForeArmAbs];
Show[plotMetaCarpAbs];
Show[{plotSternAbs,plotScapAbs,plotUpArmAbs,plotForeArmAbs,plotMetaCarpAbs}]
Scapula to UpArm (GH joint)
Scap to UpArm (GH joint)
CRScUA=Table[Inverse[DScap[[i]]].DUpArm[[i]],{i,1,n}];
RMScUA=Table[CRScUA[[i+1]].Inverse[CRScUA[[1]]],{i,1,n-1}];
Append[Transpose[Append[Transpose[{
{Cos[θ1], -Sin[θ1], 0},
{Sin[θ1], Cos[θ1], 0},
{0, 0, 1}
}].{
{Cos[θ2], 0, -Sin[θ2]},
{0, 1, 0},
{Sin[θ2], 0, Cos[θ2]}
}].{
{1, 0, 0},
{0, Cos[θ3], -Sin[θ3]},
{0, Sin[θ3], Cos[θ3]}
}],{x,y,z}],{0,0,0,1}]]//MatrixForm
ScUAjointv={
{"cos(x(1)).* cos(x(2))", "cos(x(2)).*sin(x(1))", "-sin(x(2))", "x(4)"},
{"-cos(x(3)).*sin(x(1)) - cos(x(1)).*sin(x(2)).*sin(x(3))", "cos(x(1)).*cos(x(3)) -
sin(x(1)).*sin(x(2)).*sin(x(3))", "cos(x(2)).*sin(x(3))", "x(5)"},
{"-cos(x(1)).*cos(x(3)).*sin(x(2)) + sin(x(1)).*sin(x(3))", "-
cos(x(3)).*sin(x(1)).*sin(x(2)) - cos(x(1)).*sin(x(3))", "cos(x(2)).*cos(x(3))", "x(6)"},
{0, 0, 0, 1}
});
ScUAjointv//MatrixForm
To Matlab

```

```
handrun=OpenWrite[FileNameJoin[{ ""\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking", "RMScUAFunc.m"}], PageWidth→400];
```

```
WriteString[handrun, "function G = RMScUAFunc"]
```

```
Table[WriteString[handrun, "
```

```
  G.AnsM", i, "= @AnsM", i, ";", {i, n-1}];
```

```
WriteString[handrun, "end"]
```

```
Table[WriteString[handrun, "
```

```
  function F", i, " = AnsM", i, "(x)
```

```
  F", i, "(1) = ", ScUAjointv[[1,1]], " - (" , RMScUA[[i,1,1]], ");
```

```
  F", i, "(2) = ", ScUAjointv[[1,2]], " - (" , RMScUA[[i,1,2]], ");
```

```
  F", i, "(3) = ", ScUAjointv[[1,3]], " - (" , RMScUA[[i,1,3]], ");
```

```
  F", i, "(4) = ", ScUAjointv[[1,4]], " - (" , RMScUA[[i,1,4]], ");
```

```
  F", i, "(5) = ", ScUAjointv[[2,1]], " - (" , RMScUA[[i,2,1]], ");
```

```
  F", i, "(6) = ", ScUAjointv[[2,2]], " - (" , RMScUA[[i,2,2]], ");
```

```
  F", i, "(7) = ", ScUAjointv[[2,3]], " - (" , RMScUA[[i,2,3]], ");
```

```
  F", i, "(8) = ", ScUAjointv[[2,4]], " - (" , RMScUA[[i,2,4]], ");
```

```
  F", i, "(9) = ", ScUAjointv[[3,1]], " - (" , RMScUA[[i,3,1]], ");
```

```
  F", i, "(10) = ", ScUAjointv[[3,2]], " - (" , RMScUA[[i,3,2]], ");
```

```
  F", i, "(11) = ", ScUAjointv[[3,3]], " - (" , RMScUA[[i,3,3]], ");
```

```
  F", i, "(12) = ", ScUAjointv[[3,4]], " - (" , RMScUA[[i,3,4]], ");
```

```
  end
```

```
  ", {i, n-1}];
```

```
Close[handrun]
```

```
ClearAll[i]
```

```
handrun=OpenWrite[FileNameJoin[{ ""\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking", "RMScUASol.m"}], PageWidth→400];
```

```
WriteString[handrun, "myfuns=RMScUAFunc;"]
```

```
Table[WriteString[handrun, "%% Solution ", i, "
```

```
  func = @myfuns.AnsM", i, ";
```

```
  x0 = [.1, .1, .1, 10, 10, 10];
```

```
  x", i, " = fsolve(func, x0);
```

```
  ", {i, n-1}];
```

```
WriteString[handrun, "%% MatrixForm
```

```
  XScUA=["];
```

```
Table[WriteString[handrun, "x", i, ";", {i, n-2}];
```

```
WriteString[handrun, "x", n-1];
```

```
WriteString[handrun, "];
```

```
  xlswrite('SolutionsScUA', XScUA)");
```

```
Close[handrun]
```

```
ClearAll[i]
```

```
Results
```

```

SolScUA =
Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\SV1\\Drinking\\SolutionsScUA.xls"],1];
Eigenvalues
Eigensystem[(Transpose[Table[SolScUA[[i,{1,2,3}]],{i,n-
1}]].Table[SolScUA[[i,{1,2,3}]],{i,n-1}]]/(n-1)//MatrixForm
Histograms
Histogram[{Flatten[Table[180/π SolScUA[[i,1]],{i,1,n-1}],2],
  Flatten[Table[180/π SolScUA[[i,2]],{i,1,n-1}],2],
  Flatten[Table[180/π SolScUA[[i,3]],{i,1,n-1}],2]},{1},PlotLabel→"GH Joint All
Angles",AxesLabel→{"Angles (Degrees)","Quantity"}]
Histogram[Flatten[Table[180/π SolScUA[[i,1]],{i,1,n-1}],2],{1},PlotLabel→"GH Joint
θ1 (x-axis)",AxesLabel→{"Angles (Degrees)","Quantity"}]
Histogram[Flatten[Table[180/π SolScUA[[i,2]],{i,1,n-1}],2],{.000001},PlotLabel→"GH
Joint θ2 (y-axis)",AxesLabel→{"Angles (Degrees)","Quantity"}]
Histogram[Flatten[Table[180/π SolScUA[[i,3]],{i,1,n-1}],2],{1},PlotLabel→"GH Joint
θ3 (z-axis)",AxesLabel→{"Angles (Degrees)","Quantity"}]
UpArm to Forearm (Elbow and UlnaRadius)
UpArm to Forearm (Elbow and UlnaRadius)
CRUAFa=Table[Inverse[DUpArm[[i]]].DForeArm[[i]],{i,1,n}];
RMUAFa=Table[CRUAFa[[i+1]].Inverse[CRUAFa[[1]]],{i,1,n-1}];
Show[generatePlotM[CRUAFa,Blue,5]];
SE1=makequat[{s1x,s1y,s1z},{s1x0,s1y0,s1z0}],th1,0];
SE2=makequat[{s2x,s2y,s2z},{s2x0,s2y0,s2z0}],th2,0];
dqE=quatmultSep[SE1,SE2]//Simplify;
To Matlab
Solve Axes
KEUAFAS1=dqE-HM2dq[RMUAFa[[Round[1 (n-1)/10]]]]/.{th1→TH1[[Round[1 (n-
1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KEUAFAS2=dqE-HM2dq[RMUAFa[[Round[2 (n-1)/10]]]]/.{th1→TH1[[Round[2 (n-
1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KEUAFAS3=dqE-HM2dq[RMUAFa[[Round[3 (n-1)/10]]]]/.{th1→TH1[[Round[3 (n-
1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KEUAFAS4=dqE-HM2dq[RMUAFa[[Round[4 (n-1)/10]]]]/.{th1→TH1[[Round[4 (n-
1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KEUAFAS5=dqE-HM2dq[RMUAFa[[Round[5 (n-1)/10]]]]/.{th1→TH1[[Round[5 (n-
1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KEUAFAS6=dqE-HM2dq[RMUAFa[[Round[6 (n-1)/10]]]]/.{th1→TH1[[Round[6 (n-
1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KEUAFAS7=dqE-HM2dq[RMUAFa[[Round[7 (n-1)/10]]]]/.{th1→TH1[[Round[7 (n-
1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KEUAFAS8=dqE-HM2dq[RMUAFa[[Round[8 (n-1)/10]]]]/.{th1→TH1[[Round[8 (n-
1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KEUAFAS9=dqE-HM2dq[RMUAFa[[Round[9 (n-1)/10]]]]/.{th1→TH1[[Round[9 (n-
1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};

```



```

KEUAFAS10=dqE-HM2dq[RMUAFa[[Round[n-1]]]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
KEUAFASS1=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[1 (n-1)/10]]]]2]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KEUAFASS2=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[2 (n-1)/10]]]]2]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KEUAFASS3=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[3 (n-1)/10]]]]2]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KEUAFASS4=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[4 (n-1)/10]]]]2]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KEUAFASS5=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[5 (n-1)/10]]]]2]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KEUAFASS6=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[6 (n-1)/10]]]]2]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KEUAFASS7=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[7 (n-1)/10]]]]2]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KEUAFASS8=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[8 (n-1)/10]]]]2]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KEUAFASS9=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[9 (n-1)/10]]]]2]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KEUAFASS10=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[n-1]]]]2]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","UAFaAxisF.m"}],PageWidth→4000];
WriteString[handrun,"function y = UAFaAxisF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    [s1x,s1y,s1z,s1x0,s1y0,s1z0,...
     s2x,s2y,s2z,s2x0,s2y0,s2z0,...
     ];
    Table[WriteString[handrun,"th1",Round[i*(n-1)/10], " ", "],{i,10}];
    WriteString[handrun,"...
    "]
    Table[WriteString[handrun,"th2",Round[i*(n-1)/10], " ", "],{i,9}];
    WriteString[handrun,"th2",n-1]
    WriteString[handrun,""] = cll{:};

    y=""
    WriteString[handrun,ToMatlab[{KEUAFAS1,KEUAFAS2,KEUAFAS3,KEUAFAS4,KEUAFAS5,KEUAFAS6,KEUAFAS7,KEUAFAS8,KEUAFAS9,KEUAFAS10}]];
    Close[handrun]
    handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","UAFaAxisS.m"}],PageWidth→400];

