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POSITION CONTROL STRATEGIES FOR THE BARRETT HAND

By

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A thesis submitted to the Graduate

Faculty of Idaho State University

in partial fulfillment of the requirements

for the degree of Master of Science in Measurement and Control Engineering

Idaho State University

May 2016

To the Graduate Faculty:

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Acknowledgements:

First of all, I would like to express my sincere gratitude to my major advisor Dr. Alba Perez for her continuous support during my Masters study as well thesis, for her patience, motivation and cooperation. Without her guidance and support my thesis work would have been very difficult. I would like to thank her for encouraging my research and for allowed me to work under her supervision. I would like to thanks to my committee member Dr. Marco Schoen and Dr. Maria Wong for their help during my study at ISU. I would also like to thank my classmates at ISU, especially Omid and Abhijit whose help and companionship was always a great source of inspiration for me. Thanks to the faculty of ISU's College of Science and Engineering for their support throughout my graduate studies. I have learnt many things from them. A very special thanks to my parents, uncles, relatives and my siblings especially my elder sister 'Wahida'. Without their continuous supports it would be difficult for me to accomplish my goal.

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Abstract:

The objective of the thesis is to find the position control of the Barrett Hand BH 282. Barrett Hand BH 282 is a three finger robot. Finger is denoted by F1, F2 and F3. Finger 1 and Finger 2 can rotate up to 180 degrees. However, Finger 3 is fixed. Moreover, before finding position control, there are some topics need to be determined. At first, forward and inverse kinematics of the Barrett Hand has to be solved. After that, Jacobian has to be determined. Thus, forward kinematics, inverse kinematics and Jacobian will give the tools for design a controller. In the controller section, PID controller was considered. PID is a three parameters controller. The parameters are Proportional (P), Integral (I), and derivative (D) respectively. Proportional controller effect is reducing rise time, Integral controller effect is eliminating steady state error for a constant or step input, Derivative controller effect is increasing the stability of the system or in the other sense reducing overshoot. Lastly, the trajectory planning has been determined. Cubic polynomial technique was used in determining trajectory planning. In trajectory planning section, position, velocity and acceleration was determined and then graph was plotted. In whole research, two software has been used, Mathematica and Matlab.

Chapter 1

Introduction

1.1 Robotic Hand Overview:

Robotic hand is an essentials tools for modern days. Now a day's robotics hand is using for many applications. Robotic hand can be used in space industry, in mining industry or any type of industry. Robotic hand can be designed to perform any type of desired task. As for example grasping an object by using their position. Then we can control robotic hand by using many software. Such as MATLAB, Simulink, ROS etc. Depending on the application we can design a hand to achieve our goal and desired task. There are many types of hand. Below I have described some hands. However, in my thesis I have used Barrett Hand BH 282. First of all, I would like to describe Barrett hand BH 282. After that I will describe some other hand.

1.2 Barrett Hand:

Barrett hand is designed by Barrett Hand Technology Inc. There is some type of Barrett hand such as BH 280, BH 282, BH 262. Here I am going discuss BH 282 hand. The BH 282 is a multi-fingered hand. It has three fingers. Out of three finger two of them can rotate and of of them is fixed. We can denote three finger as F1, F2 and F3. BH 282 is programmable grasper with the dexterity to secure the target of different sizes of objects, shapes, and orientations. The image of Barrette hand is given in the next page.

Figure 1.1: Barrett Hand 282

Here Finger F1 and F2 can rotate but Finger F 3 is fixed. Finger F1 and F2 can rotate 180 degrees. Finger base joint is 140 degrees, Fingertip joint 45 degrees, and finger spread is 180 degrees respectively. Barrett hand BH 282 has 4 motors and it has 8 axes. Below diagram is the cross-section of BH 282 hand.

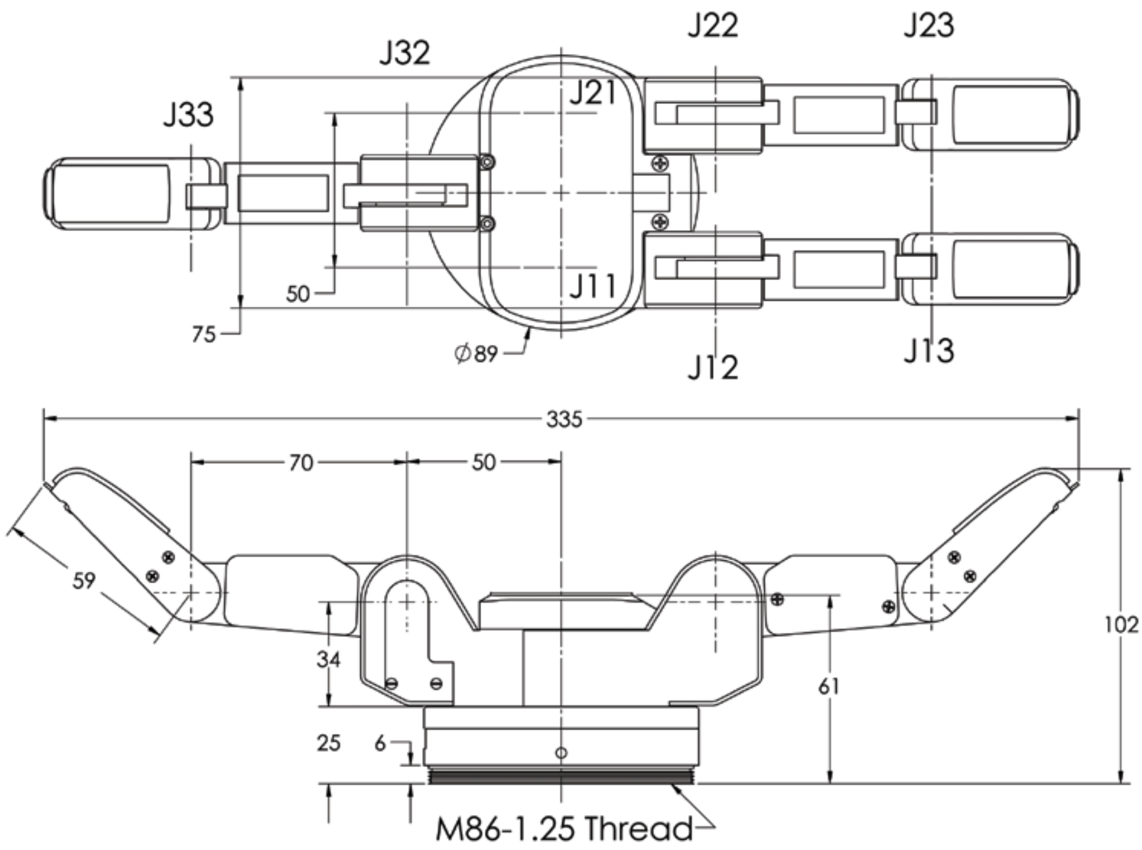

Figure 1.2: Cross-section of BH 282 hand

The specification of BH 282 hand is given below:

BH8-282 SPECIFICATIONS		
Payload		6.0 kg
Weight		980 grams
Motor Encoder Resolution		4096 counts
Motor Type		Brushless Electric
Communication		CAN, RS-232
		(USB adapters provided)
Finger Speed	Finger full open to close	1.0 sec
	Full 180 degree spread	0.5 sec
DC Operation	Voltage	20-80 VDC
	Idle/typ/peak	7/15/250 W
AC Operation	Single phase	85-260 VAC, 50/60 Hz
	Idle/typ/peak	10/20/300 W
AC Power Supply	Dimensions, L x W x H	204 x 90 x 54 mm
	Weight	0.7 kg
Kinematics	Total fingers	3 (1 fixed, 2 rotatable)
	Total hand axes	8
	Total hand motors	4
Range of motion	Finger base joint	140º
	Fingertip joint	45º
	Finger Spread	180º

Table 1.1: Specification of Barrett Hand BH 282

Moreover, BH 282 hand is totally self-contained hand. BH 282 is Communicating by high-speed CANbus or industry-standard serial communications. The integration with any arm is fast and simple. The BH8-series immediately multiplies the value of any arm requiring flexible automation. The Barrett Hand BH 282 neatly houses its own communications electronics, servo controllers, and all four brushless motors. It is three multi-jointed fingers, two have an extra degree of freedom with 180 degrees of lateral mobility supporting a large variety of grasp types. All joints have high-precision position encoders.

1.3 Barrett Hand BH 282 Software:

Barrett Hand is fully source code and examples are given and included with purchase. The default software name of the Barrett hand is PY hand. PY hand can be installed in Windows or Linux. The BH Control application fits with both Linux and Windows operating system and it has graphical user interface (GUI). By using ROS or Robotic Operating System we can determine the grasping position of BH 282 hand. Barrett hand is given in the ROS. The visual control window of the BH Control application enables control of the hand interactively. The user is capable of moving the fingers of the Barrett Hand to any desired position with a mouse.

1.4 Shadow Dexterous robotic hand:

Shadow dexterous robotic hand is very close to the human hand. It has 20 Degrees of Freedom to regenerate as nearly as possible the kinematics and dexterity of the human hand. It has 24 movements and 20 Degrees of Freedom. It has up to 129 sensors throughout the Hand. Fully ROS integration allows user to implement in different application purpose.

Figure 1.3: Shadow Dexterous robotic hand

1.5 Schunk Hand:

Schunk hand has five fingers and 20 degree of freedom. The weight of this hand is 1.3 kg and length is 242.5 mm. Palm thickness of this hand is 0.90 mm. Below the schunk figure is given:

Figure 1.4: Schunk Robotic Hand

1.6 DLR Hand II:

One of the most advanced hand is DLR hand. DLR hand has 3 joint position sensors, 3 joint torque sensors, 3 motor position/speed sensors: analog Hall sensors with interpolation 1 six-dimensional finger tip force torque sensor: strain gauge sensors, 3 motor temperature sensors, 3 sensors for temperature compensation. DLR hand has four identical finger and three degree of freedom.

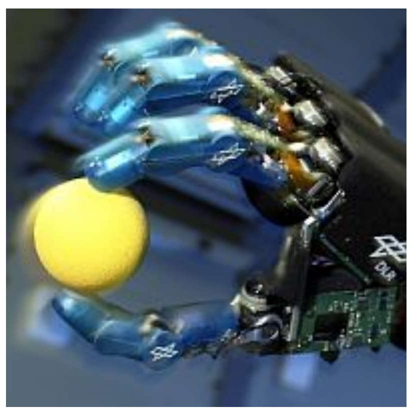

Figure 1.5: DLR Hand II

CHAPTER 2

BARRET HAND KINEMATICS

2.1 Basic Robot Kinematics:

Kinematics is the study of the motion of bodies. In kinematics we study without consideration of forces or moments. Robot kinematics is the analytical study of the motion of a robot manipulator. Formulating the suitable kinematics equation for a robot is very important. Without kinematics it is impossible to operate the robot. Thus if we want to analyze and operate a robot we must need kinematics. There are two types of kinetics, one is forward kinematics and another one is inverse kinematics. In my thesis I have found forward and inverse kinematics for Barrett Hand. And then I have found the Jacobian for Barrett Hand BH 282. In kinematics mainly we use two different types of space. One space is called Cartesian space and another is one Joint space. The transformation between two Cartesian coordinate systems can be separated into a rotation and a translation. There are many ways we can find the transformation, however the mostly used way is Denavit & Hartenberg parameter. DH parameter can show us the general transformation between two joints requires. DH parameter requires four parameters. Later in the DH parameter section I will explain four parameters. These four parameters are known as the Denavit-Hartenberg (DH) parameters. I will also discuss the Forward kinematics and Inverse kinematics.

2.1.1 Forward Kinematics:

First of all, what is forward kinematics. Forward kinematics will determine the position and the orientation of the end effector of the robot when we will provide the value of joint variables. Forward kinematics is easy to find compare to the Inverse kinematics and there is less complexity deriving the equations than inverse kinematics.

2.1.2 Inverse Kinematics:

Inverse kinematics is the reverse process of the forward kinematics. In inverse kinematics we determine the values of joint variables by providing the value of end effector's position and orientation. Finding the inverse kinematics is comparatively difficult than forward kinematics. There are two techniques in inverse kinematics. One is Analytical solution method and another one is Numerical method. In an analytical solution the joint variables are solved by the given configuration. And in numerical method the joint variables are obtained by numerical techniques. Below the figure shows us the relationship between forward and inverse kinematics.