    WriteString[handrun,"func = @UAFaAxisF;

```

```

x0 = [""]
Table[WriteString[handrun, ".1, ", {i, 2(10)+11}];
WriteString[handrun, ".1];
x = fsolve(func, x0);
"];
WriteString[handrun, "%% MatrixForm
SAUAFa = x;
xlswrite('AUAFa', SAUAFa)"];
Close[handrun]
ClearAll[i]
Solver angles
SolUAFAS=Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis
\\Vicon Nexus\\SV1\\Drinking\\AUAFa.xls"], 2];
KEUAFa1=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i, Round
[1 (n-1)/30]}];
KEUAFa2=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i, Round
[1 (n-1)/30], Round[2 (n-1)/30]}];
KEUAFa3=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i, Round
[2 (n-1)/30], Round[3 (n-1)/30]}];
KEUAFa4=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i, Round
[3 (n-1)/30], Round[4 (n-1)/30]}];
KEUAFa5=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i, Round
[4 (n-1)/30], Round[5 (n-1)/30]}];
KEUAFa6=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1

```

$y \rightarrow \text{SolUAFAS}[[2]], s1z \rightarrow \text{SolUAFAS}[[3]], s1x0 \rightarrow \text{SolUAFAS}[[4]], s1y0 \rightarrow \text{SolUAFAS}[[5]]$
 $, s1z0 \rightarrow \text{SolUAFAS}[[6]], s2x \rightarrow \text{SolUAFAS}[[7]], s2y \rightarrow \text{SolUAFAS}[[8]], s2z \rightarrow \text{SolUAFAS}[[9]]$
 $, s2x0 \rightarrow \text{SolUAFAS}[[10]], s2y0 \rightarrow \text{SolUAFAS}[[11]], s2z0 \rightarrow \text{SolUAFAS}[[12]]\}, \{i, \text{Round}$
 $[5 (n-1)/30], \text{Round}[6 (n-1)/30]\}$ };
 KEUAFA7=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [6 (n-1)/30], Round[7 (n-1)/30]}}];
 KEUAFA8=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [7 (n-1)/30], Round[8 (n-1)/30]}}];
 KEUAFA9=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [8 (n-1)/30], Round[9 (n-1)/30]}}];
 KEUAFA10=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [9 (n-1)/30], Round[10 (n-1)/30]}}];
 KEUAFA11=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [10 (n-1)/30], Round[11 (n-1)/30]}}];
 KEUAFA12=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [11 (n-1)/30], Round[12 (n-1)/30]}}];
 KEUAFA13=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[

9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [12 (n-1)/30],Round[13 (n-1)/30]}}];
 KEUAFA14=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [13 (n-1)/30],Round[14 (n-1)/30]}}];
 KEUAFA15=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [14 (n-1)/30],Round[15 (n-1)/30]}}];
 KEUAFA16=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [15 (n-1)/30],Round[16 (n-1)/30]}}];
 KEUAFA17=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [16 (n-1)/30],Round[17 (n-1)/30]}}];
 KEUAFA18=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [17 (n-1)/30],Round[18 (n-1)/30]}}];
 KEUAFA19=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [18 (n-1)/30],Round[19 (n-1)/30]}}];
 KEUAFA20=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [19 (n-1)/30],Round[20 (n-1)/30]}}];

KEUAFA21=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [20 (n-1)/30],Round[21 (n-1)/30]}];
 KEUAFA22=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [21 (n-1)/30],Round[22 (n-1)/30]}];
 KEUAFA23=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [22 (n-1)/30],Round[23 (n-1)/30]}];
 KEUAFA24=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [23 (n-1)/30],Round[24 (n-1)/30]}];
 KEUAFA25=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [24 (n-1)/30],Round[25 (n-1)/30]}];
 KEUAFA26=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [25 (n-1)/30],Round[26 (n-1)/30]}];
 KEUAFA27=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [26 (n-1)/30],Round[27 (n-1)/30]}];
 KEUAFA28=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1

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y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
,s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]]
,s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
[27 (n-1)/30],Round[28 (n-1)/30]}}];
KEUAFA29=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
,s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]]
,s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
[28 (n-1)/30],Round[29 (n-1)/30]}}];
KEUAFA30=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
,s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]]
,s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
[29 (n-1)/30],Round[n-1]}}];
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","UAFaAngleF.m"}],PageWidth→4000];
WriteString[handrun,"function y = UAFaAngleF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    ["];
    Table[WriteString[handrun,"th1",i, ", ", "{i,n-1}"];
    WriteString[handrun,"...
    "]
    Table[WriteString[handrun,"th2",i, ", ", "{i,n-2}"];
    WriteString[handrun,"th2",n-1]
    WriteString[handrun,""] = cll{:};

    y
    =" ,ToMatlab[{KEUAFA1,KEUAFA2,KEUAFA3,KEUAFA4,KEUAFA5,KEUAFA6,KEUAFA7,KEUAFA8,KEUAFA9,KEUAFA10,KEUAFA11,KEUAFA12,KEUAFA13,KEUAFA14,KEUAFA15,KEUAFA16,KEUAFA17,KEUAFA18,KEUAFA19,KEUAFA20,KEUAFA21,KEUAFA22,KEUAFA23,KEUAFA24,KEUAFA25,KEUAFA26,KEUAFA27,KEUAFA28,KEUAFA29,KEUAFA30}]];
    Close[handrun]
    handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","UAFaAngleS.m"}],PageWidth→400];

    WriteString[handrun,"func = @UAFaAngleF;
    x0 = ["]
    Table[WriteString[handrun,".1, ", "{i,2(n-1)-1}"];
    WriteString[handrun,".1];
    x = fsolve(func, x0,saoptimset('TolFun',.001));
    "];

```

```

WriteString[handrun, "%% MatrixForm
SSUAFa = x;
xlswrite('SUAFa.xlsx', SSUAFa)"];
Close[handrun]
ClearAll[i]
Results
SolUAFa =
Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\SV1\\Drinking\\SUAFa.xlsx"], 2];
SolUAFa01 = Table[ArcSin[Sin[SolUAFa[[i]]]], {i, n-1}];
SolUAFa02 = Table[ArcCos[Cos[SolUAFa[[i]]]], {i, n, 2(n-1)}];
MatSolUAFa = {SolUAFa01, SolUAFa02};
UAFaCheck = Table[(dqE-
HM2dq[RMUAFa[[Round[i]]]])/.{s1x→SolUAFAS[[1]], s1y→SolUAFAS[[2]], s1z→Sol
UAFAS[[3]], s1x0→SolUAFAS[[4]], s1y0→SolUAFAS[[5]], s1z0→SolUAFAS[[6]], s2x
→SolUAFAS[[7]], s2y→SolUAFAS[[8]], s2z→SolUAFAS[[9]], s2x0→SolUAFAS[[10]],
s2y0→SolUAFAS[[11]], s2z0→SolUAFAS[[12]], th1→SolUAFa01[[i]], th2→SolUAFa0
2[[i]]}, {i, 1, n-1}];
Mean[Flatten[UAFaCheck, 2]]
StandardDeviation[Flatten[UAFaCheck, 2]]
Eigenvalues
Eigensystem[MatSolUAFa.Transpose[MatSolUAFa]]//MatrixForm
Histograms
Histogram[{SolUAFa01 180/π,
SolUAFa02 180/π}, {1}, PlotLabel→"Elbow and UlnaRadius All
Angles", AxesLabel→{"Angles (Degrees)", "Quantity"}]
Histogram[SolUAFa01 180/π, {1}, PlotLabel→"Ulna/Radius θ", AxesLabel→{"Angles
(Degrees)", "Quantity"}]
Histogram[SolUAFa02 180/π, {1}, PlotLabel→"Elbow θ", AxesLabel→{"Angles
(Degrees)", "Quantity"}]
Wrist
Forearm to Metacarpale (Wrist)
CRW = Table[Inverse[DForeArm[[i]]].DMetaCarp[[i]], {i, 1, n}];
RMW = Table[CRW[[i+1]].Inverse[CRW[[1]]], {i, 1, n-1}];
Show[generatePlotM[CRW, Blue, 5]];
SW1 = makequat[{s1x, s1y, s1z}, {s1x0, s1y0, s1z0}], th1, 0];
SW2 = makequat[{s2x, s2y, s2z}, {s2x0, s2y0, s2z0}], th2, 0];
dqW = quatmultSep[SW1, SW2]//Simplify;
To Matlab
Solve Axes
KEWS1 = dqW - HM2dq[RMW[[Round[1 (n-1)/10]]]]/.{th1→TH1[[Round[1 (n-
1)/10]]], th2→TH2[[Round[1 (n-1)/10]]]};
KEWS2 = dqW - HM2dq[RMW[[Round[2 (n-1)/10]]]]/.{th1→TH1[[Round[2 (n-
1)/10]]], th2→TH2[[Round[2 (n-1)/10]]]};

```

```

KEWS3=dqW-HM2dq[RMW[[Round[3 (n-1)/10]]]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KEWS4=dqW-HM2dq[RMW[[Round[4 (n-1)/10]]]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KEWS5=dqW-HM2dq[RMW[[Round[5 (n-1)/10]]]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KEWS6=dqW-HM2dq[RMW[[Round[6 (n-1)/10]]]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KEWS7=dqW-HM2dq[RMW[[Round[7 (n-1)/10]]]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KEWS8=dqW-HM2dq[RMW[[Round[8 (n-1)/10]]]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KEWS9=dqW-HM2dq[RMW[[Round[9 (n-1)/10]]]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KEWS10=dqW-HM2dq[RMW[[Round[n-1]]]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking", "WAxisF.m"}],PageWidth→4000];
WriteString[handrun,"function y = WAxisF(x)

```

```

    cll = mat2cell(x,1,ones(1,length(x)));

```

```

    [s1x,s1y,s1z,s1x0,s1y0,s1z0,...
    s2x,s2y,s2z,s2x0,s2y0,s2z0,...
    "];
Table[WriteString[handrun,"th1",Round[i*(n-1)/10], " ", "{i,10}"];
WriteString[handrun,"...
" ]
Table[WriteString[handrun,"th2",Round[i*(n-1)/10], " ", "{i,9}"];
WriteString[handrun,"th2",n-1]
WriteString[handrun,""] = cll{:};

```

```

y=""
WriteString[handrun,ToMatlab[{KEWS1,KEWS2,KEWS3,KEWS4,KEWS5,KEWS6,KEWS7,KEWS8,KEWS9,KEWS10}]];
Close[handrun]
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking", "WAxisS.m"}],PageWidth→400];

```

```

WriteString[handrun,"func = @WAxisF;
x0 = []
Table[WriteString[handrun, ".1, ", {i,2(10)+11}];
WriteString[handrun, ".1];
x = fsolve(func, x0);
"];