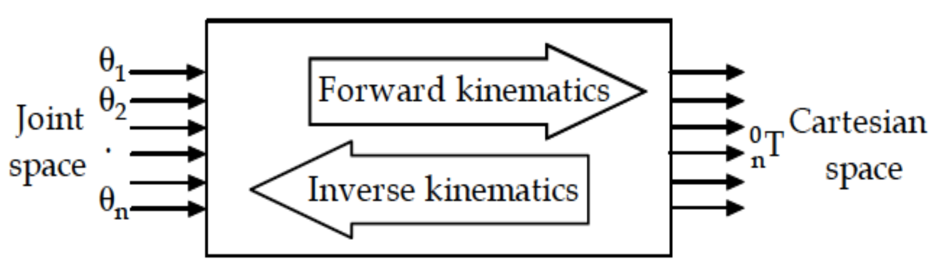

Figure 2.1: Relation between Forward and Inverse Kinematics

2.1.3 The Denavit-Hartenberg (D-H) Parameter:

In robotics finding forward kinematics is difficult. Forward kinematics problem concerned with the relationship between the position and orientation of the end effector and individual joints of the manipulator. For finding forward kinematics we need to used Denavit-Hartenberg (DH)convention. In DH convention each homogeneous transformation is basically a product of four (two rotations and two translations) basic transformation. These four quantities are called parameters, in short we can say DH Parameters. Here I am going to describe four DH Parameters. Four parameters are θ_i, a_i, d_i, and α_i. These parameters are associated with the link and joint of the manipulator.

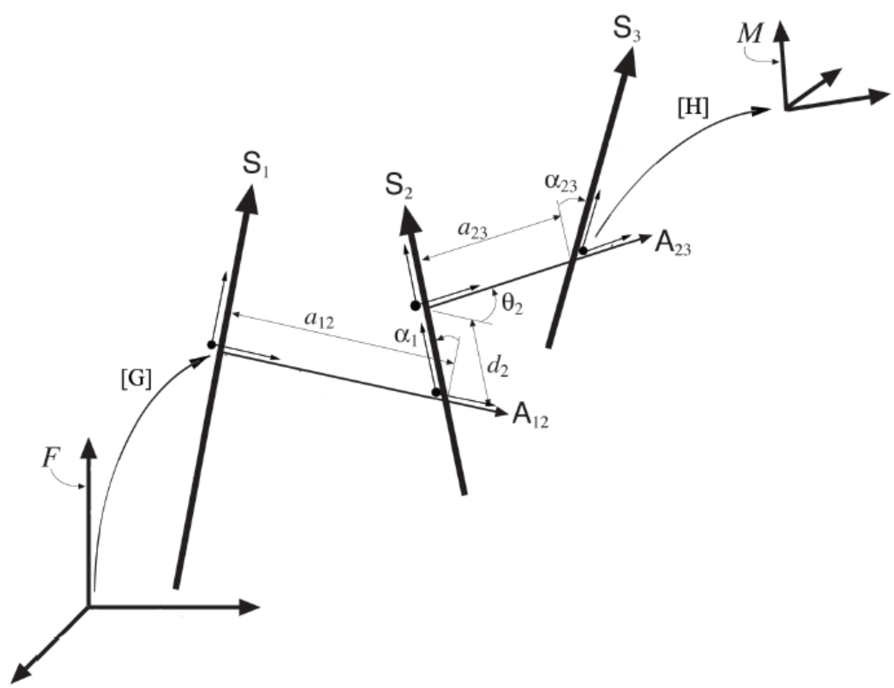

Fig 2.2: Local Transformation along the links of a robot.

As I have described that only four parameters are needed to transform from local frame to local frame. Two of them are transformation along the link, which is X axis displacement and two are Z axis displacement. Here description of the parameter is given below

#1. Link Length a_i : the distance between joint axes S_{i-1} and S_i measured from common normal line $A_{i-1,i.}$

#2. Twist Angle α_i : An angle between joint axes S_{i-1} and S_i measured from common normal line $A_{i-1,i.}$

#3. Joint Angle θ_i : An angle between previous common normal line $A_{i-1,i}$ and next common normal line $A_{i,i+1.}$ If the joint angle is revolute joint, then joint angle includes the variables values of the joint rotation.

#4: Offset d_i : the distance between previous common normal line and $A_{i-1,i}$ and next common normal line $A_{i,i+1.}$ If the joint is prismatic joint, then offset includes the variable values of the slide.

2.1.4 Jacobian:

Jacobian is basically set of partial derivatives or partial differential equations. Jacobian matrix is very useful tools. Jacobian's is really important in robot control and modelling. From Jacobian we can get linear and angular velocity or end effector orientation and position. In other way we can also get joint angles from Jacobian. By using Jacobian, we can also obtain the torque. The form of Jacobian is looks like this $\dot{X} = J\dot{\theta}$ where $\dot{X}$ represents linear and angular velocity, J is the Jacobian matrix and $\dot{\theta}$ is joint angle.

2.2 Forward Kinematics of Barrett Hand:

2.2.1: Forward Kinematics for Finger 1:

By using calculated forward Kinematics in this section we will determine fingertip position and orientation with respect to the palm. The following homogeneous transform equation is used to determine the transforms between axes k and k-1.

$$^{k-1}T_k = \begin{bmatrix} C\theta_k & -s\theta_k & 0 & a_{k-1} \\ s\theta_k c\alpha_{k-1} & c\theta_k c\alpha_{k-1} & -s\alpha_{k-1} & -s\alpha_{k-1} d_k \\ s\theta_k s\alpha_{k-1} & c\theta_k s\alpha_{k-1} & c\alpha_{k-1} & c\alpha_{k-1} d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation 2.1: Homogeneous Transform Between {K-1} and {k}………. (1)

Where:

a_{k-1} = distance from z_{k-1} to z_k measured along x_{k-1}

a_{k-1} = angle between z_{k-1} to z_k measured about x_{k-1}

d_k = distance from x_{k-1} to x_k measured along z_k

θ_k = angle between x_{k-1} to x_k measured about z_k

$$c\theta_k = \cos(\theta_k)$$

$$s\theta_k = \sin(\theta_k)$$

The forward kinematics are determined using the following equation:

$$^{w}T_{T} = ^{w}T_{0} {}^{0}T_{1} {}^{1}T_{2} {}^{2}T_{3} {}^{3}T_{T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

DH Parameters values applicable for all finger is given below

Parameter	Value
A_1	50mm
A_2	70mm
A_3	50mm
D_0	25mm
D_3	9.5mm
Φ_2	2.46°
Φ_3	50°

Table 2.1: DH Parameter Value

Also, DH Link parameter is given below

Joint	a_{k-1}	a_{k-1}	d_k	θ_k
1	0	Π	0	Θ_{J41}
2	A_1	$-\pi/2$	0	$\Theta_{J11} + \Phi_2$
3	A_2	0	0	$\Theta_{J12} + \Phi_3$
T	A_3	$-\pi/2$	D_3	0

Table 2.2: DH Link parameter for finger 1

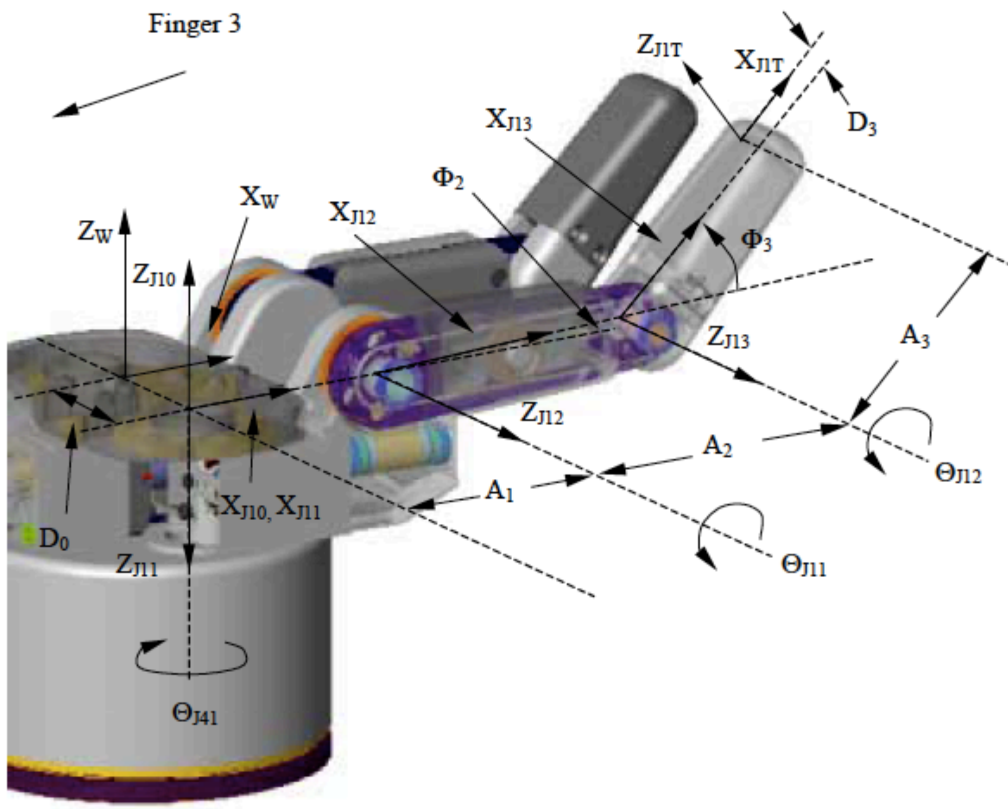

Figure 2.3: Finger F1 DH frame

Now, the relationship between frame 0 and the world coordinate frame is obtained by using:

$$ {}^{W}_{0}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -D_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{.......................................} \quad (3) $$

From homogeneous transform in equation 1 and the link parameters for finger F1 in Table 2 and from equation 2 we will obtain the forward kinematics for finger 1 or F1.

$$ {}^{W}_{0}T = \begin{bmatrix} C_4C_{ab} & S_4 & -C_4S_{ab} & A_3c_4c_{ab} + D_3(-c_4s_{ab}) + A_2c_4c_a + A_1c_4 \\ -S_4C_{ab} & C_4 & S_4S_{ab} & A_3(-s_4c_{ab}) + D_3s_4s_{ab} - A_2s_4s_{ab} - A_2s_4c_a - A_1s_4 - D_0 \\ S_{ab} & 0 & C_{ab} & A_3s_{ab} + D_3c_{ab} + A_2s_a \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Equation 2.2 - Forward Kinematics for Finger F1……………………….. (4)

Where:

$$ a = \Theta_{J11} + \Phi_2 $$

$$ a = \Theta_{J12} + \Phi_3 $$

$$ C_{ab} = \cos\,(a + b) $$

$$ s_{ab} = \sin(a + b) $$

$$ c_4 = \cos(\Theta_{J41}) $$

$$ s_4 = \sin(\Theta_{J41}) $$

2.2.2 Forward Kinematics for Finger 2:

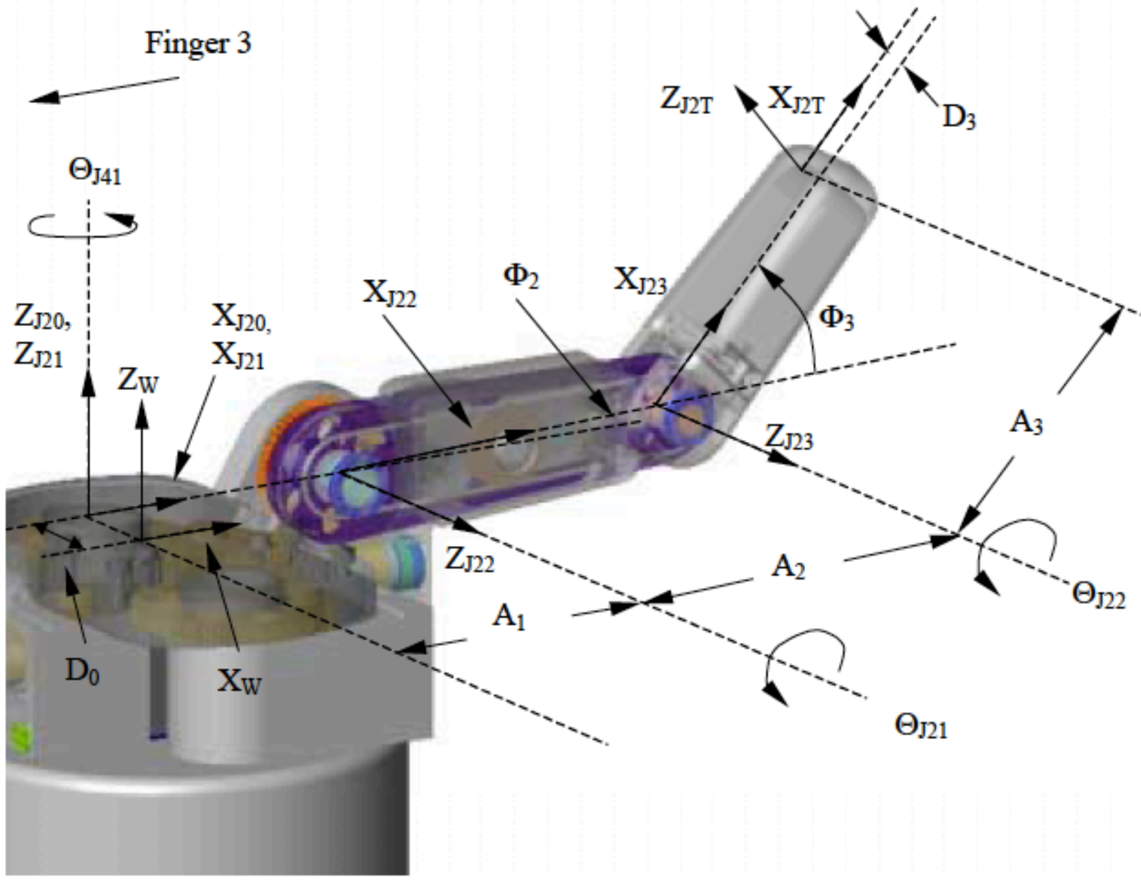

Figure 2.4: DH Frame for Finger 2

DH Link parameter for finger 2 is below

Joint	a_{k-1}	α_{k-1}	d_k	θ_k
1	0	0	0	Θ_{J41}
2	A_1	$\pi/2$	0	$\Theta_{J21} + \Phi_2$
3	A_2	0	0	$\Theta_{J22} + \Phi_3$
T	A_3	$-\pi/2$	D_3	0

Table 2.3: DH Parameters Frame for Finger 2

The relationship between frame 0 and the world coordinate frame is determined by using:

$$ {}^{W}_{0}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -D_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

We will obtain the forward kinematics by using homogeneous transform in Equation 1 and the link parameters for finger F2 in Table 4.

$$ {}^{W}_{0}T = \begin{bmatrix} c_4 c_{ab} & -s_4 & -c_4 s_{ab} & A_3 c_4 c_{ab} + D_3(-c_4 s_{ab}) + A_2 c_4 c_a + A_1 c_4 \\ s_4 c_{ab} & c_4 & -s_4 s_{ab} & A_3 s_4 c_{ab} + D_3(-s_4 s_{ab}) + A_2 s_4 c_a + A_1 s_4 + D_0 \\ s_{ab} & 0 & c_{ab} & A_3 s_{ab} + D_3 c_{ab} + A_2 s_a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation 2.3: Forward Kinematics for Finger F2

Where:

$$a = \Theta_{J21} + \Phi_2$$

$$b = \Theta_{J22} + \Phi_3$$

$$c_{ab} = \cos(a + b)$$

$$s_{ab} = \sin(a + b)$$

$$c_4 = \cos(\Theta_{J41})$$

$$s_4 = \sin(\Theta_{J41})$$

2.2.3: Forward Kinematics for Finger F3:

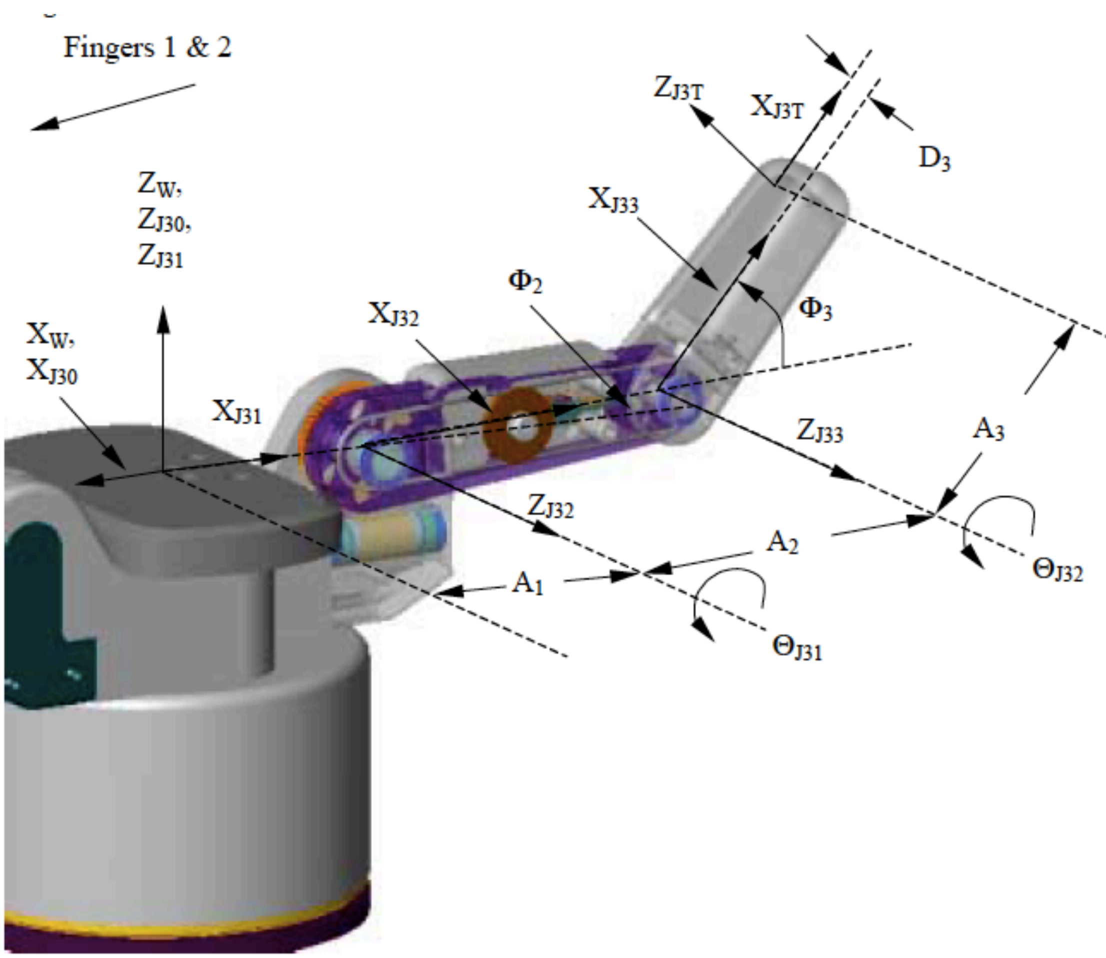

Figure 2.5: DH Frame for Finger F3

For the finger 3, here spread motion QJ41 is only affects the motions for fingers F1 and F2. Thus, we have added an extra frame for finger F3 for remaining consistency between all of the finger joint variables.

Here D-H Link Parameters for Finger F3 is given below:

Joint	a_{k-1}	α_{k-1}	d_k	θ_k
1	0	0	0	π
2	A_1	$\pi/2$	0	$\Theta_{J31} + \Phi_2$
3	A_2	0	0	$\Theta_{J32} + \Phi_3$
T	A_3	$-\pi/2$	D_3	0

Table 2.4: DH Parameter Frame for Finger F 3

The relationship between frame 0 and the world coordinate frame is determined by using:

$$ {}^{W}_{0}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots\dots\dots\dots (7)$$

The forward kinematics is obtained by from euqation1 and the link parameters for finger F 3 in table 5. By using Equation 5, the forward kinematics for finger 3 is:

$$ {}^{W}_{T}T = \begin{bmatrix} -C_{ab} & 0 & s_{ab} & -A_3 c_{ab} + D_3 s_{ab} - A_2 c_a - A_1 \\ 0 & 1 & 0 & 0 \\ S_{ab} & 0 & C_{ab} & A_3 s_{ab} + D_3 c_{ab} + A_2 s_a \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Equation 2.4 - Forward Kinematics for Finger F3

Where:

$$a = \Theta_{\mathrm{J31}} + \Phi_2$$

$$b = \Theta_{\mathrm{J32}} + \Phi_3$$

$$c_{ab} = \cos(a + b)$$

$$s_{ab} = \sin(a + b)$$

2.3 Inverse Kinematics for Barrett Hand:

2.3.1 Inverse Kinematics for Finger 1:

From finger 1 forward kinematics we have known the following values.

$$P_{12} = S_4\ ,\quad S_4 = \sin\,(\theta J41),\quad P_{22} = c_4,\quad C_4 = \cos\,(\theta J41)$$

$$\frac{P_{12}}{P_{22}} = \frac{\sin(\theta J41)}{\cos(\theta J41)} = \tan(\theta J41)$$

$$\theta_{J41} = \tan^{-1}\left(\frac{p_{12}}{p_{22}}\right) \qquad (8)$$

Now,

$$P_{21} = -S_4 c_{ab},\ \ S_4 = \sin\left(\theta_{J41}\right), C_{ab} = -\frac{P_{21}}{S_4}\ , P_{23} = S_4 S_{ab},\quad S_{ab} = \frac{p_{23}}{s_4}$$

$$P_{34} = A_3 S_{ab} + D_3 C_{ab} + A_2 S_a.$$

$$S_a = \frac{P_{34} - A_3 S_{ab} - D_3 C_{ab}}{A_2}$$

$$S_a = \frac{P_{34} - A_3\left(\frac{P_{23}}{S_4}\right) - D_3\left(-\frac{P_{21}}{S_4}\right)}{A_2}$$

$$a = \sin^{-1}\left\{\frac{P_{34} - A_3\left(\frac{P_{23}}{s_4}\right) - D_3\left(-\frac{P_{21}}{s_4}\right)}{A_2}\right\}$$

Again,

$$P_{14} = A_3 c_4 c_{ab} + D_3(-c_4 s_{ab}) + A_2 c_4 c_a + A_1 s_4$$

$$c_a = \frac{P_{14} - A_3 C_4 C_{ab} - D_3(-c_4 s_{ab}) - A_1 s_4}{A_2 c_4} \quad \text{thus} \quad c_a = \frac{P_{14} - A_3 C_4\left(-\frac{P_{21}}{S_4}\right) - D_3\left(-C_4 \frac{P_{23}}{S_4}\right) - A_1 S_4}{A_2 C_4}$$

$$a = \mathrm{Cos}^{-1}\left\{\frac{\mathrm{P}_{14} - \mathrm{A}_3\mathrm{C}_4\left(-\frac{\mathrm{P}_{21}}{\mathrm{S}_4}\right) - \mathrm{D}_3\left(-\mathrm{C}_4\frac{\mathrm{P}_{23}}{S_4}\right) - \mathrm{A}_1\mathrm{S}_4^{\mathrm{A}}}{\mathrm{A}_2\mathrm{C}_4}\right\}$$

$$\mathrm{P}_{21} = -\mathrm{S}_4\mathrm{C}_{\mathrm{ab}}, \mathrm{P}_{23} = \mathrm{S}_4\mathrm{S}_{\mathrm{ab}}, \quad \frac{\mathrm{P}_{23}}{\mathrm{P}_{21}} = \frac{S_4S_{ab}}{-S_4C_{ab}} = -\mathrm{tan}_{\mathrm{ab}}$$

$$\mathrm{ab} = \tan^{-1}\left(\frac{-\mathrm{P}_{23}}{\mathrm{P}_{2\mathrm{L}}}\right) \quad \text{where [ab = a + b]}$$

$$\mathrm{b} = \tan^{-1}\left(\frac{-\mathrm{P}_{23}}{\mathrm{P}_{21}}\right) - \left\{\mathrm{Sin}^{-1}\frac{\mathrm{P}_{34} - \mathrm{A}_3\left(\frac{\mathrm{P}_{23}}{S_4}\right) - \mathrm{D}_3\left(-\frac{\mathrm{P}_{21}}{S_4}\right)}{\mathrm{A}_2}\right\}$$

Given, $a = \theta_{J11} + \theta_2$.

$$\theta_{J11} = \sin^{-1}\{\frac{\mathrm{P}_{34} - \mathrm{A}_3\left(\frac{\mathrm{P}_{23}}{s_4}\right) - D_3\left(-\frac{P_{21}}{s_4}\right)}{A_2}\} - \theta_2 \quad \text{..} (9)$$

Again calculation for θ_{J12}

$\mathrm{b} = \theta_{J12} + \theta_3$

$\theta_{J12} = \mathrm{b} - \theta_3$

$$\theta_{J12} = \left\{\tan^{-1}\left(\frac{-\mathrm{P}_{23}}{\mathrm{P}_{21}}\right) - \sin^{-1}\left\{\frac{P_{34} - A_3\left(\frac{P_{23}}{S_4}\right) - D_3\left(-\frac{P_{21}}{S_4}\right)}{\mathrm{A}_2}\right\}\right\} - \theta_3 \text{.........................}(10)$$

Therefore equation 8, 9 and 10 is inverse kinematics for $\theta_{J41}, \theta_{J11}$ *and* θ_{J12} respectively.