```



```

WriteString[handrun,"%% MatrixForm
SAW = x;
xlswrite('AW',SAW)"];
Close[handrun]
ClearAll[i]
Solver angles
SolWS=Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vi
con Nexus\\SV1\\Drinking\\AW.xls"],2];
KEW1=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[1 (n-1)/30]}];
KEW2=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[1 (n-1)/30],Round[2 (n-1)/30]}];
KEW3=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[2 (n-1)/30],Round[3 (n-1)/30]}];
KEW4=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[3 (n-1)/30],Round[4 (n-1)/30]}];
KEW5=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[4 (n-1)/30],Round[5 (n-1)/30]}];
KEW6=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[5 (n-1)/30],Round[6 (n-1)/30]}];
KEW7=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[6 (n-1)/30],Round[7 (n-1)/30]}];
KEW8=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol

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$WS[[2]],s1z \rightarrow SolWS[[3]],s1x0 \rightarrow SolWS[[4]],s1y0 \rightarrow SolWS[[5]],s1z0 \rightarrow SolWS[[6]],s2x \rightarrow SolWS[[7]],s2y \rightarrow SolWS[[8]],s2z \rightarrow SolWS[[9]],s2x0 \rightarrow SolWS[[10]],s2y0 \rightarrow SolWS[[11]],s2z0 \rightarrow SolWS[[12]]\},\{i, Round[7 (n-1)/30], Round[8 (n-1)/30]\}$ };
 KEW9=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[8 (n-1)/30], Round[9 (n-1)/30]\}];
 KEW10=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[9 (n-1)/30], Round[10 (n-1)/30]\}];
 KEW11=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[10 (n-1)/30], Round[11 (n-1)/30]\}];
 KEW12=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[11 (n-1)/30], Round[12 (n-1)/30]\}];
 KEW13=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[12 (n-1)/30], Round[13 (n-1)/30]\}];
 KEW14=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[13 (n-1)/30], Round[14 (n-1)/30]\}];
 KEW15=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[14 (n-1)/30], Round[15 (n-1)/30]\}];
 KEW16=Table[dqW-
 HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]]\},\{i, Round[15 (n-1)/30], Round[16 (n-1)/30]\}];

KEW17=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[16 (n-1)/30],Round[17 (n-1)/30]}];
KEW18=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[17 (n-1)/30],Round[18 (n-1)/30]}];
KEW19=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[18 (n-1)/30],Round[19 (n-1)/30]}];
KEW20=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[19 (n-1)/30],Round[20 (n-1)/30]}];
KEW21=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[20 (n-1)/30],Round[21 (n-1)/30]}];
KEW22=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[21 (n-1)/30],Round[22 (n-1)/30]}];
KEW23=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[22 (n-1)/30],Round[23 (n-1)/30]}];
KEW24=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[23 (n-1)/30],Round[24 (n-1)/30]}];
KEW25=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x

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→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[24 (n-1)/30],Round[25 (n-1)/30]}];
KEW26=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[25 (n-1)/30],Round[26 (n-1)/30]}];
KEW27=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[26 (n-1)/30],Round[27 (n-1)/30]}];
KEW28=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[27 (n-1)/30],Round[28 (n-1)/30]}];
KEW29=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[28 (n-1)/30],Round[29 (n-1)/30]}];
KEW30=Table[dqW-
HM2dq[RMW[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolWS[[1]],s1y→Sol
WS[[2]],s1z→SolWS[[3]],s1x0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x
→SolWS[[7]],s2y→SolWS[[8]],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[1
1]],s2z0→SolWS[[12]]},{i,Round[29 (n-1)/30],Round[n-1]}];
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","WAngleF.m"}],PageWidth→4000];
WriteString[handrun,"function y = WAngleF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    ["];
    Table[WriteString[handrun,"th1",i, ", ", "],{i,n-1}];
    WriteString[handrun,"...
    "]
    Table[WriteString[handrun,"th2",i, ", ", "],{i,n-2}];
    WriteString[handrun,"th2",n-1]
    WriteString[handrun,""] = cll{:};

    y
    =" ,ToMatlab[{KEW1,KEW2,KEW3,KEW4,KEW5,KEW6,KEW7,KEW8,KEW9,KEW1
0,KEW11,KEW12,KEW13,KEW14,KEW15,KEW16,KEW17,KEW18,KEW19,KEW20

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,KEW21,KEW22,KEW23,KEW24,KEW25,KEW26,KEW27,KEW28,KEW29,KEW30}
]];
Close[handrun]
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","WAngleS.m"}],PageWidth→400];

WriteString[handrun,"func = @WAngleF;
x0 = []
Table[WriteString[handrun, ".1, "],{i,2(n-1)-1}];
WriteString[handrun, ".1];
x = fsolve(func, x0,saoptimset('TolFun',.001));
"];
WriteString[handrun,"%% MatrixForm
xlswrite('SW.xlsx',x)"];
Close[handrun]
ClearAll[i]
Results
SolW =
Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\SV1\\Drinking\\SW.xlsx"],2];
SolW01=Table[ArcSin[Sin[SolW[[i]]]],{i,n-1}];
SolW02=Table[ArcSin[Sin[SolW[[i]]]],{i,n,2(n-1)}];
MatSolW={SolW01,SolW02};
SolUAFa01=Table[ArcSin[Sin[SolUAFa[[i]]]],{i,n-1}];
SolUAFa02=Table[ArcCos[Cos[SolUAFa[[i]]]],{i,n,2(n-1)}];
MatSolUAFa={SolUAFa01,SolUAFa02};
WCheck=Table[(dqW-
HM2dq[RMW[[Round[i]]]])/.{s1x→SolWS[[1]],s1y→SolWS[[2]],s1z→SolWS[[3]],s1x
0→SolWS[[4]],s1y0→SolWS[[5]],s1z0→SolWS[[6]],s2x→SolWS[[7]],s2y→SolWS[[8]
],s2z→SolWS[[9]],s2x0→SolWS[[10]],s2y0→SolWS[[11]],s2z0→SolWS[[12]],th1→So
lW01[[i]],th2→SolW02[[i]]},{i,1,n-1}];
Mean[Flatten[WCheck,2]]
StandardDeviation[Flatten[WCheck,2]]
Eigenvalues
Eigensystem[MatSolW.Transpose[MatSolW]]//MatrixForm
Histograms
Histogram[{SolW01 180/π,
SolW02 180/π},{1},PlotLabel→"Wrist All Angles",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Histogram[SolW01 180/π,{1},PlotLabel→"Wrist 01",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Histogram[SolW02 180/π,{1},PlotLabel→"Wrist 02",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Thumb Joint 1
Setup
```

```

CRTJ1=Table[Inverse[DMetaCarp[[i]].DThumb[[i]],{i,1,n}];
RMTJ1=Table[CRTJ1[[i+1]].Inverse[CRTJ1[[1]]],{i,1,n-1}];
Show[generatePlotM[CRTJ1,Blue,5];
STJ11=makequat[{s1x,s1y,s1z},{s1x0,s1y0,s1z0}],01,0;
STJ12=makequat[{s2x,s2y,s2z},{s2x0,s2y0,s2z0}],02,0;
dqTJ1=quatmultSep[STJ11,STJ12]//Simplify;

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To Matlab

Solve Axes

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KETJ1S1=dqE-HM2dq[RMTJ1[[Round[1 (n-1)/10]]]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KETJ1S2=dqE-HM2dq[RMTJ1[[Round[2 (n-1)/10]]]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KETJ1S3=dqE-HM2dq[RMTJ1[[Round[3 (n-1)/10]]]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KETJ1S4=dqE-HM2dq[RMTJ1[[Round[4 (n-1)/10]]]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KETJ1S5=dqE-HM2dq[RMTJ1[[Round[5 (n-1)/10]]]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KETJ1S6=dqE-HM2dq[RMTJ1[[Round[6 (n-1)/10]]]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KETJ1S7=dqE-HM2dq[RMTJ1[[Round[7 (n-1)/10]]]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KETJ1S8=dqE-HM2dq[RMTJ1[[Round[8 (n-1)/10]]]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KETJ1S9=dqE-HM2dq[RMTJ1[[Round[9 (n-1)/10]]]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KETJ1S10=dqE-HM2dq[RMTJ1[[Round[n-1]]]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
KETJ1SS1=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[1 (n-1)/10]]]]2]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KETJ1SS2=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[2 (n-1)/10]]]]2]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KETJ1SS3=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[3 (n-1)/10]]]]2]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KETJ1SS4=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[4 (n-1)/10]]]]2]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KETJ1SS5=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[5 (n-1)/10]]]]2]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KETJ1SS6=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[6 (n-1)/10]]]]2]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KETJ1SS7=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[7 (n-1)/10]]]]2]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};

```

```

KETJ1SS8=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[8 (n-1)/10]]]]2]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KETJ1SS9=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[9 (n-1)/10]]]]2]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KETJ1SS10=Total[Total[dqE2]]-Total[Total[HM2dq[RMTJ1[[Round[n-1]]]]2]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking","TJ1AxisF.m"}],PageWidth→4000];
WriteString[handrun,"function y = TJ1AxisF(x)

```

```

    cll = mat2cell(x,1,ones(1,length(x)));

```

```

    [s1x,s1y,s1z,s1x0,s1y0,s1z0,...
    s2x,s2y,s2z,s2x0,s2y0,s2z0,...
    ];
Table[WriteString[handrun,"th1",Round[i*(n-1)/10], " ",{i,10}];
WriteString[handrun,"...
"]
Table[WriteString[handrun,"th2",Round[i*(n-1)/10]," ",{i,9}];
WriteString[handrun,"th2",n-1]
WriteString[handrun,""] = cll{:};

```

```

y=""
WriteString[handrun,ToMatlab[{KETJ1S1,KETJ1S2,KETJ1S3,KETJ1S4,KETJ1S5,KETJ1S6,KETJ1S7,KETJ1S8,KETJ1S9,KETJ1S10}]];
Close[handrun]
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking","TJ1AxisS.m"}],PageWidth→400];

```

```

WriteString[handrun,"func = @TJ1AxisF;
x0 = []
Table[WriteString[handrun,".1, ",{i,2(10)+11}];
WriteString[handrun,".1];
x = fsolve(func, x0);
"];
WriteString[handrun,"%% MatrixForm
SATJ1 = x;
xlswrite('ATJ1',SATJ1)"];
Close[handrun]
ClearAll[i]
Solver angles
SolTJ1S=Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking\\ATJ1.xls"],2];
KETJ11=Table[dqE-
HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[

```

$6]], s2x \rightarrow \text{SolTJ1S}[[7]], s2y \rightarrow \text{SolTJ1S}[[8]], s2z \rightarrow \text{SolTJ1S}[[9]], s2x0 \rightarrow \text{SolTJ1S}[[10]], s2y0 \rightarrow \text{SolTJ1S}[[11]], s2z0 \rightarrow \text{SolTJ1S}[[12]]], \{i, \text{Round}[1 (n-1)/30]\}$ };
 KETJ12=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[1 (n-1)/30],Round[2 (n-1)/30]}];
 KETJ13=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[2 (n-1)/30],Round[3 (n-1)/30]}];
 KETJ14=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[3 (n-1)/30],Round[4 (n-1)/30]}];
 KETJ15=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[4 (n-1)/30],Round[5 (n-1)/30]}];
 KETJ16=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[5 (n-1)/30],Round[6 (n-1)/30]}];
 KETJ17=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[6 (n-1)/30],Round[7 (n-1)/30]}];
 KETJ18=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[7 (n-1)/30],Round[8 (n-1)/30]}];
 KETJ19=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]],{i,Round[8 (n-1)/30],Round[9 (n-1)/30]}];
 KETJ110=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→Sol

$lTJ1S[[2]], s1z \rightarrow SolTJ1S[[3]], s1x0 \rightarrow SolTJ1S[[4]], s1y0 \rightarrow SolTJ1S[[5]], s1z0 \rightarrow SolTJ1S[[6]], s2x \rightarrow SolTJ1S[[7]], s2y \rightarrow SolTJ1S[[8]], s2z \rightarrow SolTJ1S[[9]], s2x0 \rightarrow SolTJ1S[[10]], s2y0 \rightarrow SolTJ1S[[11]], s2z0 \rightarrow SolTJ1S[[12]]\}, \{i, Round[9 (n-1)/30], Round[10 (n-1)/30]\}$;
 KETJ111=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[10 (n-1)/30], Round[11 (n-1)/30]\};
 KETJ112=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[11 (n-1)/30], Round[12 (n-1)/30]\};
 KETJ113=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[12 (n-1)/30], Round[13 (n-1)/30]\};
 KETJ114=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[13 (n-1)/30], Round[14 (n-1)/30]\};
 KETJ115=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[14 (n-1)/30], Round[15 (n-1)/30]\};
 KETJ116=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[15 (n-1)/30], Round[16 (n-1)/30]\};
 KETJ117=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[16 (n-1)/30], Round[17 (n-1)/30]\};
 KETJ118=Table[dqE-
 HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]\},{i, Round[17 (n-1)/30], Round[18 (n-1)/30]\};


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6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0
→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]},{i,Round[26 (n-1)/30],Round[27 (n-1)/30]}];
KETJ128=Table[dqE-
HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→So
lTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[
6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0
→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]},{i,Round[27 (n-1)/30],Round[28 (n-1)/30]}];
KETJ129=Table[dqE-
HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→So
lTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[
6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0
→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]},{i,Round[28 (n-1)/30],Round[29 (n-1)/30]}];
KETJ130=Table[dqE-
HM2dq[RMTJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolTJ1S[[1]],s1y→So
lTJ1S[[2]],s1z→SolTJ1S[[3]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[
6]],s2x→SolTJ1S[[7]],s2y→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0
→SolTJ1S[[11]],s2z0→SolTJ1S[[12]]},{i,Round[29 (n-1)/30],Round[n-1]}];
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","TJ1AngleF.m"}],PageWidth→4000];
WriteString[handrun,"function y = TJ1AngleF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    ["];
    Table[WriteString[handrun,"th1",i," ", " "],{i,n-1}];
    WriteString[handrun,"...
    "]
    Table[WriteString[handrun,"th2",i," ", " "],{i,n-2}];
    WriteString[handrun,"th2",n-1]
    WriteString[handrun,""] = cll{:};

    y
    =",ToMatlab[{KETJ11,KETJ12,KETJ13,KETJ14,KETJ15,KETJ16,KETJ17,KETJ18,KE
TJ19,KETJ110,KETJ111,KETJ112,KETJ113,KETJ114,KETJ115,KETJ116,KETJ117,K
ETJ118,KETJ119,KETJ120,KETJ121,KETJ122,KETJ123,KETJ124,KETJ125,KETJ126
,KETJ127,KETJ128,KETJ129,KETJ130}]];
    Close[handrun]
    handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","TJ1AngleS.m"}],PageWidth→400];

    WriteString[handrun,"func = @TJ1AngleF;
    x0 = ["]
    Table[WriteString[handrun,".1, "],{i,2(n-1)-1}];
    WriteString[handrun,".1];
    x = fsolve(func, x0,saoptimset('TolFun',.001));

```

```

"];
WriteString[handrun,"%% MatrixForm
SSTJ1 = x;
xlswrite('STJ1.xlsx',SSTJ1)"];
Close[handrun]
ClearAll[i]
Results
SolTJ1 =
Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\SV1\\Drinking\\STJ1.xlsx"],2];
SolTJ101=Table[ArcSin[Sin[SolTJ1[[i]]]],{i,n-1}];
SolTJ102=Table[ArcSin[Sin[SolTJ1[[i]]]],{i,n,2(n-1)}];
MatSolTJ1={SolTJ101,SolTJ102};
TJ1Check=Table[(dqTJ1-
HM2dq[RMTJ1[[Round[i]]]]/.{s1x→SolTJ1S[[1]],s1y→SolTJ1S[[2]],s1z→SolTJ1S[[3
]],s1x0→SolTJ1S[[4]],s1y0→SolTJ1S[[5]],s1z0→SolTJ1S[[6]],s2x→SolTJ1S[[7]],s2y
→SolTJ1S[[8]],s2z→SolTJ1S[[9]],s2x0→SolTJ1S[[10]],s2y0→SolTJ1S[[11]],s2z0→So
lTJ1S[[12]],01→SolTJ101[[i]],02→SolTJ102[[i]]},{i,1,n-1}];
Mean[Flatten[TJ1Check,2]]
StandardDeviation[Flatten[TJ1Check,2]]
Eigenvalues
Eigensystem[MatSolTJ1.Transpose[MatSolTJ1]]//MatrixForm
Histograms
Histogram[{SolTJ101 180/π,
SolTJ102 180/π},{1},PlotLabel→"Thumb Joint 1 All Angles",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Histogram[SolTJ101 180/π,{1},PlotLabel→"01",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Histogram[SolTJ102 180/π,{1},PlotLabel→"02",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Index Joint 1
Setup
CRInJ1=Table[Inverse[DMetaCarp[[i]]].DIndex[[i]],{i,1,n}];
RMInJ1=Table[CRInJ1[[i+1]].Inverse[CRInJ1[[1]]],{i,1,n-1}];
Show[generatePlotM[CRInJ1,Blue,5]];
SInJ11=makequat[{s1x,s1y,s1z},{s1x0,s1y0,s1z0}},01,0];
SInJ12=makequat[{s2x,s2y,s2z},{s2x0,s2y0,s2z0}},02,0];
dqInJ1=quatmultSep[SInJ11,SInJ12]//Simplify;

To Matlab
Solve Axes
KEInJ1S1=dqE-HM2dq[RMInJ1[[Round[1 (n-1)/10]]]]/.{th1→TH1[[Round[1 (n-
1)/10]]],th2→TH2[[Round[1 (n-1)/10]]];
KEInJ1S2=dqE-HM2dq[RMInJ1[[Round[2 (n-1)/10]]]]/.{th1→TH1[[Round[2 (n-
1)/10]]],th2→TH2[[Round[2 (n-1)/10]]];

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KEInJ1S3=dqE-HM2dq[RMInJ1[[Round[3 (n-1)/10]]]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KEInJ1S4=dqE-HM2dq[RMInJ1[[Round[4 (n-1)/10]]]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KEInJ1S5=dqE-HM2dq[RMInJ1[[Round[5 (n-1)/10]]]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KEInJ1S6=dqE-HM2dq[RMInJ1[[Round[6 (n-1)/10]]]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KEInJ1S7=dqE-HM2dq[RMInJ1[[Round[7 (n-1)/10]]]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KEInJ1S8=dqE-HM2dq[RMInJ1[[Round[8 (n-1)/10]]]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KEInJ1S9=dqE-HM2dq[RMInJ1[[Round[9 (n-1)/10]]]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KEInJ1S10=dqE-HM2dq[RMInJ1[[Round[n-1]]]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
KEInJ1SS1=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[1 (n-1)/10]]]]2]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KEInJ1SS2=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[2 (n-1)/10]]]]2]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KEInJ1SS3=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[3 (n-1)/10]]]]2]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KEInJ1SS4=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[4 (n-1)/10]]]]2]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KEInJ1SS5=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[5 (n-1)/10]]]]2]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KEInJ1SS6=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[6 (n-1)/10]]]]2]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KEInJ1SS7=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[7 (n-1)/10]]]]2]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KEInJ1SS8=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[8 (n-1)/10]]]]2]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KEInJ1SS9=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[9 (n-1)/10]]]]2]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KEInJ1SS10=Total[Total[dqE2]]-Total[Total[HM2dq[RMInJ1[[Round[n-1]]]]2]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
handrun=OpenWrite[FileNameJoin[{""\Users\John\Documents\Mechanical_Engineerin
g\Thesis\Vicon Nexus\SV1\Drinking","InJ1AxisF.m"}],PageWidth→4000];
WriteString[handrun,"function y = InJ1AxisF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    [s1x,s1y,s1z,s1x0,s1y0,s1z0,...
    s2x,s2y,s2z,s2x0,s2y0,s2z0,...
    "];

```

```

Table[WriteString[handrun,"th1",Round[i*(n-1)/10], " ", "],{i,10}];
WriteString[handrun,"...
"]
Table[WriteString[handrun,"th2",Round[i*(n-1)/10], " ", "],{i,9}];
WriteString[handrun,"th2",n-1]
WriteString[handrun,"] = cll{:}];

y="]
WriteString[handrun,ToMatlab[{KEInJ1S1,KEInJ1S2,KEInJ1S3,KEInJ1S4,KEInJ1S5,KEInJ1S6,KEInJ1S7,KEInJ1S8,KEInJ1S9,KEInJ1S10}]];
Close[handrun]
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking","InJ1AxisS.m"}],PageWidth→400];

WriteString[handrun,"func = @InJ1AxisF;
x0 = []
Table[WriteString[handrun,".1, "],{i,2(10)+11}];
WriteString[handrun,".1];
x = fsolve(func, x0);
"];
WriteString[handrun,"%% MatrixForm
SAInJ1 = x;
xlswrite('AInJ1',SAInJ1)"];
Close[handrun]
ClearAll[i]
Solver angles
SolInJ1S=Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking\\AInJ1.xls"],2];
KEInJ11=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[1 (n-1)/30]}];
KEInJ12=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[1 (n-1)/30],Round[2 (n-1)/30]}];
KEInJ13=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[2 (n-1)/30],Round[3 (n-1)/30]}];

```

KEInJ14=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
 olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
 1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[3 (n-1)/30],Round[4 (n-
 1)/30]};
 KEInJ15=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
 olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
 1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[4 (n-1)/30],Round[5 (n-
 1)/30]};
 KEInJ16=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
 olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
 1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[5 (n-1)/30],Round[6 (n-
 1)/30]};
 KEInJ17=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
 olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
 1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[6 (n-1)/30],Round[7 (n-
 1)/30]};
 KEInJ18=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
 olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
 1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[7 (n-1)/30],Round[8 (n-
 1)/30]};
 KEInJ19=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
 olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
 1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[8 (n-1)/30],Round[9 (n-
 1)/30]};
 KEInJ110=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
 olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
 1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[9 (n-1)/30],Round[10 (n-
 1)/30]};
 KEInJ111=Table[dqE-
 HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S

],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[17 (n-1)/30],Round[18 (n-1)/30]}];

KEInJ119=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[18 (n-1)/30],Round[19 (n-1)/30]}];

KEInJ120=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[19 (n-1)/30],Round[20 (n-1)/30]}];

KEInJ121=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[20 (n-1)/30],Round[21 (n-1)/30]}];

KEInJ122=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[21 (n-1)/30],Round[22 (n-1)/30]}];

KEInJ123=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[22 (n-1)/30],Round[23 (n-1)/30]}];

KEInJ124=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[23 (n-1)/30],Round[24 (n-1)/30]}];

KEInJ125=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→SolInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[24 (n-1)/30],Round[25 (n-1)/30]}];

```

KEInJ126=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[25 (n-1)/30],Round[26 (n-
1)/30]};
KEInJ127=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[26 (n-1)/30],Round[27 (n-
1)/30]};
KEInJ128=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[27 (n-1)/30],Round[28 (n-
1)/30]};
KEInJ129=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[28 (n-1)/30],Round[29 (n-
1)/30]};
KEInJ130=Table[dqE-
HM2dq[RMInJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolInJ1S[[1]],s1y→S
olInJ1S[[2]],s1z→SolInJ1S[[3]],s1x0→SolInJ1S[[4]],s1y0→SolInJ1S[[5]],s1z0→SolInJ
1S[[6]],s2x→SolInJ1S[[7]],s2y→SolInJ1S[[8]],s2z→SolInJ1S[[9]],s2x0→SolInJ1S[[10]
],s2y0→SolInJ1S[[11]],s2z0→SolInJ1S[[12]]},{i,Round[29 (n-1)/30],Round[n-1]};
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","InJ1AngleF.m"}],PageWidth→4000];
WriteString[handrun,"function y = InJ1AngleF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    ["];
    Table[WriteString[handrun,"th1",i, ", ", "],{i,n-1}];
    WriteString[handrun,"...
    "]
    Table[WriteString[handrun,"th2",i, ", ", "],{i,n-2}];
    WriteString[handrun,"th2",n-1]
    WriteString[handrun,""] = cll{:};

    y
    =",ToMatlab[{KEInJ11,KEInJ12,KEInJ13,KEInJ14,KEInJ15,KEInJ16,KEInJ17,KEInJ1

```

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8,KEInJ19,KEInJ110,KEInJ111,KEInJ112,KEInJ113,KEInJ114,KEInJ115,KEInJ116,KEInJ117,KEInJ118,KEInJ119,KEInJ120,KEInJ121,KEInJ122,KEInJ123,KEInJ124,KEInJ125,KEInJ126,KEInJ127,KEInJ128,KEInJ129,KEInJ130}]]];
Close[handrun]
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking","InJ1AngleS.m"}],PageWidth→400];

WriteString[handrun,"func = @InJ1AngleF;
x0 = []
Table[WriteString[handrun, ".1, ", {i, 2(n-1)-1}];
WriteString[handrun, ".1];
x = fsolve(func, x0, saoptimset("TolFun", .001));
"];
WriteString[handrun, "%% MatrixForm
SSInJ1 = x;
xlswrite('SInJ1.xlsx', SSInJ1)"];
Close[handrun]
ClearAll[i]
Results
SolInJ1 =
Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking\\SInJ1.xlsx"], 2];
SolInJ101=Table[ArcSin[Sin[SolInJ1[[i]]]], {i, n-1}];
SolInJ102=Table[ArcSin[Sin[SolInJ1[[i]]]], {i, n, 2(n-1)}];
MatSolInJ1={SolInJ101, SolInJ102};
InJ1Check=Table[(dqInJ1-
HM2dq[RMInJ1[[Round[i]]]])/.{s1x→SolInJ1S[[1]], s1y→SolInJ1S[[2]], s1z→SolInJ1S[[3]], s1x0→SolInJ1S[[4]], s1y0→SolInJ1S[[5]], s1z0→SolInJ1S[[6]], s2x→SolInJ1S[[7]], s2y→SolInJ1S[[8]], s2z→SolInJ1S[[9]], s2x0→SolInJ1S[[10]], s2y0→SolInJ1S[[11]], s2z0→SolInJ1S[[12]], 01→SolInJ101[[i]], 02→SolInJ102[[i]]}, {i, 1, n-1}];
Mean[Flatten[InJ1Check, 2]]
StandardDeviation[Flatten[InJ1Check, 2]]
Eigenvalues
Eigensystem[MatSolInJ1.Transpose[MatSolInJ1]]//MatrixForm
Histograms
Histogram[{SolInJ101 180/π,
SolInJ102 180/π}, {1}, PlotLabel→"Index Finger Joint 1 All
Angles", AxesLabel→{"Angles (Degrees)", "Quantity"}]
Histogram[SolInJ101 180/π, {1}, PlotLabel→"01", AxesLabel→{"Angles
(Degrees)", "Quantity"}]
Histogram[SolInJ102 180/π, {1}, PlotLabel→"02", AxesLabel→{"Angles
(Degrees)", "Quantity"}]
Ring Joint 1
Setup
CRRJ1=Table[Inverse[DMetaCarp[[i]]].DRing[[i]], {i, 1, n}];
RMRJ1=Table[CRRJ1[[i+1]].Inverse[CRRJ1[[1]]], {i, 1, n-1}];

```

```
Show[generatePlotM[CRRJ1,Blue,5]];
SRJ11=makequat[{s1x,s1y,s1z},{s1x0,s1y0,s1z0}],01,0);
SRJ12=makequat[{s2x,s2y,s2z},{s2x0,s2y0,s2z0}],02,0);
dqRJ1=quatmultSep[SRJ11,SRJ12]/Simplify;
```