2.3.2 Inverse Kinematics for Finger 2:

We will obtain inverse kinematics for finger 2 from forward kinematics of finger 2. From forward kinematics of finger 2 we have obtained following identities:

$\mathrm{P_{21}} = \mathrm{S_4 C_{ab}}, \quad \mathrm{C_{ab}} = \frac{\mathrm{P_{21}}}{S_4}, \ \mathrm{P_{23}} = -\mathrm{S_4 S_{ab}}, \ S_{ab} = -\frac{\mathrm{P_{23}}}{S_4}, \qquad \frac{\mathrm{P_{23}}}{\mathrm{P_{21}}} = \frac{-\mathrm{S_4 S_{ab}}}{\mathrm{S_4 C_{ab}}} = -\tan_{\mathrm{ab}}.$

$\tan_{\mathrm{ab}} = -\frac{\mathrm{P_{23}}}{\mathrm{P_{21}}} \qquad \tan_{\mathrm{ab}} = \tan(a+b)$

Thus, $ab = \tan^{-1}\left(-\frac{\mathrm{P_{23}}}{\mathrm{P_{21}}}\right)$. $[ab = a + b]$

Now,

$\mathrm{P_{34}} = \mathrm{A_3 S_{ab}} + \mathrm{D_3 C_{ab}} + \mathrm{A_2 S_a}.$

$$S_a = \frac{P_{34} - A_3 S_{ab} - D_3 C_{ab}}{A_2}$$

$a = Sin^{-1}\left\{\frac{P_{34} - A_3\left(-\frac{P_{23}}{S_4}\right) - D_3\left(\frac{P_{21}}{S_4}\right)}{\mathrm{A_2}}\right\}, \ a = \theta_{J21} + \theta_2$

$\theta_{J21} = \mathrm{a} - \theta_2$. Thus,

$\theta_{J21} = \mathrm{Sin}^{-1}\left\{\frac{\mathrm{P_{34}} - \mathrm{A_3}\left(-\frac{\mathrm{P_{23}}}{S_4}\right) - \mathrm{D_3}\left(\frac{\mathrm{P_{21}}}{S_4}\right)}{\mathrm{A_2}}\right\}\theta_2$…………………………………….. (11)

$a + b = \tan^1\left(-\frac{\mathrm{P_{23}}}{\mathrm{P_{21}}}\right)$

$\mathrm{b} = \tan^{-1}\left(-\frac{\mathrm{P_{23}}}{\mathrm{P_{21}}}\right) - \mathrm{Sin}^{-1}\left\{\frac{\mathrm{P_{23}} - \mathrm{A_3}\left(-\frac{\mathrm{P_{23}}}{S_4}\right) - \mathrm{D_3}\left(\frac{\mathrm{P_{21}}}{\mathrm{S_4}}\right)}{\mathrm{A_2}}\right\}.$

$\theta_{J22} = \mathrm{b} - \theta_3.$

$\theta_{J22} = \tan^{-1}\left(-\frac{P_{23}}{P_{21}}\right) - Sin^{-1}\left\{\frac{\mathrm{P_{34}} - \mathrm{A_3}\left(-\frac{\mathrm{P_{23}}}{S_4}\right) - \mathrm{D_3}\left(\frac{\mathrm{P_{21}}}{\mathrm{S_4}}\right)}{A_2}\right\} -$

θ_3…………………………………(12)

Therefore equation 11 and 12 is inverse kinematics for $\theta_{J21} and\ \theta_{J22}$ respectively.

2.3.3 Inverse Kinematics for Finger 3:

We will obtain inverse kinematics for finger 3 from forward kinematics of finger 3. From forward kinematics of finger 3 we have obtained following identities:

$P_{13} = S_{ab}, P_{33} = C_{ab}, \frac{P_{13}}{P_{33}} = \frac{S_{ab}}{C_{ab}} = \tan_{ab}$

$ab = \tan^{-1}\left(\frac{P_{13}}{P_{33}}\right).$ Where [ab=a+b]

Now, $P_{14} = -A_3 C_{ab} + D_3 S_{ab} - A_2 C_a - A_1.$

$C_a = \frac{-A_3 P_{33} + D_3 P_{13} - A_1 - P_{14}}{A_2}.$

$$a = \cos^{-1}\left\{\frac{-A_3 P_{33} + D_3 P_{13} - A_1 - P_{14}}{A_2}\right\}$$

$$a + b = \tan^{-1}\left(\frac{P_{13}}{P_{33}}\right)$$

$b = \tan^{-1}\left(\frac{P_{13}}{P_{33}}\right) - a.$

$$b = \tan^{-1}\left(\frac{P_{13}}{P_{33}}\right) - \cos^{-1}\left\{\frac{-A_3 P_{33} + D_3 P_{13} - A_1 - P_4}{A_2}\right\}$$

Now,

$a = \theta_{J31} + \theta_2$

$\theta_{J31} = a - \theta_2$

$$\theta_{J31} = \cos^{-1}\left\{\frac{-A_3 P_{33} + D_3 D_{13} - A_1 - P_{14}}{A_2}\right\} - \theta_2 \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (13)$$

b=$\theta_{J32} + \theta_3$

$\theta_{J32} = b - \theta_3$

$$\theta_{J32} = \tan^{-1}\left(\frac{P_{13}}{P_{33}}\right) - \cos^{-1}\left\{\frac{-A_3 P_{33} + D_3 P_{13} - A_1 - P_1 4}{A_2}\right\} - \theta_3 \ldots\ldots\ldots\ldots\ldots\ldots. (14)$$

Therefore equation 13 and 14 is inverse kinematics for θ_{J31} *and* θ_{J32}.

Thus, we have found our inverse kinematics for finger 1, 2 and 3. Here all of the inverse kinematics is given.

$$\theta_{J41} = \tan^{-1}\left(\frac{P_{12}}{P_{21}}\right)$$

$$\theta_{J11} = \sin^{-1}\left\{\frac{P_{34} - A_3\left(\frac{P_{23}}{S_4}\right) - D_3\left(-\frac{P_{21}}{S_4}\right)}{A_2}\right\} - \theta_2.$$

$$\theta_{J12} = \tan^{-1}\left(-\frac{P_{23}}{P_{21}}\right) - \sin^{-1}\left\{\frac{P_{34} - A_3\left(\frac{P_{23}}{S_4}\right) - D_3\left(-\frac{P_{21}}{S_4}\right)}{A_2}\right\} - \theta_3.$$

$$\theta_{J21} = \sin^{-1}\left\{\frac{P_{34} - A_3\left(-\frac{P_{23}}{S_4}\right) - D_3\left(\frac{P_{21}}{S_4}\right)}{A_2}\right\} - \theta_2.$$

$$\theta_{J22} = \tan^{-1}\left(-\frac{P_{23}}{P_{21}}\right) - \sin^{-1}\left(\frac{p_{34} - A_3\left(-\frac{P_{23}}{S_4}\right) - D_3\left(\frac{P_{21}}{S_4}\right)}{A_2}\right) - \theta_3.$$

$$\theta_{J31} = \cos^{-1}\left\{\frac{-A_3P_{33} + D_3P_{13} - A_1 - P_{14}}{A_2}\right\} - \theta_2.$$

$$\theta_{J32} = \tan^{-1}\left(\frac{P_{13}}{P_{33}}\right) - \cos^{-1}\left(\frac{-A_3P_{33} + D_3P_{13} - A_1 - P_{14}}{A_2}\right) - \theta_3$$

2.4: Jacobian for Barrett Hand:

Here in this section I will discuss about about Jacobian for Barrett Hand. I have determined Jacobian for finger 1, 2 and 2 respectively. After finding Jacobian we will determine the torque and force of each finger.

2.4.1: Jacobian for Finger 1:

Here I will discuss the procedure of the finding Jacobian for finger 1. However, I have done all of matrix calculation in Mathematica. Matrix calculation is really intricate thus I have used Mathematica for making calculation easy. I have also denoted some terms in different way in Mathematica but I will explain everything in this section. Considering below figure and finger 1:

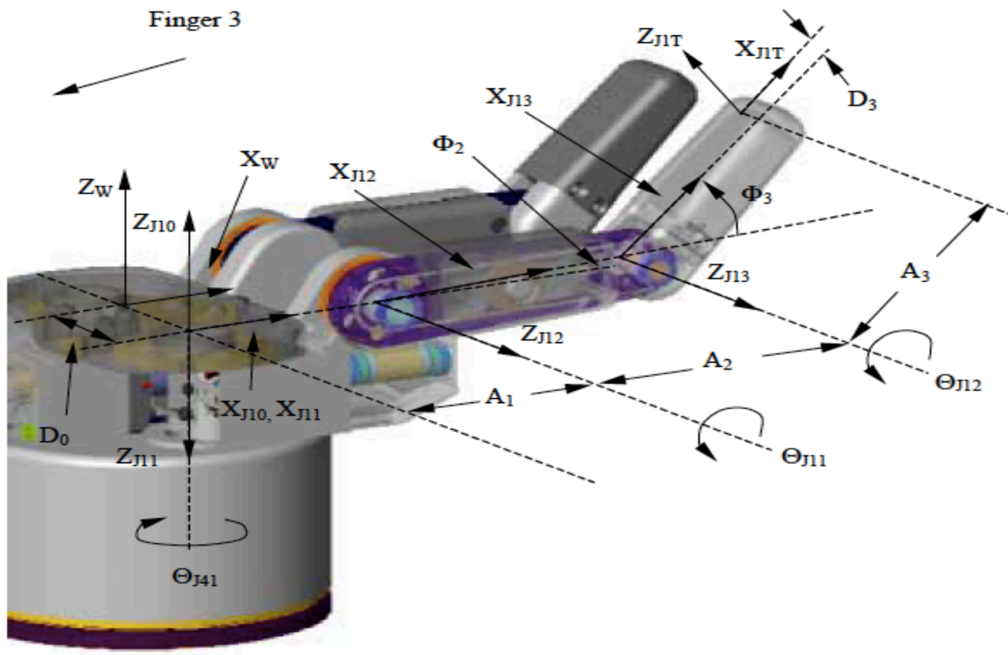

Figure 2.6: Barrett Hand Finger 1(right) and 2 (left)

In appendix, all Mathematica code is provided. Now we will consider the joint 1 of finger 1. The direction of the finger is denoted by V11. The matrix form of V11 is

$$\text{Direction } (V_1)_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

We need to convert it into World frame thus the world frame direction will be

$$(V_1)_W = {}^{W}T_O{}^{O}T_1(V_1)_1$$

Now the Point is $$(P_1)_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Again, we need to convert it into World frame thus the world frame point will be

$$(P_1)_W = \begin{pmatrix} 0 \\ -25 \\ 0 \end{pmatrix}$$

Moreover, ${}^{0}T_1$ and wT_0 matrix is given below:

$$^{0}T_1 = \begin{bmatrix} \cos\theta_{J41} & -\sin\theta_{J41} & 0 \\ -\sin\theta_{J41} & -\cos\theta_{J41} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$^{W}T_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the cross product of $(V_1)_w$ and $(P_1)_w$ is

$$(V_1)_W \times (P_1)_W = \begin{bmatrix} -25 \\ 0 \\ 0 \end{bmatrix}$$

Now we can calculate $(V_1)_W = ^{W} T_o {^o}T_1 (V_1)_1$

$$(V_1)_W = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Again, $(P_1)_W = ^{W} T_0 \times (P_1)_0$

$$(P_1)_W = \begin{bmatrix} 0 \\ -25 \\ 0 \end{bmatrix}$$

Thus, the Jacobian for joint 1 of finger 1 is

$$J_{\theta 41} = \begin{bmatrix} V_1 \\ (P_1)_W \times (V_1)_W \end{bmatrix}$$

Here cross product of $(P_1)_W \times (V_1)_W$ is denoted by $(Q_1)_W$.