To Matlab
Solve Axes

```
KERJ1S1=dqE-HM2dq[RMRJ1[[Round[1 (n-1)/10]]]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KERJ1S2=dqE-HM2dq[RMRJ1[[Round[2 (n-1)/10]]]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KERJ1S3=dqE-HM2dq[RMRJ1[[Round[3 (n-1)/10]]]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KERJ1S4=dqE-HM2dq[RMRJ1[[Round[4 (n-1)/10]]]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KERJ1S5=dqE-HM2dq[RMRJ1[[Round[5 (n-1)/10]]]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KERJ1S6=dqE-HM2dq[RMRJ1[[Round[6 (n-1)/10]]]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KERJ1S7=dqE-HM2dq[RMRJ1[[Round[7 (n-1)/10]]]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KERJ1S8=dqE-HM2dq[RMRJ1[[Round[8 (n-1)/10]]]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KERJ1S9=dqE-HM2dq[RMRJ1[[Round[9 (n-1)/10]]]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KERJ1S10=dqE-HM2dq[RMRJ1[[Round[n-1]]]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
KERJ1SS1=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[1 (n-1)/10]]]]2]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KERJ1SS2=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[2 (n-1)/10]]]]2]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KERJ1SS3=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[3 (n-1)/10]]]]2]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KERJ1SS4=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[4 (n-1)/10]]]]2]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KERJ1SS5=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[5 (n-1)/10]]]]2]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KERJ1SS6=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[6 (n-1)/10]]]]2]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KERJ1SS7=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[7 (n-1)/10]]]]2]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KERJ1SS8=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[8 (n-1)/10]]]]2]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
```

```

KERJ1SS9=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[9 (n-1)/10]]]]2]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KERJ1SS10=Total[Total[dqE2]]-Total[Total[HM2dq[RMRJ1[[Round[n-1]]]]2]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking","RJ1AxisF.m"}],PageWidth→4000];
WriteString[handrun,"function y = RJ1AxisF(x)

```

```

    cll = mat2cell(x,1,ones(1,length(x)));

```

```

    [s1x,s1y,s1z,s1x0,s1y0,s1z0,...
     s2x,s2y,s2z,s2x0,s2y0,s2z0,...
     "];
Table[WriteString[handrun,"th1",Round[i*(n-1)/10], " ", "},{i,10}];
WriteString[handrun,"...
" ]
Table[WriteString[handrun,"th2",Round[i*(n-1)/10], " ", "},{i,9}];
WriteString[handrun,"th2",n-1]
WriteString[handrun,"] = cll{:}];

```

```

y=""
WriteString[handrun,ToMatlab[{KERJ1S1,KERJ1S2,KERJ1S3,KERJ1S4,KERJ1S5,KERJ1S6,KERJ1S7,KERJ1S8,KERJ1S9,KERJ1S10}]];
Close[handrun]
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking","RJ1AxisS.m"}],PageWidth→400];

```

```

WriteString[handrun,"func = @RJ1AxisF;
x0 = []
Table[WriteString[handrun,".1, "],{i,2(10)+11}];
WriteString[handrun,".1];
x = fsolve(func, x0);
"];
WriteString[handrun,"%% MatrixForm
SARJ1 = x;
xlswrite('ARJ1',SARJ1)"];
Close[handrun]
ClearAll[i]
Solver angles
SolRJ1S=Flatten[Import[{"\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon Nexus\\SV1\\Drinking\\ARJ1.xls"},2];
KERJ11=Table[dqE-
HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]},{i,Round[1 (n-1)/30]}];

```


$[[6]], s2x \rightarrow \text{SolRJ1S}[[7]], s2y \rightarrow \text{SolRJ1S}[[8]], s2z \rightarrow \text{SolRJ1S}[[9]], s2x0 \rightarrow \text{SolRJ1S}[[10]], s2y0 \rightarrow \text{SolRJ1S}[[11]], s2z0 \rightarrow \text{SolRJ1S}[[12]]\}, \{i, \text{Round}[9 (n-1)/30], \text{Round}[10 (n-1)/30]\}$ };
 KERJ111=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[10 (n-1)/30], Round[11 (n-1)/30]\}];
 KERJ112=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[11 (n-1)/30], Round[12 (n-1)/30]\}];
 KERJ113=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[12 (n-1)/30], Round[13 (n-1)/30]\}];
 KERJ114=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[13 (n-1)/30], Round[14 (n-1)/30]\}];
 KERJ115=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[14 (n-1)/30], Round[15 (n-1)/30]\}];
 KERJ116=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[15 (n-1)/30], Round[16 (n-1)/30]\}];
 KERJ117=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[16 (n-1)/30], Round[17 (n-1)/30]\}];
 KERJ118=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]\}, {i, Round[17 (n-1)/30], Round[18 (n-1)/30]\}];
 KERJ119=Table[dqE-
 HM2dq[RMRJ1[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→S


```

KERJ128=Table[dqE-
HM2dq[RMRJ1[Round[i]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→S
olRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S
[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2
y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]},{i,Round[27 (n-1)/30],Round[28 (n-1)/30]}];
KERJ129=Table[dqE-
HM2dq[RMRJ1[Round[i]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→S
olRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S
[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2
y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]},{i,Round[28 (n-1)/30],Round[29 (n-1)/30]}];
KERJ130=Table[dqE-
HM2dq[RMRJ1[Round[i]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolRJ1S[[1]],s1y→S
olRJ1S[[2]],s1z→SolRJ1S[[3]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S
[[6]],s2x→SolRJ1S[[7]],s2y→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2
y0→SolRJ1S[[11]],s2z0→SolRJ1S[[12]]},{i,Round[29 (n-1)/30],Round[n-1]}];
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","RJ1AngleF.m"}],PageWidth→4000];
WriteString[handrun,"function y = RJ1AngleF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    ["];
    Table[WriteString[handrun,"th1",i, ", ", "],{i,n-1}];
    WriteString[handrun,"...
    "]
    Table[WriteString[handrun,"th2",i, ", ", "],{i,n-2}];
    WriteString[handrun,"th2",n-1]
    WriteString[handrun,"] = cll{:}];

    y
    =" ,ToMatlab[{KERJ11,KERJ12,KERJ13,KERJ14,KERJ15,KERJ16,KERJ17,KERJ18,K
ERJ19,KERJ110,KERJ111,KERJ112,KERJ113,KERJ114,KERJ115,KERJ116,KERJ117
,KERJ118,KERJ119,KERJ120,KERJ121,KERJ122,KERJ123,KERJ124,KERJ125,KERJ
126,KERJ127,KERJ128,KERJ129,KERJ130}]];
    Close[handrun]
    handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\SV1\\Drinking","RJ1AngleS.m"}],PageWidth→400];

    WriteString[handrun,"func = @RJ1AngleF;
    x0 = ["]
    Table[WriteString[handrun, ".1, ", "],{i,2(n-1)-1}];
    WriteString[handrun, ".1];
    x = fsolve(func, x0,saoptimset('TolFun',.001));
    "];
    WriteString[handrun,"%% MatrixForm
    SSRJ1 = x;

```

```

xlswrite('SRJ1.xlsx',SSRJ1");
Close[handrun]
ClearAll[i]
Results
SolRJ1 =
Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\SV1\\Drinking\\SRJ1.xlsx"],2];
SolRJ101=Table[SolRJ1[[i]],{i,n-1}];
SolRJ102=Table[SolRJ1[[i]],{i,n,2(n-1)}];
MatSolRJ1={SolRJ101,SolRJ102};
RJ1Check=Table[(dqRJ1-
HM2dq[RMRJ1[[Round[i]]]])/.{s1x→SolRJ1S[[1]],s1y→SolRJ1S[[2]],s1z→SolRJ1S[[3
]],s1x0→SolRJ1S[[4]],s1y0→SolRJ1S[[5]],s1z0→SolRJ1S[[6]],s2x→SolRJ1S[[7]],s2y
→SolRJ1S[[8]],s2z→SolRJ1S[[9]],s2x0→SolRJ1S[[10]],s2y0→SolRJ1S[[11]],s2z0→S
olRJ1S[[12]],01→SolRJ101[[i]],02→SolRJ102[[i]]},{i,1,n-1}];
Mean[Flatten[RJ1Check,2]]
StandardDeviation[Flatten[RJ1Check,2]]
Eigenvalues
Eigensystem[MatSolRJ1.Transpose[MatSolRJ1]]//MatrixForm
Histograms
Histogram[{SolRJ101 180/π,
SolRJ102 180/π},{1},PlotLabel→"Ring Finger Joint 1 All
Angles",AxesLabel→{"Angles (Degrees)","Quantity"}]
Histogram[SolRJ101 180/π,{2},PlotLabel→"01",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Histogram[SolRJ102 180/π,{1},PlotLabel→"02",AxesLabel→{"Angles
(Degrees)","Quantity"}]
Thumb Joint 2
Solution
TJ2=Table[180/ π (ArcSin[Norm[Cross[DataThumbS3M1[[i]]-
DataThumbS4M1[[i]],DataThumbS3M1[[i]]-
DataThumbS2M1[[i]]]/(Norm[DataThumbS3M1[[i]]-DataThumbS2M1[[i]]]
Norm[DataThumbS3M1[[i]]-DataThumbS4M1[[i]]]))),{i,n}];
Results

Histograms
Histogram[TJ2,{1},AxesLabel→{"Angles (°)","Frequency"},PlotLabel→"Second Joint
in Thumb"]
Index Joint 2 and 3
Solution
InJ2=Table[180/ π (ArcSin[Norm[Cross[DataIndexS2M1[[i]]-
DataIndexS3M1[[i]],DataIndexS2M1[[i]]-
DataIndexS1M2[[i]]]/(Norm[DataIndexS1M1[[i]]-DataIndexS1M2[[i]]]
Norm[DataIndexS2M1[[i]]-DataIndexS3M1[[i]]]))),{i,n}];

```

```
InJ3=Table[180/  $\pi$  (ArcSin[Norm[Cross[DataIndexS3M1[[i]]-
DataIndexS4M1[[i]],DataIndexS3M1[[i]]-
DataIndexS2M1[[i]]]]/(Norm[DataIndexS3M1[[i]]-DataIndexS2M1[[i]]]
Norm[DataIndexS3M1[[i]]-DataIndexS4M1[[i]]]])),{i,n}];
```

Results

Eigen System

```
Eigensystem[{InJ2,InJ3}.Transpose[{InJ2,InJ3}]]//MatrixForm
```

Histograms

```
Histogram[{InJ2,InJ3},{1},AxesLabel→{"Angles (°)","Frequency"},PlotLabel→"Joints
in Index Finger"]
```

```
Histogram[InJ2,{1},AxesLabel→{"Angles (°)","Frequency"},PlotLabel→"Second Joint
in Index Finger"]
```

```
Histogram[InJ3,{1},AxesLabel→{"Angles (°)","Frequency"},PlotLabel→"Third Joint in
Index Finger"]
```

Ring Joint 2 and 3

Solution

```
RJ2=Table[180/  $\pi$  (ArcSin[Norm[Cross[DataRingS2M1[[i]]-
DataRingS3M1[[i]],DataRingS2M1[[i]]-
DataRingS1M2[[i]]]]/(Norm[DataRingS1M1[[i]]-DataRingS1M2[[i]]]
Norm[DataRingS2M1[[i]]-DataRingS3M1[[i]]]])),{i,n}];
```

```
RJ3=Table[180/  $\pi$  (ArcSin[Norm[Cross[DataRingS3M1[[i]]-
DataRingS4M1[[i]],DataRingS3M1[[i]]-
DataRingS2M1[[i]]]]/(Norm[DataRingS3M1[[i]]-DataRingS2M1[[i]]]
Norm[DataRingS3M1[[i]]-DataRingS4M1[[i]]]])),{i,n}];
```

Results

Eigen System

```
Eigensystem[{RJ2,RJ3}.Transpose[{RJ2,RJ3}]]//MatrixForm
```

Histograms

```
Histogram[{RJ2,RJ3},{1},AxesLabel→{"Angles (°)","Frequency"},PlotLabel→"Joints
in Ring Finger"]
```

```
Histogram[RJ2,{1},AxesLabel→{"Angles (°)","Frequency"},PlotLabel→"Second Joint
in Ring Finger"]
```

```
Histogram[RJ3,{1},AxesLabel→{"Angles (°)","Frequency"},PlotLabel→"Third Joint in
Ring Finger"]
```

PCA

```
 $\theta$ Sol=Table[{SolScUA[[i,1]],SolScUA[[i,2]],SolScUA[[i,3]],SolUAFa $\theta$ 1[[i]],SolUAFa $\theta$ 
2[[i]],SolW $\theta$ 1[[i]],SolW $\theta$ 2[[i]],SolTJ1 $\theta$ 1[[i]],SolTJ1 $\theta$ 2[[i]], $\pi$ /180
TJ2[[i]],SolInJ1 $\theta$ 1[[i]],SolInJ1 $\theta$ 2[[i]], $\pi$ /180 InJ2[[i]], $\pi$ /180
InJ3[[i]],SolRJ1 $\theta$ 1[[i]],SolRJ1 $\theta$ 2[[i]], $\pi$ /180 RJ2[[i]], $\pi$ /180 RJ3[[i]]},{i,n-1}];
```

```
Dimensions[ $\theta$ Sol]
```

```
Length[ $\theta$ Sol]
```

```
 $\theta$ SolAve=Mean[ $\theta$ Sol];
```

```
 $\theta$ SolTran=Transpose[ $\theta$ Sol];
```

```

Table1=Table[0SolTran[[1,i]]-0SolAve[[1]],{i,Length[0Sol]};
Table2=Table[0SolTran[[2,i]]-0SolAve[[2]],{i,Length[0Sol]};
Table3=Table[0SolTran[[3,i]]-0SolAve[[3]],{i,Length[0Sol]};
Table4=Table[0SolTran[[4,i]]-0SolAve[[4]],{i,Length[0Sol]};
Table5=Table[0SolTran[[5,i]]-0SolAve[[5]],{i,Length[0Sol]};
Table6=Table[0SolTran[[6,i]]-0SolAve[[6]],{i,Length[0Sol]};
Table7=Table[0SolTran[[7,i]]-0SolAve[[7]],{i,Length[0Sol]};
Table8=Table[0SolTran[[8,i]]-0SolAve[[8]],{i,Length[0Sol]};
Table9=Table[0SolTran[[9,i]]-0SolAve[[9]],{i,Length[0Sol]};
Table10=Table[0SolTran[[10,i]]-0SolAve[[10]],{i,Length[0Sol]};
Table11=Table[0SolTran[[11,i]]-0SolAve[[11]],{i,Length[0Sol]};
Table12=Table[0SolTran[[12,i]]-0SolAve[[12]],{i,Length[0Sol]};
Table13=Table[0SolTran[[13,i]]-0SolAve[[13]],{i,Length[0Sol]};
Table14=Table[0SolTran[[14,i]]-0SolAve[[14]],{i,Length[0Sol]};
Table15=Table[0SolTran[[15,i]]-0SolAve[[15]],{i,Length[0Sol]};
Table16=Table[0SolTran[[16,i]]-0SolAve[[16]],{i,Length[0Sol]};
Table17=Table[0SolTran[[17,i]]-0SolAve[[17]],{i,Length[0Sol]};
Table18=Table[0SolTran[[18,i]]-0SolAve[[18]],{i,Length[0Sol]};
Cmat={
  {Table1.Table1, Table2.Table1, Table3.Table1, Table4.Table1, Table5.Table1,
    Table6.Table1, Table7.Table1, Table8.Table1, Table9.Table1, Table10.Table1,
    Table11.Table1, Table12.Table1, Table13.Table1, Table14.Table1, Table15.Table1,
    Table16.Table1, Table17.Table1, Table18.Table1},
  {Table1.Table2, Table2.Table2, Table3.Table2, Table4.Table2, Table5.Table2,
    Table6.Table2, Table7.Table2, Table8.Table2, Table9.Table2, Table10.Table2,
    Table11.Table2, Table12.Table2, Table13.Table2, Table14.Table2, Table15.Table2,
    Table16.Table2, Table17.Table2, Table18.Table2},
  {Table1.Table3, Table2.Table3, Table3.Table3, Table4.Table3, Table5.Table3,
    Table6.Table3, Table7.Table3, Table8.Table3, Table9.Table3, Table10.Table3,
    Table11.Table3, Table12.Table3, Table13.Table3, Table14.Table3, Table15.Table3,
    Table16.Table3, Table17.Table3, Table18.Table3},
  {Table1.Table4, Table2.Table4, Table3.Table4, Table4.Table4, Table5.Table4,
    Table6.Table4, Table7.Table4, Table8.Table4, Table9.Table4, Table10.Table4,
    Table11.Table4, Table12.Table4, Table13.Table4, Table14.Table4, Table15.Table4,
    Table16.Table4, Table17.Table4, Table18.Table4},
  {Table1.Table5, Table2.Table5, Table3.Table5, Table4.Table5, Table5.Table5,
    Table6.Table5, Table7.Table5, Table8.Table5, Table9.Table5, Table10.Table5,
    Table11.Table5, Table12.Table5, Table13.Table5, Table14.Table5, Table15.Table5,
    Table16.Table5, Table17.Table5, Table18.Table5},
  {Table1.Table6, Table2.Table6, Table3.Table6, Table4.Table6, Table5.Table6,
    Table6.Table6, Table7.Table6, Table8.Table6, Table9.Table6, Table10.Table6,
    Table11.Table6, Table12.Table6, Table13.Table6, Table14.Table6, Table15.Table6,
    Table16.Table6, Table17.Table6, Table18.Table6},
  {Table1.Table7, Table2.Table7, Table3.Table7, Table4.Table7, Table5.Table7,
    Table6.Table7, Table7.Table7, Table8.Table7, Table9.Table7, Table10.Table7,

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Table11.Table7, Table12.Table7, Table13.Table7, Table14.Table7, Table15.Table7, Table16.Table7, Table17.Table7, Table18.Table7},

{Table1.Table8, Table2.Table8, Table3.Table8, Table4.Table8, Table5.Table8, Table6.Table8, Table7.Table8, Table8.Table8, Table9.Table8, Table10.Table8, Table11.Table8, Table12.Table8, Table13.Table8, Table14.Table8, Table15.Table8, Table16.Table8, Table17.Table8, Table18.Table8},

{Table1.Table9, Table2.Table9, Table3.Table9, Table4.Table9, Table5.Table9, Table6.Table9, Table7.Table9, Table8.Table9, Table9.Table9, Table10.Table9, Table11.Table9, Table12.Table9, Table13.Table9, Table14.Table9, Table15.Table9, Table16.Table9, Table17.Table9, Table18.Table9},

{Table1.Table10, Table2.Table10, Table3.Table10, Table4.Table10, Table5.Table10, Table6.Table10, Table7.Table10, Table8.Table10, Table9.Table10, Table10.Table10, Table11.Table10, Table12.Table10, Table13.Table10, Table14.Table10, Table15.Table10, Table16.Table10, Table17.Table10, Table18.Table10},

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{Table1.Table12, Table2.Table12, Table3.Table12, Table4.Table12, Table5.Table12, Table6.Table12, Table7.Table12, Table8.Table12, Table9.Table12, Table10.Table12, Table11.Table12, Table12.Table12, Table13.Table12, Table14.Table12, Table15.Table12, Table16.Table12, Table17.Table12, Table18.Table12},

{Table1.Table13, Table2.Table13, Table3.Table13, Table4.Table13, Table5.Table13, Table6.Table13, Table7.Table13, Table8.Table13, Table9.Table13, Table10.Table13, Table11.Table13, Table12.Table13, Table13.Table13, Table14.Table13, Table15.Table13, Table16.Table13, Table17.Table13, Table18.Table13},

{Table1.Table14, Table2.Table14, Table3.Table14, Table4.Table14, Table5.Table14, Table6.Table14, Table7.Table14, Table8.Table14, Table9.Table14, Table10.Table14, Table11.Table14, Table12.Table14, Table13.Table14, Table14.Table14, Table15.Table14, Table16.Table14, Table17.Table14, Table18.Table14},

{Table1.Table15, Table2.Table15, Table3.Table15, Table4.Table15, Table5.Table15, Table6.Table15, Table7.Table15, Table8.Table15, Table9.Table15, Table10.Table15, Table11.Table15, Table12.Table15, Table13.Table15, Table14.Table15, Table15.Table15, Table16.Table15, Table17.Table15, Table18.Table15},

{Table1.Table16, Table2.Table16, Table3.Table16, Table4.Table16, Table5.Table16, Table6.Table16, Table7.Table16, Table8.Table16, Table9.Table16, Table10.Table16, Table11.Table16, Table12.Table16, Table13.Table16, Table14.Table16, Table15.Table16, Table16.Table16, Table17.Table16, Table18.Table16},

{Table1.Table17, Table2.Table17, Table3.Table17, Table4.Table17, Table5.Table17, Table6.Table17, Table7.Table17, Table8.Table17, Table9.Table17, Table10.Table17, Table11.Table17, Table12.Table17, Table13.Table17, Table14.Table17, Table15.Table17, Table16.Table17, Table17.Table17, Table18.Table17},

{Table1.Table18, Table2.Table18, Table3.Table18, Table4.Table18, Table5.Table18, Table6.Table18, Table7.Table18, Table8.Table18, Table9.Table18, Table10.Table18, Table11.Table18, Table12.Table18, Table13.Table18, Table14.Table18, Table15.Table18, Table16.Table18, Table17.Table18, Table18.Table18}

```

};
Eigen=Eigensystem[Cmat];
EigenVal=Eigen[[1]];
EigenVec=Eigen[[2]];
EigenVal//MatrixForm
Val=7;
Sum[EigenVal[[i]],{i,1,Val}]/Sum[EigenVal[[i]],{i,1,18}]*100
y=Table[{Table1[[j]],Table2[[j]],Table3[[j]],Table4[[j]],Table5[[j]],Table6[[j]],Table7[[j]],Table8[[j]],Table9[[j]],Table10[[j]],Table11[[j]],Table12[[j]],Table13[[j]],Table14[[j]],Table15[[j]],Table16[[j]],Table17[[j]],Table18[[j]]},{j,n-1}];
w=Table[Table[y[[j]].EigenVec[[i]],{i,Val}},{j,n-1}];
w//MatrixForm;
Xcor=Table[θSolAve+Sum[w[[j,i]]EigenVec[[i]],{i,Val}},{j,n-1}];
Xcor//MatrixForm;
Mean[Mean[θSol-Xcor]]
Mean[StandardDeviation[θSol-Xcor]]
Needs["PlotLegends`"]
ListPlot[Table[Table[180/π Xcor[[i,j]],{i,n-1}},{j,18}],PlotLegend→{"GH θx","GH θy","GH θz","URRot","Elbow","W θ1","W θ2","TJ1 θ1","TJ1 θ2","TJ2","InJ1 θ1","InJ1 θ2","InJ2","InJ3","RJ1 θ1","RJ1 θ2","RJ2","RJ3"},LegendPosition→{1.1,-0.4},PlotMarkers→Automatic]
SSyn1=110;
Graphics3D[{PointSize→.0085,Point[Data[[SSyn1,{3,4,5}]]],Point[Data[[SSyn1,{21,22,23}]]],Point[Data[[SSyn1,{33,34,35}]]],Point[(Data[[SSyn1,{39,40,41}]]+Data[[SSyn1,{48,49,50}]])/2],Point[Data[[SSyn1,{66,67,68}]]],Point[Data[[SSyn1,{69,70,71}]]],Point[Data[[SSyn1,{72,73,74}]]],Point[Data[[SSyn1,{75,76,77}]]],Point[Data[[SSyn1,{78,79,80}]]],Point[Data[[SSyn1,{81,82,83}]]],Point[Data[[SSyn1,{84,85,86}]]],Point[Data[[SSyn1,{87,88,89}]]],Point[Data[[SSyn1,{90,91,92}]]],Point[Data[[SSyn1,{93,94,95}]]],Point[Data[[SSyn1,{96,97,98}]]],Darker[Purple,Thick,Line[{Data[[SSyn1,{3,4,5}]],Data[[SSyn1,{21,22,23}]]}],Darker[Blue,Line[{Data[[SSyn1,{21,22,23}]],Data[[SSyn1,{33,34,35}]]}],Darker[Green,Line[{Data[[SSyn1,{33,34,35}]],(Data[[SSyn1,{39,40,41}]]+Data[[SSyn1,{48,49,50}]])/2}],Darker[Yellow,.4],Line[{(Data[[SSyn1,{39,40,41}]]+Data[[SSyn1,{48,49,50}]])/2,Data[[SSyn1,{66,67,68}]]}],Darker[Yellow,.25],Line[{Data[[SSyn1,{66,67,68}]],Data[[SSyn1,{69,70,71}]]}],Yellow,Line[{Data[[SSyn1,{69,70,71}]],Data[[SSyn1,{72,73,74}]]}],Darker[Orange,.6],Line[{(Data[[SSyn1,{39,40,41}]]+Data[[SSyn1,{48,49,50}]])/2,Data[[SSyn1,{75,76,77}]]}],Darker[Orange,.4],Line[{Data[[SSyn1,{75,76,77}]],Data[[SSyn1,{78,79,80}]]}],Darker[Orange,.2],Line[{Data[[SSyn1,{78,79,80}]],Data[[SSyn1,{81,82,83}]]}],Orange,Line[{Data[[SSyn1,{81,82,83}]],Data[[SSyn1,{84,85,86}]]}],Darker[Red,.6],Line[{(Data[[SSyn1,{39,40,41}]]+Data[[SSyn1,{48,49,50}]])/2,Data[[SSyn1,{87,88,89}]]}],Darker[Red,.4],Line[{Data[[SSyn1,{87,88,89}]],Data[[SSyn1,{90,91,92}]]}],Darker[Red,.2],Line[{Data[[SSyn1,{90,91,92}]],Data[[SSyn1,{93,94,95}]]}],Red,Line[{Data[[SSyn1,{93,94,95}]],Data[[SSyn1,{96,97,98}]]}],Axes→True]

```

Mathematica Code Appended for Longer Trials

This code here replaces the code for Solve Axes, Solve Angle and the data acquisition for the results Section.

Written by John Roylance:

Solve Axes