Thus $(Q_1)_W = (P_1)_W \times (V_1)_W = \begin{bmatrix} 0 \\ -25 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = (Q_1)_W = \begin{bmatrix} -25 \\ 0 \\ 0 \end{bmatrix}$

Thus, the Jacobian of joint 1 of finger 1 is denoted by $J_{\theta 41}$ in the figure, however fro simplification I have denoted it by $J_{\theta 01}$

$$(J_{\theta 01}) = \begin{Bmatrix} 0 \\ 0 \\ -1 \\ -25 \\ 0 \\ 0 \end{Bmatrix}$$

Now, for finger 1 joint 2: The procedure is exactly same as joint 1 finger 1. There is only some additional matrix need to multiply. For finding $(T_1)_2$ matrix just need to plug $-\pi/2$ value in equation (1) and following matrix obtained

$$(T_1)_2 = \begin{bmatrix} \cos(\theta_{11} + \varphi_2) & -\sin(\theta_{11} + \varphi_2) & 0 \\ 0 & 0 & 1 \\ -\sin(\theta_{11} + Q_2) & -\cos(\theta_{11} + Q) & 0 \end{bmatrix}$$

Once again direction for joint 2 is $(V_2)_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Thus, $(V_2)_w = {}^W T_o {}^T O_1 (T_1)_2 (V_2)_2 = \begin{bmatrix} -\sin\theta 41 \\ -\cos\theta 41 \\ 0 \end{bmatrix}$

Point of joint 2 is $(P_2)_1 = \begin{bmatrix} 70 \\ 0 \\ 0 \end{bmatrix}$

Then, $(P_2)_W = {}^W T_0 {}^1 T_0 (P_2)_1 = \begin{bmatrix} 70 & \cos\theta 41 \\ -70 & \sin\theta 41 \\ & 0 \end{bmatrix}$

Therefore, $(Q_2)_w$ is $(Q2)w = (P_2)_W + (P_1)_W = \begin{bmatrix} 70 \cos\theta 41 \\ -25 - 70 \sin\theta 41 \\ 0 \end{bmatrix}$

Now, the cross product of $(V_2)_w \times (Q_2)_W$. $= \begin{bmatrix} 0 \\ 0 \\ 70+25\ sin\theta 41 \end{bmatrix}$

Therefore, the Jacobian for θ_{J11} is

$$\theta_{J11} = \begin{bmatrix} -\sin\theta 41 \\ -\cos\theta 41 \\ 0 \\ 0 \\ 0 \\ 70 + 25 \sin\theta\ 41 \end{bmatrix}$$

In addition, with joint 1 and 2, I need to calculate Jacobian for Joint 3. So all calculation below is for joint 3.

The Matrix, $(T_2)_3 = \begin{bmatrix} \cos(\theta_{12} + \theta_3) & -\sin\ (\theta_{12} + \theta_1 & 0 \\ \sin(\theta_{12} + \theta_3) & \cos(\theta_{12} + \theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The direction of $(V_3)_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

We need to find

$$(V_3)_W = {}^W T_0 {}^1T_0 (T_1)_2 (T_2)_3 (V_3)_3$$

$$= \begin{bmatrix} -\sin\theta 41 \\ -\cos\theta 41 \\ 0 \end{bmatrix}$$

Point $(P_3)_2 = \begin{bmatrix} 70 \\ 0 \\ 0 \end{bmatrix}$

Thus, $(P_3)_W = {}^W T_0 {}^1T_0 (T_1)_2 (P_3)_2 \quad = \begin{bmatrix} 70\cos(.04+\theta_{11})\cos(\theta 41) \\ -70\cos(.04+\theta_{11})\sin(\theta 41) \\ 70\sin(.04+\theta_{11}) \end{bmatrix}$

Now, we need to add $(P_1)_W + (P_2)_W + (P_3)_W$ for finding $(Q_3)_W$

Thus $(Q_3)_W = (P_1)_W + (P_2)_W + (P_3)_W$

$$(Q_3)_W = \begin{bmatrix} 70\cos\theta 41 + 70\cos(.41+\theta_{11})\cos(\theta 41) \\ -25 - 70\sin(\theta 41) - 70\cos(.41+\theta_{11})\sin(\theta 41) \\ 70\sin(.042+\theta_{11}) \end{bmatrix}.$$

Now, we need to do the cross product of $(Q_3)_W$ and $(V_3)_W$

$$(V_3)_W \times (Q_3)_W = \begin{bmatrix} -70\cos(\theta_{41})\sin(.04+\theta_{11}) \\ 70\sin(.04+\theta_{11})\sin(\theta_{41}) \\ 70\cos^2(\theta_{41}) + 70\cos(.04+\theta_{11})\cos^2(\theta_{41}) + 25\sin(\theta_{41}) + \\ 70\sin^2(\theta_{41}) + 70\cos(.04+\theta_{11})\sin^2(\theta_{41}) \end{bmatrix}$$

Thus, Jacobian for joint 3 finger 1 is

$$J_{12} = \begin{bmatrix} -\sin\theta 41 \\ -\cos\theta 41 \\ -70\cos\theta 41\sin(0.04+\theta_{11}) \\ 70\sin(0.04+\theta_{11})\sin(\theta 411) + 25\sin\theta\, 411 \end{bmatrix}$$

Therefore, Jacobian for finger 1:

$$J = \begin{bmatrix} 0 & -\sin\theta 41 & -\sin\theta 41 \\ 0 & -\cos\theta 41 & -\cos\theta 41 \\ -1 & 0 & 0 \\ -25 & 0 & -70\cos\theta 41\sin(.04+\theta_{11}) \\ 0 & 0 & 70\sin(.04+\theta_{11})\sin\theta 41 \\ 0 & 70+25\sin\theta 41 & 70+69.9\cos\theta_{11}-3.04\sin\theta_{11}+25\sin\theta_{41} \end{bmatrix}$$

2.4.2: JACOBIAN FOR FINGER 2:

The process for finding Jacobian for finger 2 is exactly same as finger 1. There is only some additional matrix calculation. I have provided all of the calculation in appendix. The Jacobian for finger 2 is given below:

$$J = \begin{bmatrix} 0 & \sin\theta 41 & \sin\theta 41 \\ 0 & -\cos\theta 41 & -\cos\theta 41 \\ 1 & 0 & 0 \\ -25 & 0 & -70\cos\theta 41\sin(.04+\theta_{11}) \\ 0 & 0 & -70\sin(.04+\theta_{11})\sin\theta 41 \\ 0 & 50+25\sin\theta 41 & 69.9\cos\theta_{21}-3.04\sin\theta_{21}+50+25\sin\theta_{41} \end{bmatrix}$$

2.4.3: JACOBIAN FOR FINGER 3:

The process for finding Jacobian for finger 3 is exactly same as finger 1 and 2. There is only some additional matrix calculation. I have provided all of the calculation in appendix. The Jacobian for finger 3 is given below:

$$J = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 70\,\mathrm{Sin}(\theta 31 + 0.04) \\ 0 & 0 \\ 50 & 50 + 70\,\mathrm{Cos}(\theta 31 + 0.04) \end{bmatrix}$$

CHAPTER 3:

BARRETT HAND CONTROL STRATAGIES, THEORY AND IMPLEMENTATION

3.1 CONTROL STRATAGIES:

To find the Barrett hand control strategies we need to find the torque of each finger. There are two kind of techniques. One is Static force techniques and another is Dynamic force techniques. In my thesis I have used static force techniques. By using Jacobian, I have found the torque of each finger. I will explain this in another section. After finding the torque then I have used PID controller for controlling the hand. And make sure the error is zero so that we can go the desired position. In the next section I am going to describe static force.

3.2 STATIC FORCE:

Static force is used when the response is slow. In the other hand, we can say that static force is used for slow motion. When we want our robotic hand to do some task in slow motion, we can use the static force. However, for the fast response we need dynamics force.

There is a relationship between static force and torques. When the manipulator interacts with the environment then there is some force and moments generated at the end effector of the manipulator. These actually generate the torques. Let consider F which is wrench. In the F there are two components. One is force of vector and another is moment of a vectors.

Let F=[F_x, F_y, F_z,m_x, m_y, m_z]. Where "F" is the force along x, y and z axis and moment "m" along x, y and z axis. Now we can denote τ as a torque vector for joints. Therefore, we can relate

$\tau= (J^T)F$

Where J is the Jacobian and F is force.

Below I have considered finger 1:

J has already determined. I have done the calculation in Mathematica, here I have just given some simple form of that example.

Force is given f={2,4,5}

Distance is given r={50,70,50}

Now we need to calculate Moments m. We know that moment is the cross product of distance vector and force vector. Thus m=r*f. Therefore, we have obtained m= {150,-150,60}

Thus the wrench F={150,-150,60,2,4,5}

Now we can calculate τ.

Tau (τ)=K.F /.{θ41->π,θ11->π}

Where K is transpose of Jacobian J and F is wrench. Given value for θ41=π,θ11=π we have obtained the value of torques τ which is

(-110, 200, -155.687). The complete calculation is given in the Mathematica file in the appendix B. As we have found the τ now we can design the PID controller.

3.3 PID CONTROLLER:

If a controller has three elements which is Proportional (P), Integral (I), and Derivative (D) and if these three elements are in parallel then it is called PID controller. PID controller is very popular because of its simplicity. Moreover, PID controller is the extreme case of Lead-Lag controller. For Barrett Hand control strategies, I have used PID controller. Here I am going to provide a brief about PID controller. PID controller means Proportional Integral Derivative controller. The PID controller is determined by below

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt}$$

where u(t) is input or reference signal. K_P, K_i, and K_d is the gain for proportional, integral and derivative. By using Matlab we can increase or decrease the value of K_P, K_i, and K_d. The general block diagram of PID controller is given below:

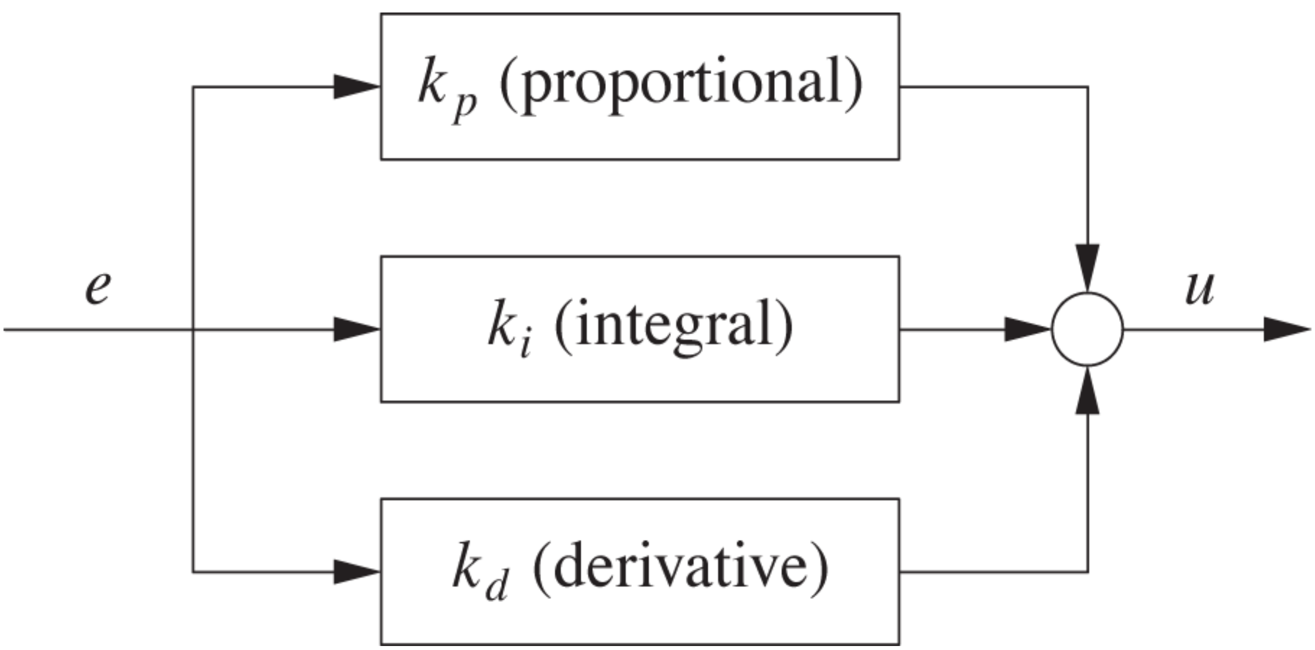

Figure 3.1: PID Controller General Block Diagram

Here in the block diagram K_P, K_i, and K_d is connected with the plant and then we will obtain the output. We can also use the feedback system with PID controller so that we can minimize the error and find our desired output.