```
KEUAFAS1=dqE-HM2dq[RMUAFa[[Round[1 (n-1)/10]]]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KEUAFAS2=dqE-HM2dq[RMUAFa[[Round[2 (n-1)/10]]]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KEUAFAS3=dqE-HM2dq[RMUAFa[[Round[3 (n-1)/10]]]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KEUAFAS4=dqE-HM2dq[RMUAFa[[Round[4 (n-1)/10]]]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KEUAFAS5=dqE-HM2dq[RMUAFa[[Round[5 (n-1)/10]]]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KEUAFAS6=dqE-HM2dq[RMUAFa[[Round[6 (n-1)/10]]]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KEUAFAS7=dqE-HM2dq[RMUAFa[[Round[7 (n-1)/10]]]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KEUAFAS8=dqE-HM2dq[RMUAFa[[Round[8 (n-1)/10]]]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KEUAFAS9=dqE-HM2dq[RMUAFa[[Round[9 (n-1)/10]]]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
KEUAFAS10=dqE-HM2dq[RMUAFa[[Round[n-1]]]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
KEUAFASS1=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[1 (n-1)/10]]]]2]]/.{th1→TH1[[Round[1 (n-1)/10]]],th2→TH2[[Round[1 (n-1)/10]]]};
KEUAFASS2=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[2 (n-1)/10]]]]2]]/.{th1→TH1[[Round[2 (n-1)/10]]],th2→TH2[[Round[2 (n-1)/10]]]};
KEUAFASS3=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[3 (n-1)/10]]]]2]]/.{th1→TH1[[Round[3 (n-1)/10]]],th2→TH2[[Round[3 (n-1)/10]]]};
KEUAFASS4=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[4 (n-1)/10]]]]2]]/.{th1→TH1[[Round[4 (n-1)/10]]],th2→TH2[[Round[4 (n-1)/10]]]};
KEUAFASS5=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[5 (n-1)/10]]]]2]]/.{th1→TH1[[Round[5 (n-1)/10]]],th2→TH2[[Round[5 (n-1)/10]]]};
KEUAFASS6=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[6 (n-1)/10]]]]2]]/.{th1→TH1[[Round[6 (n-1)/10]]],th2→TH2[[Round[6 (n-1)/10]]]};
KEUAFASS7=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[7 (n-1)/10]]]]2]]/.{th1→TH1[[Round[7 (n-1)/10]]],th2→TH2[[Round[7 (n-1)/10]]]};
KEUAFASS8=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[8 (n-1)/10]]]]2]]/.{th1→TH1[[Round[8 (n-1)/10]]],th2→TH2[[Round[8 (n-1)/10]]]};
KEUAFASS9=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[9 (n-1)/10]]]]2]]/.{th1→TH1[[Round[9 (n-1)/10]]],th2→TH2[[Round[9 (n-1)/10]]]};
```

```

KEUAFAS10=Total[Total[dqE2]]-Total[Total[HM2dq[RMUAFa[[Round[n-1]]]]2]]/.{th1→TH1[[Round[n-1]]],th2→TH2[[Round[n-1]]]};
handrun=OpenWrite[FileNameJoin[{{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAxisF.m"}},PageWidth→4000];
WriteString[handrun,"function y = UAFaAxisF(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    [s1x,s1y,s1z,s1x0,s1y0,s1z0,...
    s2x,s2y,s2z,s2x0,s2y0,s2z0,...
    "];
    Table[WriteString[handrun,"th1",Round[i*(n-1)/10], " ", "],{i,10}];
    WriteString[handrun,"...
    "]
    Table[WriteString[handrun,"th2",Round[i*(n-1)/10], " ", "],{i,9}];
    WriteString[handrun,"th2",n-1]
    WriteString[handrun,""] = cll{:};

    y="]
    WriteString[handrun,ToMatlab[{KEUAFAS1,KEUAFAS2,KEUAFAS3,KEUAFAS4,K
    EUAFAS5,KEUAFAS6,KEUAFAS7,KEUAFAS8,KEUAFAS9,KEUAFAS10}]];
    Close[handrun]
    \\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
    Nexus\\NS2\\Writing\\UAFaAxisF.m
    handrun=OpenWrite[FileNameJoin[{{"\\Users\\John\\Documents\\Mechanical_Engineerin
    g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAxisS.m"}},PageWidth→400];

    WriteString[handrun,"func = @UAFaAxisF;
    x0 = []
    Table[WriteString[handrun,".1, "],{i,2(10)+11}];
    WriteString[handrun,".1];
    x = fsolve(func, x0);
    "];
    WriteString[handrun,"%% MatrixForm
    SAUAFa = x;
    xlswrite('AUAFa',SAUAFa)"];
    Close[handrun]
    ClearAll[i]
    \\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
    Nexus\\NS2\\Writing\\UAFaAxisS.m
    Solver angles
    SolUAFAS=Flatten[Import["\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis
    \\Vicon Nexus\\NS2\\Writing\\AUAFa.xls"],2];
    KEUAFAS1=Table[dqE-
    HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
    y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]

```


,s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [1 (n-1)/30]}];

KEUAFa2=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [1 (n-1)/30], Round[2 (n-1)/30]}];

KEUAFa3=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [2 (n-1)/30], Round[3 (n-1)/30]}];

KEUAFa4=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [3 (n-1)/30], Round[4 (n-1)/30]}];

KEUAFa5=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [4 (n-1)/30], Round[5 (n-1)/30]}];

KEUAFa6=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [5 (n-1)/30], Round[6 (n-1)/30]}];

KEUAFa7=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [6 (n-1)/30], Round[7 (n-1)/30]}];

KEUAFa8=Table[dqE-
HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round [7 (n-1)/30], Round[8 (n-1)/30]}];

9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [7 (n-1)/30],Round[8 (n-1)/30]}}];
 KEUAFA9=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [8 (n-1)/30],Round[9 (n-1)/30]}}];
 KEUAFA10=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [9 (n-1)/30],Round[10 (n-1)/30]}}];
 KEUAFA11=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [10 (n-1)/30],Round[11 (n-1)/30]}}];
 KEUAFA12=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [11 (n-1)/30],Round[12 (n-1)/30]}}];
 KEUAFA13=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [12 (n-1)/30],Round[13 (n-1)/30]}}];
 KEUAFA14=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [13 (n-1)/30],Round[14 (n-1)/30]}}];
 KEUAFA15=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
 [14 (n-1)/30],Round[15 (n-1)/30]}}];

KEUAFA16=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [15 (n-1)/30], Round[16 (n-1)/30]}];
 KEUAFA17=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [16 (n-1)/30], Round[17 (n-1)/30]}];
 KEUAFA18=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [17 (n-1)/30], Round[18 (n-1)/30]}];
 KEUAFA19=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [18 (n-1)/30], Round[19 (n-1)/30]}];
 KEUAFA20=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [19 (n-1)/30], Round[20 (n-1)/30]}];
 KEUAFA21=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [20 (n-1)/30], Round[21 (n-1)/30]}];
 KEUAFA22=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]
],s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [21 (n-1)/30], Round[22 (n-1)/30]}];
 KEUAFA23=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1

$y \rightarrow \text{SolUAFAS}[[2]], s1z \rightarrow \text{SolUAFAS}[[3]], s1x0 \rightarrow \text{SolUAFAS}[[4]], s1y0 \rightarrow \text{SolUAFAS}[[5]]$
 $, s1z0 \rightarrow \text{SolUAFAS}[[6]], s2x \rightarrow \text{SolUAFAS}[[7]], s2y \rightarrow \text{SolUAFAS}[[8]], s2z \rightarrow \text{SolUAFAS}[[9]]$
 $, s2x0 \rightarrow \text{SolUAFAS}[[10]], s2y0 \rightarrow \text{SolUAFAS}[[11]], s2z0 \rightarrow \text{SolUAFAS}[[12]]\}, \{i, \text{Round}$
 $[22 (n-1)/30], \text{Round}[23 (n-1)/30]\}$];
 KEUAFA24=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [23 (n-1)/30], Round[24 (n-1)/30]};
 KEUAFA25=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [24 (n-1)/30], Round[25 (n-1)/30]};
 KEUAFA26=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [25 (n-1)/30], Round[26 (n-1)/30]};
 KEUAFA27=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [26 (n-1)/30], Round[27 (n-1)/30]};
 KEUAFA28=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [27 (n-1)/30], Round[28 (n-1)/30]};
 KEUAFA29=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[
 9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]}, {i, Round
 [28 (n-1)/30], Round[29 (n-1)/30]};
 KEUAFA30=Table[dqE-
 HM2dq[RMUAFa[[Round[i]]]]/.{th1→TH1[[i]],th2→TH2[[i]],s1x→SolUAFAS[[1]],s1
 y→SolUAFAS[[2]],s1z→SolUAFAS[[3]],s1x0→SolUAFAS[[4]],s1y0→SolUAFAS[[5]]
 },s1z0→SolUAFAS[[6]],s2x→SolUAFAS[[7]],s2y→SolUAFAS[[8]],s2z→SolUAFAS[[

```

9]],s2x0→SolUAFAS[[10]],s2y0→SolUAFAS[[11]],s2z0→SolUAFAS[[12]]},{i,Round
[29 (n-1)/30],Round[n-1]}}];
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAngleF1.m"}],PageWidth→4000];
WriteString[handrun,"function y = UAFaAngleF1(x)

```

```

    cll = mat2cell(x,1,ones(1,length(x)));

```

```

["];
Table[WriteString[handrun,"th1",i, " ", " "],{i,1,Round[4 (n-1)/30]}}];
WriteString[handrun,"...
"]
Table[WriteString[handrun,"th2",i," ", " "],{i,1,Round[4 (n-1)/30]-1}];
WriteString[handrun,"th2",Round[4 (n-1)/30]]
WriteString[handrun,"] = cll{:}];

```

```

    y =",ToMatlab[{KEUAFA1,KEUAFA2,KEUAFA3,KEUAFA4}]];
Close[handrun]
\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\NS2\\Writing\\UAFaAngleF1.m
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAngleF2.m"}],PageWidth→4000];
WriteString[handrun,"function y = UAFaAngleF2(x)

```

```

    cll = mat2cell(x,1,ones(1,length(x)));

```

```

["];
Table[WriteString[handrun,"th1",i, " ", " "],{i,Round[4 (n-1)/30],Round[8 (n-1)/30]}}];
WriteString[handrun,"...
"]
Table[WriteString[handrun,"th2",i," ", " "],{i,Round[4 (n-1)/30],Round[8 (n-1)/30]-1}];
WriteString[handrun,"th2",Round[8 (n-1)/30]]
WriteString[handrun,"] = cll{:}];

```

```

    y =",ToMatlab[{KEUAFA5,KEUAFA6,KEUAFA7,KEUAFA8}]];
Close[handrun]
\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
Nexus\\NS2\\Writing\\UAFaAngleF2.m
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAngleF3.m"}],PageWidth→4000];
WriteString[handrun,"function y = UAFaAngleF3(x)

```