3.4 POSITION CONTROL:

The purpose of the PID controller in the Barrett hand is to minimize the error and find the desired position of the end effector. By varying K_P, K_i, and K_d and some other value we can minimize the overshoot as well as we can minimize the error. By this process I have achieved the desired position for Barrett Hand.

3.5 PID Controller Design for Barrett Hand:

In Barrett hand we have already known that there are three fingers. Each finger has two joints. Therefore, we have three fingers and six joints. Moreover, there are four motors in the Barrett hand. One motor is located on the base of the hand and rest of the motors has attached to the bottom of the finger. Theses motors actually run three fingers. In my PID controller design I have found the transfer function for each motor. By using those transfer function, we can control the position of each finger. However, in that case we need to find six different transfer function and we have to implement all of them separately by controlling three fingers at a time. At the beginning of PID controller design we will consider that the motor is a servo motor. Thus, we will follow servomechanism. In addition, we have to consider some facts. Here I have described the facts. First of all, I have considered actuator as an armature control DC servomotor. The arm or finger is connected to the motor via gears. The gear ratio should be considered here. The gear ratio is denoted by “n” in controller design, n=r1/r2. Now, we have to consider motor parameters. Motor input voltage is E_a, armature current I_a, developed torque T, θ_m is motor shaft angle, θ_L is the angle of the arm. J is moment of inertia and B is the total friction coefficient. From the Barrett hand motor specification, I have obtained all values. Such as inductance L=1.2, resistance R=4.9, voltage= 19.8, current I=4, inertia J=0.0002, friction coefficient B=0.0105, gear ration n=r1/r2=16/30, torque sensitivity K_t =2.5, and K_m=0.018. After determining the transfer function, I have obtained third order, however in that case for the simplicity I have ignore inductance and hence I have obtained second order transfer function. Moreover, θ_c is the desired angle and θ_L is the actual angle of the robot finger. Compensator transfer function is

$$G_c(S)=K_p+ K_d S$$

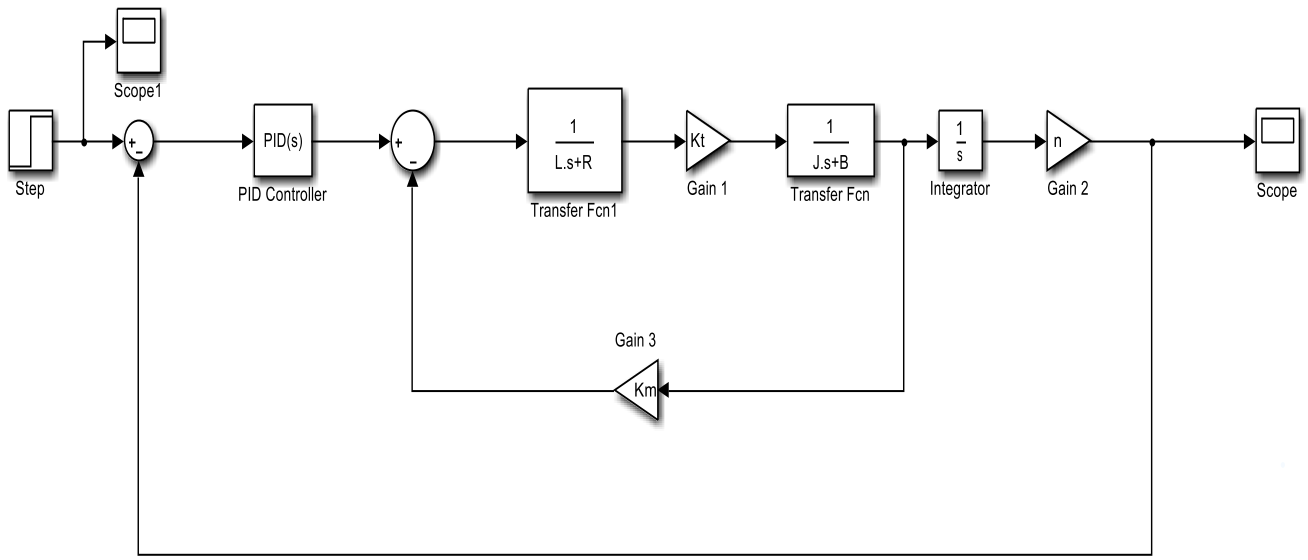

Figure 3.2: PID controller for Finger 1 Joint 1

In appendix B I have enclosed Matlab code. In the PID controller I have provided step input. After tuning the PID controller and run the Matlab file with all required values I have obtained the graph. Here I have attached the graph.

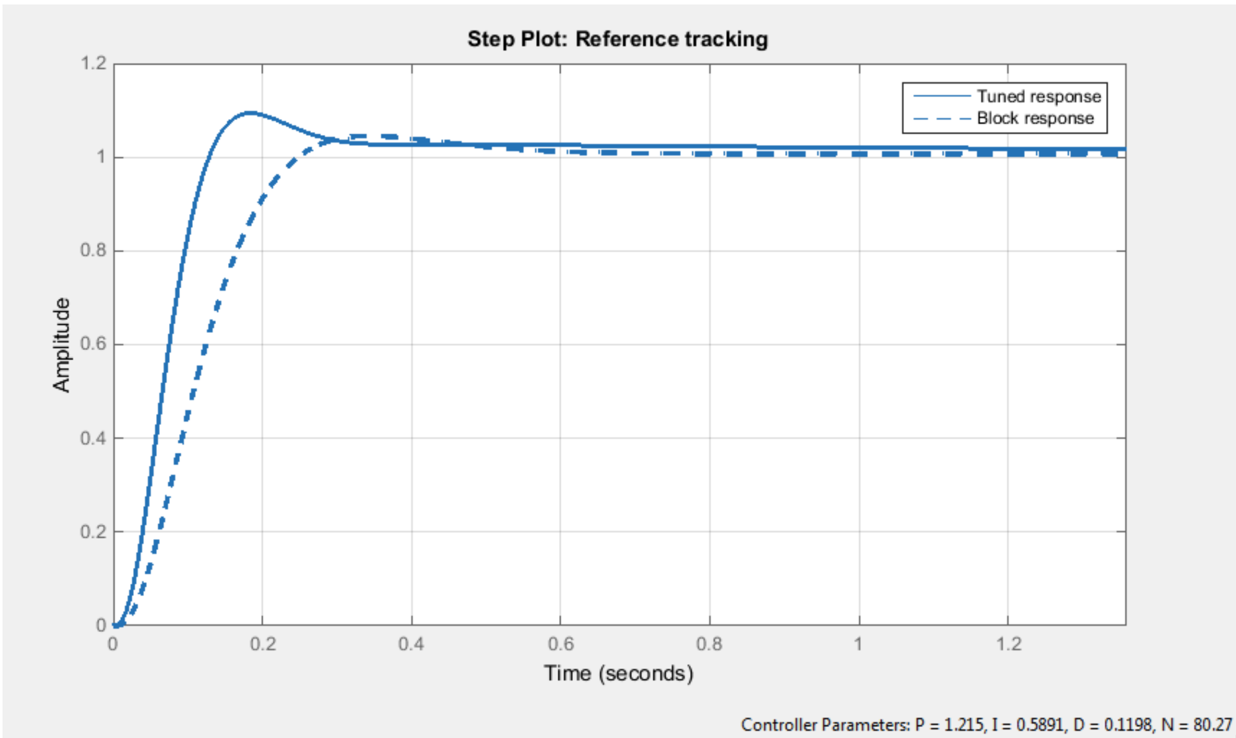

Figure 3.3: PID controller after tuned.

Here in this figure, the dotted line is block response and continuous line is tuned response.

The value of P is 1.25, I is 0.5, D is 0.1198 and N is 80.

I have considered step input; thus the input graph is below.

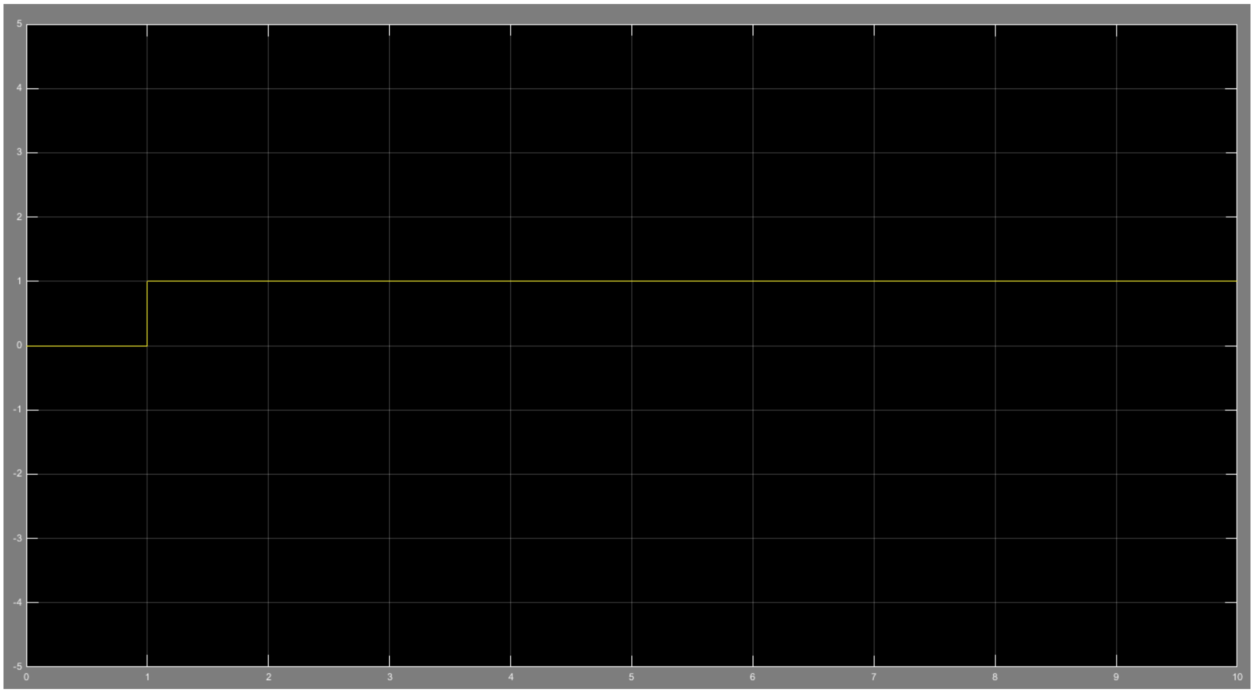

Figure 3.4: Step input graph.

Now, here below is the step output graph.

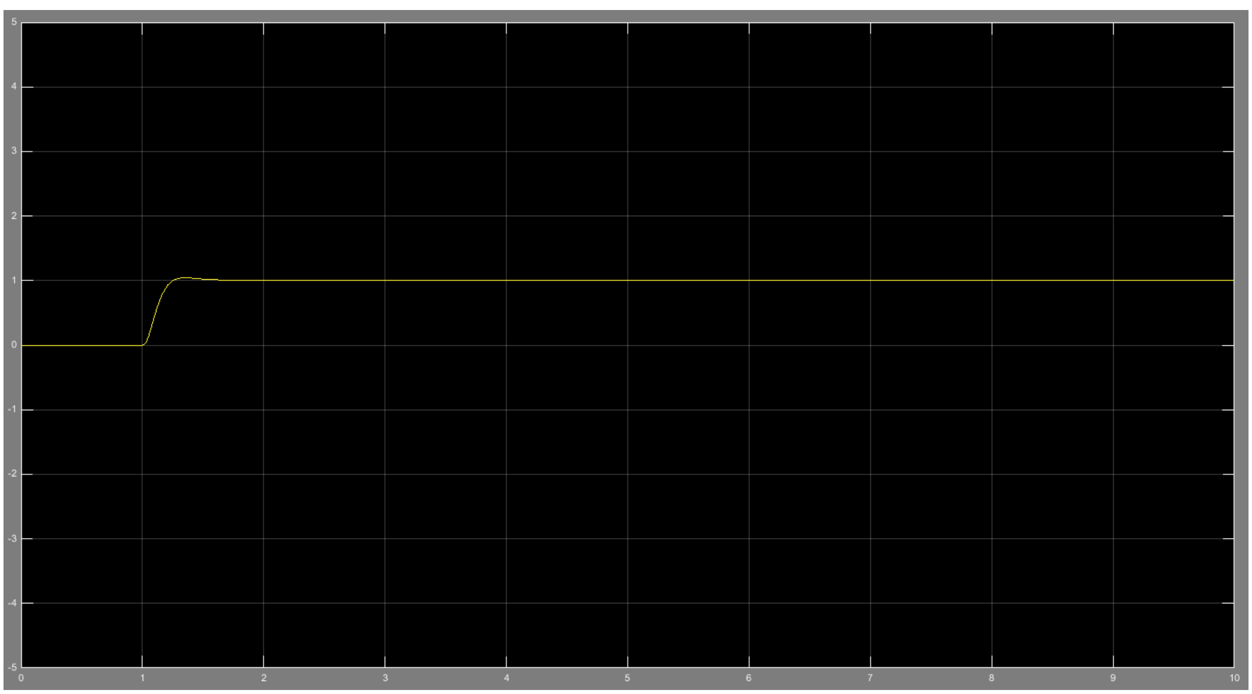

Figure 3.5: Step output graph

Moreover, from the PID controller I have obtained the Bode plot. Here below the bode is given.

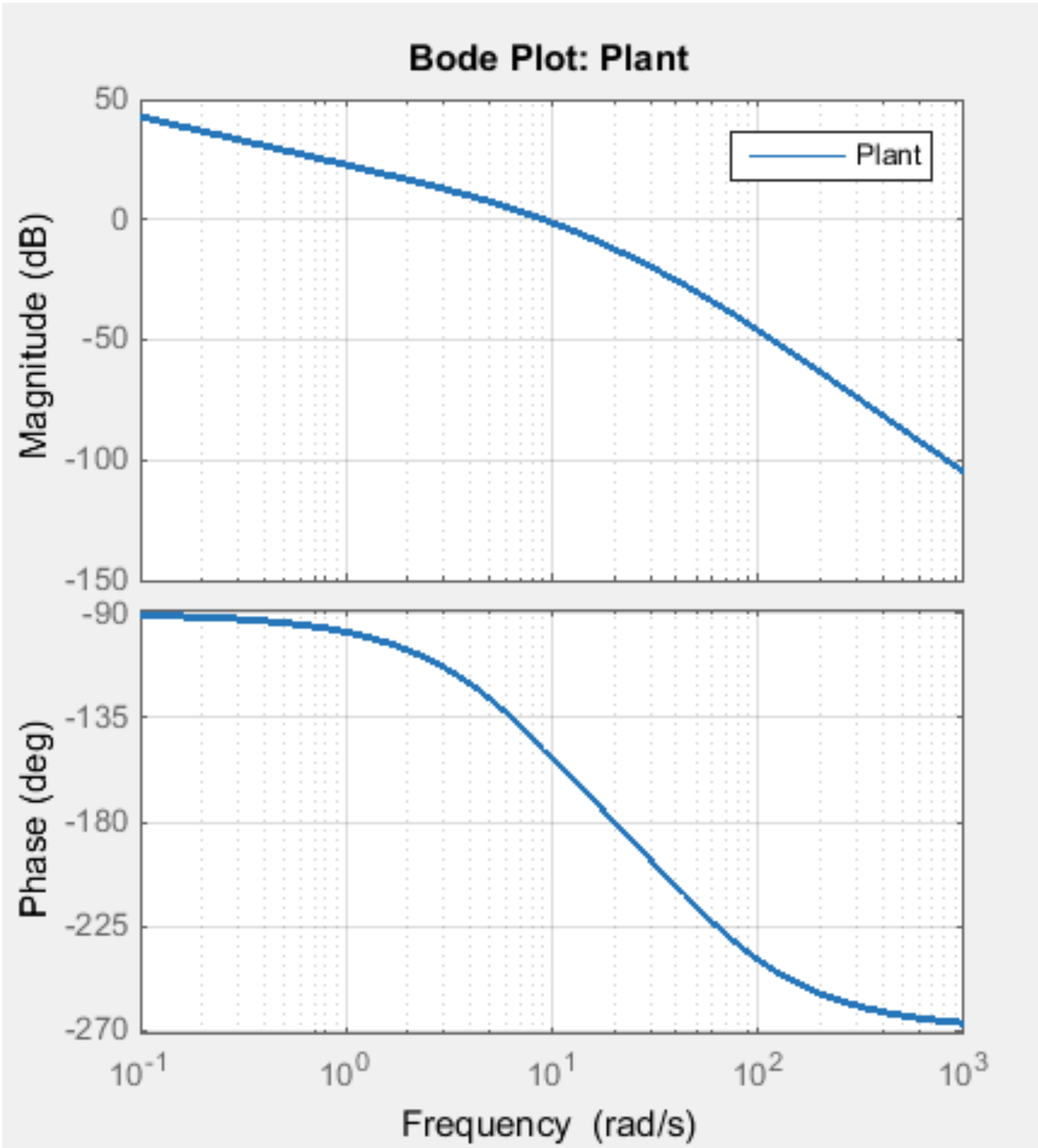

Figure 3.6: Bode Plot

Therefore, by using these PID technic I can determine the position or angle of the finger. The same technique will be also applicable to Finger 2 and Finger 3. Barrett hand finger 1 and 2 can be controlled by the technique.

Chapter 4

Barrett Hand Trajectory Planning:

4.1 Introduction:

By using trajectory planning we can determine the position, velocity and acceleration of the robotic hand. There are many ways to determine those parameters. As for example Joint space and Cartesian space. I have followed joint space mechanism. In joint space, I have used Cubic Polynomials technique.

4.2 Cubic Polynomials:

In cubic polynomials technique to find position, velocity and acceleration, I need to find the initial and final position. Initial and final position can be obtained from the forward and inverse kinematics.

4.3: Cubic Polynomials for Finger 1:

From the forward kinematics of finger 1, We have seen three positions which is θ_{J41} , θ_{J11} , and θ_{J12}. We need to provide a possible value of these three angles. I have assumed three values which are $\pi/18$, $\pi/9$, and $\pi/6$. Put these three values in forward kinematic then one matrix will be found.

The matrix is given below:

−0.21248012956429566	0.17364817766693033	−0.9616124504878741	93.18852554685857
0.03746597970661569	0.984807753012208	0.1695582200061951	−41.431651346351224
0.9764468725460508	0.	−0.21575797805651686	73.51532730569997
0	0.	0.	1.

Now, plug those values in Inverse kinematics then we will find the initial position. Similarly, for the final position I have assumed some values which are 4π/18, 45π/180, and 50π/180 and fond another matrix. Which is

−0.6457878252355835	0.6427876096865393	−0.4120463245934376	38.35323139553366
0.5418803259739559	0.766044443118978	0.3457479189943073	−57.18218231844918
0.5378882756668476	0.	−0.8430161344245704	70.462145942951
0.	0.	0.	1.

and now plug those values in Inverse kinematics and we will find the final position. Calculation of those matrix in Mathematica is in appendix.

<u>Finger 1 and joint 1:</u>

$P_{12} = .173648$

$P_{21} = .037466$

$P_{22} = .984808$

$P_{23} = 0169558$

$P_{34} = 73.5153$

$$\theta_{J41} = \frac{\pi}{18} \qquad \varphi_2 = \frac{246\pi}{18000}$$

$$\theta_{J11} = \frac{\pi}{9} \qquad \varphi_3 = \frac{5\pi}{18}$$

$$\theta_{J12} = \frac{\pi}{6}$$

We obtained initial position,

$\theta_{J41} = 10°, \theta_{J11} = 20°, \theta_{J12} = 152°$

For the Final position and Forward kinematics value,

$$\theta_{J41} = \frac{4\pi}{18}, \qquad \theta_{J11} = \frac{45\text{pi}}{180} \qquad \theta_{J12} = \frac{50\text{pi}}{180}$$

Then, we obtained, Final position,

$\theta_{J41} = 40°, \ \theta_{J11} = 45° \quad \theta_{J12} = 132°$

We assume initial time is 0 sec and final time is 4 sec. $t = 0$; $tf = 4$ sec .

Plot against tome is equation is $\theta_{J41} = (t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$.

So, $\theta_{J41}(0) = 10$.

Thus, $10 = c_0$, thus $\theta_{J41}(t) = c_1 + 2c_2 t + 3c_3 t^2$.

$\theta_{J41}(0), c_1 = 0$

$\theta_{J41}(t_f) = c_0 + c_1 t_f + c_2 t_f^2 + c_3 t_f^3$.

$\theta_{J41}(t_f) = 40°$.

$40 = 10 + 0 + 16c_2 + 64c_3$.

$16c_2 + 64c_3 = 30$.---(1)

$\theta_{J41}(t_f) = c_1 + 2c_2 t_f + 3c_3 t_f^2$.

$0 = 8c_2 + 48c_3$.

$8c_2 + 48c_3 = 0$---(ii)

From equation 1 and 2, we obtained C_2 and C_3 values.

Therefore, the value of $c_0 = 10$; $c_1 = 0$; $c_2 = 5.625$; $c_3 = -0.9375$.

Now we have Position, Velocity and Acceleration equation.

Position $\theta(t) = 10 + 5.625t^2 - .9375t^3$.

Velocity $\dot{\theta}(t) = 11.25t - 2.8125t^2$.

Acceleration $\ddot{\theta}(t) = -11.25 - 5.625t$.

By using Matlab, I have found the graph of these three functions. In appendix the Inverse kinematics Matlab code and these three functions has given.

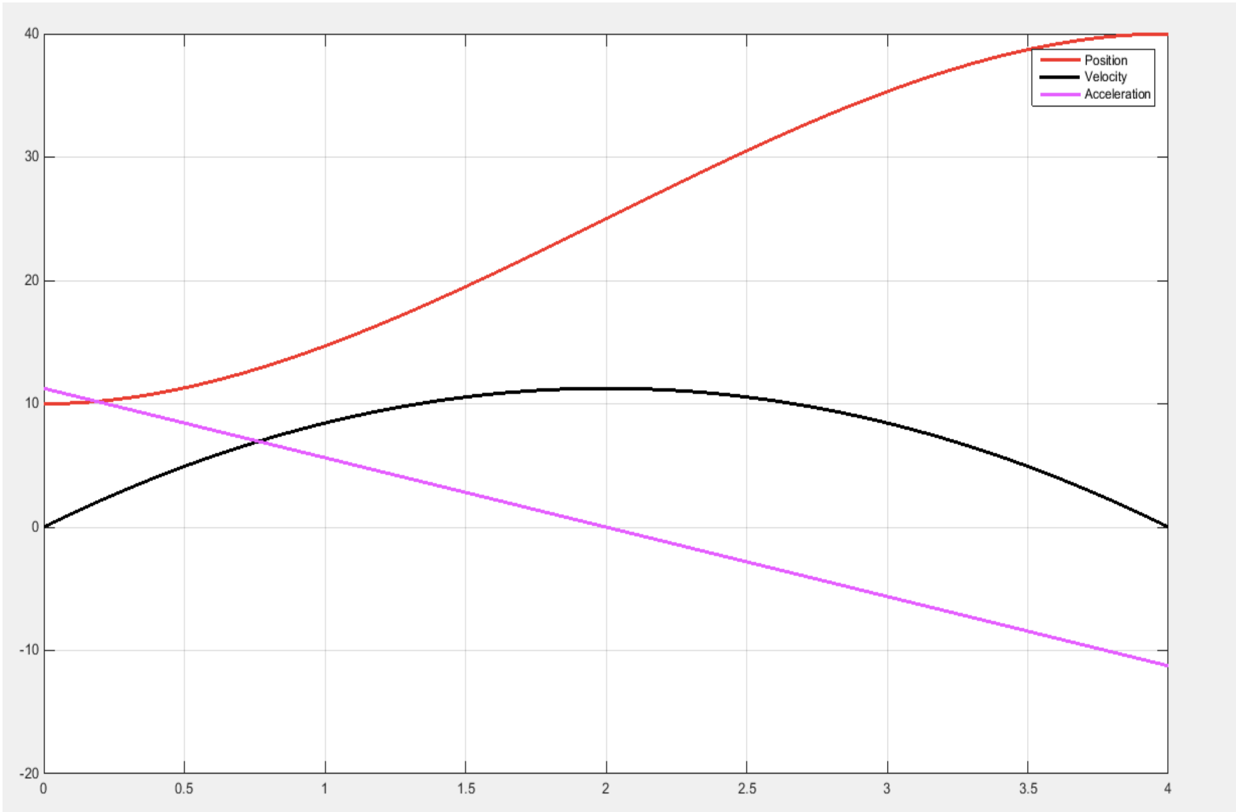

Figure 4.1: Position, Velocity and Acceleration for Finger 1 Joint 1.