```

    cll = mat2cell(x,1,ones(1,length(x)));

```

```

["];
Table[WriteString[handrun,"th1",i, " ", " "],{i,Round[8 (n-1)/30],Round[12 (n-1)/30]}}];

```

```

WriteString[handrun,"...
"]
Table[WriteString[handrun,"th2",i," "],{i,Round[8 (n-1)/30],Round[12 (n-1)/30]-1}];
WriteString[handrun,"th2",Round[12 (n-1)/30]]
WriteString[handrun,"] = cll{:}];

y =",ToMatlab[{KEUAFA9,KEUAFA10,KEUAFA11,KEUAFA12}]];
Close[handrun]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleF3.m
handrun=OpenWrite[FileNameJoin[{{"\\Users\John\Documents\Mechanical_Engineerin
g\Thesis\Vicon Nexus\NS2\Writing", "UAFaAngleF4.m"}},PageWidth→4000];
WriteString[handrun,"function y = UAFaAngleF4(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    ["];
Table[WriteString[handrun,"th1",i," "],{i,Round[12 (n-1)/30],Round[16 (n-1)/30]}];
WriteString[handrun,"...
"]
Table[WriteString[handrun,"th2",i," "],{i,Round[12 (n-1)/30],Round[16 (n-1)/30]-1}];
WriteString[handrun,"th2",Round[16 (n-1)/30]]
WriteString[handrun,"] = cll{:}];

y =",ToMatlab[{KEUAFA13,KEUAFA14,KEUAFA15,KEUAFA16}]];
Close[handrun]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleF4.m
handrun=OpenWrite[FileNameJoin[{{"\\Users\John\Documents\Mechanical_Engineerin
g\Thesis\Vicon Nexus\NS2\Writing", "UAFaAngleF5.m"}},PageWidth→4000];
WriteString[handrun,"function y = UAFaAngleF5(x)

    cll = mat2cell(x,1,ones(1,length(x)));

    ["];
Table[WriteString[handrun,"th1",i," "],{i,Round[16 (n-1)/30],Round[20 (n-1)/30]}];
WriteString[handrun,"...
"]
Table[WriteString[handrun,"th2",i," "],{i,Round[16 (n-1)/30],Round[20 (n-1)/30]-1}];
WriteString[handrun,"th2",Round[20 (n-1)/30]]
WriteString[handrun,"] = cll{:}];

y =",ToMatlab[{KEUAFA17,KEUAFA18,KEUAFA19,KEUAFA20}]];
Close[handrun]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleF5.m

```

```
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing", "UAFaAngleF6.m"}], PageWidth→4000];
WriteString[handrun, "function y = UAFaAngleF6(x)
```

```
    cll = mat2cell(x, 1, ones(1, length(x)));
```

```
    ["];
```

```
Table[WriteString[handrun, "th1", i, " ", "], {i, Round[20 (n-1)/30], Round[24 (n-1)/30]}];
```

```
WriteString[handrun, "...
```

```
"]
```

```
Table[WriteString[handrun, "th2", i, " ", "], {i, Round[20 (n-1)/30], Round[24 (n-1)/30-1]}];
```

```
WriteString[handrun, "th2", Round[24 (n-1)/30]]
```

```
WriteString[handrun, "] = cll{:}];
```

```
    y = "ToMatlab[{KEUAFA21, KEUAFA22, KEUAFA23, KEUAFA24}];
```

```
Close[handrun]
```

```
\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
```

```
Nexus\\NS2\\Writing\\UAFaAngleF6.m
```

```
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing", "UAFaAngleF7.m"}], PageWidth→4000];
```

```
WriteString[handrun, "function y = UAFaAngleF7(x)
```

```
    cll = mat2cell(x, 1, ones(1, length(x)));
```

```
    ["];
```

```
Table[WriteString[handrun, "th1", i, " ", "], {i, Round[24 (n-1)/30], Round[28 (n-1)/30]}];
```

```
WriteString[handrun, "...
```

```
"]
```

```
Table[WriteString[handrun, "th2", i, " ", "], {i, Round[24 (n-1)/30], Round[28 (n-1)/30-1]}];
```

```
WriteString[handrun, "th2", Round[28 (n-1)/30]]
```

```
WriteString[handrun, "] = cll{:}];
```

```
    y = "ToMatlab[{KEUAFA25, KEUAFA26, KEUAFA27, KEUAFA28}];
```

```
Close[handrun]
```

```
\\Users\\John\\Documents\\Mechanical_Engineering\\Thesis\\Vicon
```

```
Nexus\\NS2\\Writing\\UAFaAngleF7.m
```

```
handrun=OpenWrite[FileNameJoin[{"\\Users\\John\\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing", "UAFaAngleF8.m"}], PageWidth→4000];
```

```
WriteString[handrun, "function y = UAFaAngleF8(x)
```

```
    cll = mat2cell(x, 1, ones(1, length(x)));
```

```
    ["];
```

```
Table[WriteString[handrun, "th1", i, " ", "], {i, Round[28 (n-1)/30], n-1}];
```

```
WriteString[handrun, "...
```

```
"]
```

```

Table[WriteString[handrun,"th2",i," ",{i,Round[28 (n-1)/30],n-2}];
WriteString[handrun,"th2",n-1]
WriteString[handrun,""] = cl[{}];

y=",ToMatlab[{KEUAFA29,KEUAFA30}]];
Close[handrun]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleF8.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAngleS1.m"}],PageWidth→400];

WriteString[handrun,"func = @UAFaAngleF1;
x0 = []
Table[WriteString[handrun, ".1, ", {i,2((Round[4 (n-1)/30]))-1}];
WriteString[handrun, ".1];
x = fsolve(func, x0,saoptimset('TolFun',.001));
"];
WriteString[handrun,"%% MatrixForm
SSUAFa1 = x;
xlswrite('SUAFa1.xlsx',SSUAFa1)"];
Close[handrun]
ClearAll[i]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleS1.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAngleS2.m"}],PageWidth→400];

WriteString[handrun,"func = @UAFaAngleF2;
x0 = []
Table[WriteString[handrun, ".1, ", {i,2((Round[8 (n-1)/30]+1)-(Round[4 (n-1)/30]))-1}];
WriteString[handrun, ".1];
x = fsolve(func, x0,saoptimset('TolFun',.001));
"];
WriteString[handrun,"%% MatrixForm
SSUAFa2 = x;
xlswrite('SUAFa2.xlsx',SSUAFa2)"];
Close[handrun]
ClearAll[i]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleS2.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\\Mechanical_Engineerin
g\\Thesis\\Vicon Nexus\\NS2\\Writing","UAFaAngleS3.m"}],PageWidth→400];

WriteString[handrun,"func = @UAFaAngleF3;
x0 = []
Table[WriteString[handrun, ".1, ", {i,2((Round[12 (n-1)/30]+1)-(Round[8 (n-1)/30]))-1}];

```



```

WriteString[handrun, ".1"];
x = fsolve(func, x0, saoptimset("TolFun", .001));
"];
WriteString[handrun, "%% MatrixForm
SSUAFa3 = x;
xlswrite('SUAFa3.xlsx', SSUAFa3)"];
Close[handrun]
ClearAll[i]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleS3.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\\Mechanical_Engineerin
g\\Thesis\Vicon Nexus\NS2\Writing", "UAFaAngleS4.m"}], PageWidth→400];

WriteString[handrun, "func = @UAFaAngleF4;
x0 = []
Table[WriteString[handrun, ".1, ", {i, 2((Round[16 (n-1)/30]+1)-(Round[12 (n-1)/30]))-
1}];
WriteString[handrun, ".1"];
x = fsolve(func, x0, saoptimset("TolFun", .001));
"];
WriteString[handrun, "%% MatrixForm
SSUAFa4 = x;
xlswrite('SUAFa4.xlsx', SSUAFa4)"];
Close[handrun]
ClearAll[i]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleS4.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\\Mechanical_Engineerin
g\\Thesis\Vicon Nexus\NS2\Writing", "UAFaAngleS5.m"}], PageWidth→400];

WriteString[handrun, "func = @UAFaAngleF5;
x0 = []
Table[WriteString[handrun, ".1, ", {i, 2((Round[20 (n-1)/30]+1)-(Round[16 (n-1)/30]))-
1}];
WriteString[handrun, ".1"];
x = fsolve(func, x0, saoptimset("TolFun", .001));
"];
WriteString[handrun, "%% MatrixForm
SSUAFa5 = x;
xlswrite('SUAFa5.xlsx', SSUAFa5)"];
Close[handrun]
ClearAll[i]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleS5.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\\Mechanical_Engineerin
g\\Thesis\Vicon Nexus\NS2\Writing", "UAFaAngleS6.m"}], PageWidth→400];

```

```

WriteString[handrun,"func = @UAFaAngleF6;
x0 = []
Table[WriteString[handrun, ".1, ", {i, 2, ((Round[24 (n-1)/30]+1)-(Round[20 (n-1)/30]))-1}];
WriteString[handrun, ".1];
x = fsolve(func, x0, saoptimset('TolFun', .001));
"];
WriteString[handrun, "%% MatrixForm
SSUAFa6 = x;
xlswrite('SUAFa6.xlsx', SSUAFa6)"];
Close[handrun]
ClearAll[i]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleS6.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\Mechanical_Engineerin
g\Thesis\Vicon Nexus\NS2\Writing", "UAFaAngleS7.m"}], PageWidth→400];

WriteString[handrun,"func = @UAFaAngleF7;
x0 = []
Table[WriteString[handrun, ".1, ", {i, 2, ((Round[28 (n-1)/30]+1)-(Round[24 (n-1)/30]))-1}];
WriteString[handrun, ".1];
x = fsolve(func, x0, saoptimset('TolFun', .001));
"];
WriteString[handrun, "%% MatrixForm
SSUAFa7 = x;
xlswrite('SUAFa7.xlsx', SSUAFa7)"];
Close[handrun]
ClearAll[i]
\Users\John\Documents\Mechanical_Engineering\Thesis\Vicon
Nexus\NS2\Writing\UAFaAngleS7.m
handrun=OpenWrite[FileNameJoin[{"\\Users\John\Documents\Mechanical_Engineerin
g\Thesis\Vicon Nexus\NS2\Writing", "UAFaAngleS8.m"}], PageWidth→400];

WriteString[handrun,"func = @UAFaAngleF8;
x0 = []
Table[WriteString[handrun, ".1, ", {i, 2, ((Round[n])-(Round[28 (n-1)/30]))-1}];
WriteString[handrun, ".1];
x = fsolve(func, x0, saoptimset('TolFun', .001));
"];
WriteString[handrun, "%% MatrixForm
SSUAFa8 = x;
xlswrite('SUAFa8.xlsx', SSUAFa8)"];
Close[handrun]
ClearAll[i]

```



```

{Flatten[Append[
  {Flatten[Append[
    {Flatten[Append[
      {Flatten[Append[{Table[ArcCos[Cos[SolUAFa1[[i]]],{i,Length[SolUAFa1]/2+1,Length
[SolUAFa1]}]}],
Table[ArcCos[Cos[SolUAFa2[[i]]],{i,Length[SolUAFa2]/2+1,Length[SolUAFa2]}]],2]
},
Table[ArcCos[Cos[SolUAFa3[[i]]],{i,Length[SolUAFa2]/2+1,Length[SolUAFa3]}]],2]
},
Table[ArcCos[Cos[SolUAFa4[[i]]],{i,Length[SolUAFa4]/2+1,Length[SolUAFa4]}]],2]
},
Table[ArcCos[Cos[SolUAFa5[[i]]],{i,Length[SolUAFa5]/2+1,Length[SolUAFa5]}]],2]
},
Table[ArcCos[Cos[SolUAFa6[[i]]],{i,Length[SolUAFa6]/2+1,Length[SolUAFa6]}]],2]
},
Table[ArcCos[Cos[SolUAFa7[[i]]],{i,Length[SolUAFa7]/2+1,Length[SolUAFa7]}]],2]
},
Table[ArcCos[Cos[SolUAFa8[[i]]],{i,Length[SolUAFa8]/2+1,Length[SolUAFa8]}]],2];
SolUAFa02=Table[SolUAFa022[[i]],{i,1,n+6}];
MatSolUAFa={SolUAFa01,SolUAFa02};

```