<u>Finger 1 and Joint 2:</u>

I have followed the exact procedure same as finger 1 and joint 1. For the finger 1 and joint 2, the initial position and final positions are 20 degrees and 40 degrees respectively. Mathematical calculation is given below:

$\theta_{J11} = (t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3.$ $\quad t = 0,$ $\quad t_f = 4$ sec.

$\theta_{J11}(0) = 20 \quad \therefore c_0 = 20\,.$

$\theta_{J11}(t) = c_1 + 2c_2 t + 3c_3 t^3\,.$

$\theta_{J11}(0) = 0 \quad \therefore c_1 = 0\,.$

$\theta_{J11}(t_f) = c_0 + c_1 t_f + c_2 t_f^2 + c_3 t_f^3$.

$\theta_{J11}(t_f) = 45$.

$=> 45 = 20 + 0 + 16c_2 + 64c_3$.

$16c_2 + 64c_3 = 25$---(i).

$\theta_{J11}(t_f) = c_1 + 2c_2 t_f + 3c_3 t_f^2 \quad \theta_{J11}(t_f) = 0$.

$8c_2 + 48c_3 = 0$.---(ii) .

$c_0 = 20$; $c_1 = 0$; $c_2 = 4.7$; $c_3 = -0.8$

Thus,

Position $\theta(t) = 20 + 4.7\mathrm{t}^2 - 0.8\mathrm{t}^3$.

Velocity $\dot{\theta}(t) = 9.4\mathrm{t} - 2.7\mathrm{t}^2$.

Acceleration $\ddot{\theta}(t) = 9.4 - 4.8\mathrm{t}$

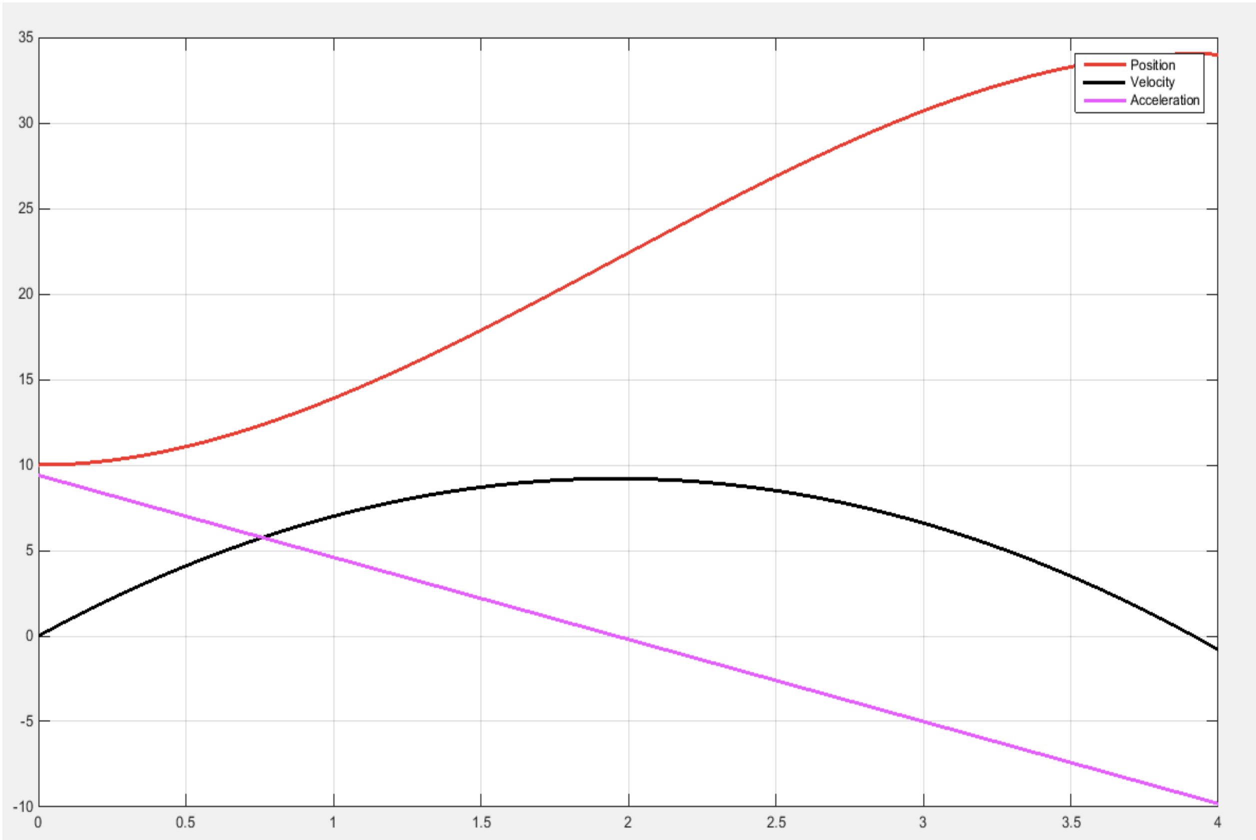

Figure 4.2: Position, Velocity and Acceleration for Finger 1 Joint 2.

Finger 1 Joint 3:

Initial Position is -152 and Final position -132

$\theta_{J12}(0) = -152 \quad t = 0, \quad t_f = 4$

$c(0) = -152.$

$\dot{\theta}_{J12}(\theta) = 0, c_1 = 0 .$

$\theta_{J12}(t_f) = -132. t_f = 4$

$=> -132 = -152 + 0 + 16c_2 + 64c_3 .$

$16c_2 + 64c_3 = 20$---(i).

$\dot{\theta}_{J12}(t_f) = 0$

$8c_2 + 48c_3 = 0$---(ii).

$c_o = -152 ; c_1 = 0 ; c_2 = 3.75 ; c_3 = -.625.$

Thus, Position $\dot{\theta}_{J12}(t) = -152 + 3.75t^2 - .625t^3$.

Velocity $\dot{\theta}_{J12}(t) = 7.5t - 1.875t^2$.

Acceleration $\dot{\theta}_{J12}(t) = 7.5 - 3.75t.$

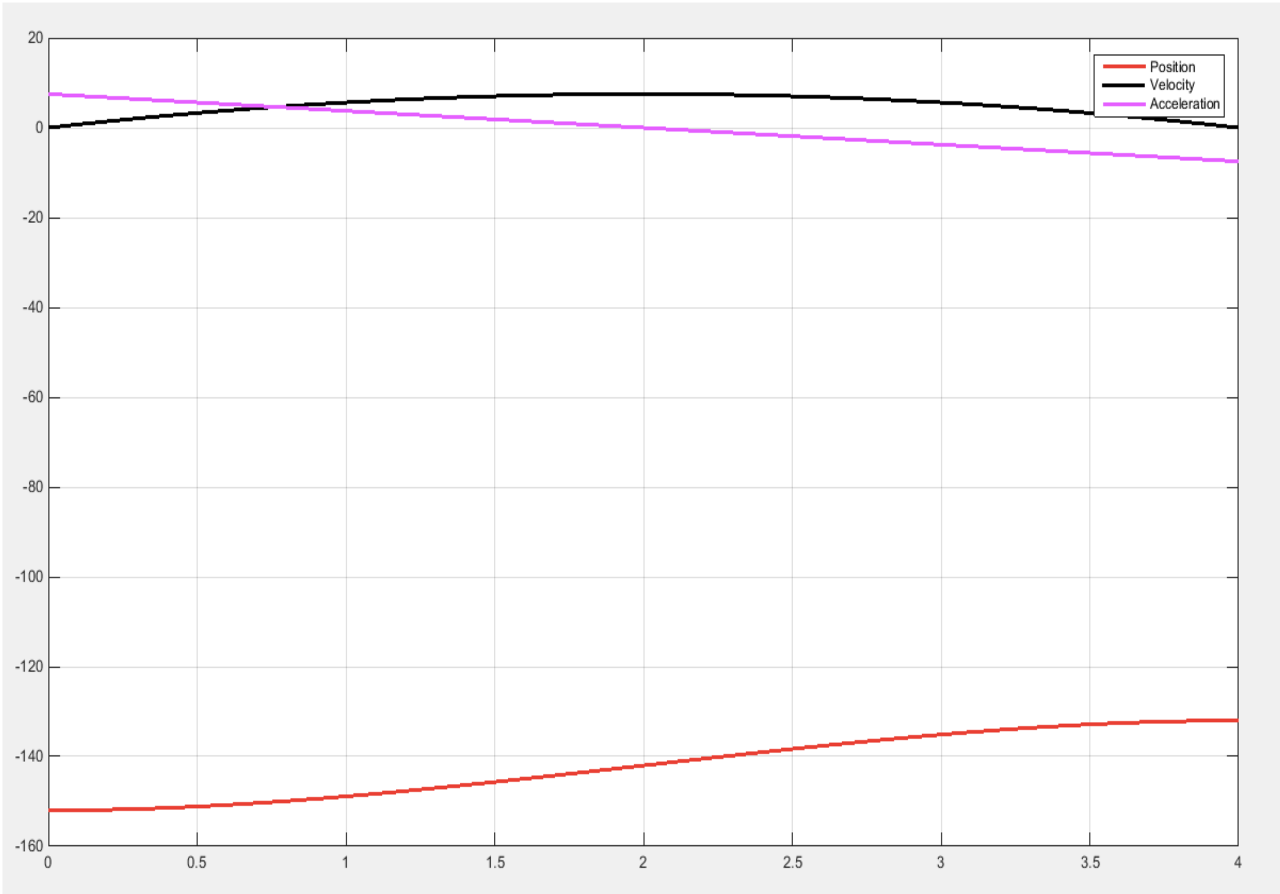

Figure 4.3: Figure: Position, Velocity and Acceleration for Finger 1 Joint 3.

4.4: Cubic Polynomials for Finger 2:

Initial position is 10 and Final position is 40. After calculation I have obtained the finger 2 joint 1 and finger 2 joint 2 is same as finger 1 joint 1 and finger 1 joint 2. However, the finger 2 and joint 3 is different. Thus for,

<u>Finger 2 Joint 3:</u>

Initial position= -150 and Final position= -130.

$\theta_{J22}(0) = -150. \quad t = 0, \; t_f = 4$

$c_0 = -150$

$\theta_{J22}(0) = 0 \qquad c_1 = 0$.

$\theta_{J22}(t_f) = -130 \;\; t_f = 4.$

$-16c_2 + 64c4 = 20$ ---(i)

$\theta_{J2z}(t_f) = 0$,

$8c_2 + 48c_3 = 0.$

$co = -150; c_1 = 0; c_2 = 3.75; c_3 = -.625.$

Thus,

Position $\theta_{J22}(t) = -150 + 3.75t^2 - .625t^3$.

Velocity $\dot{\theta}_{J22}(t) = 7.5t - 1.875t^2$

Acceleration is $\ddot{\theta}_{J22}(+) = 7.5 - 3.75t$

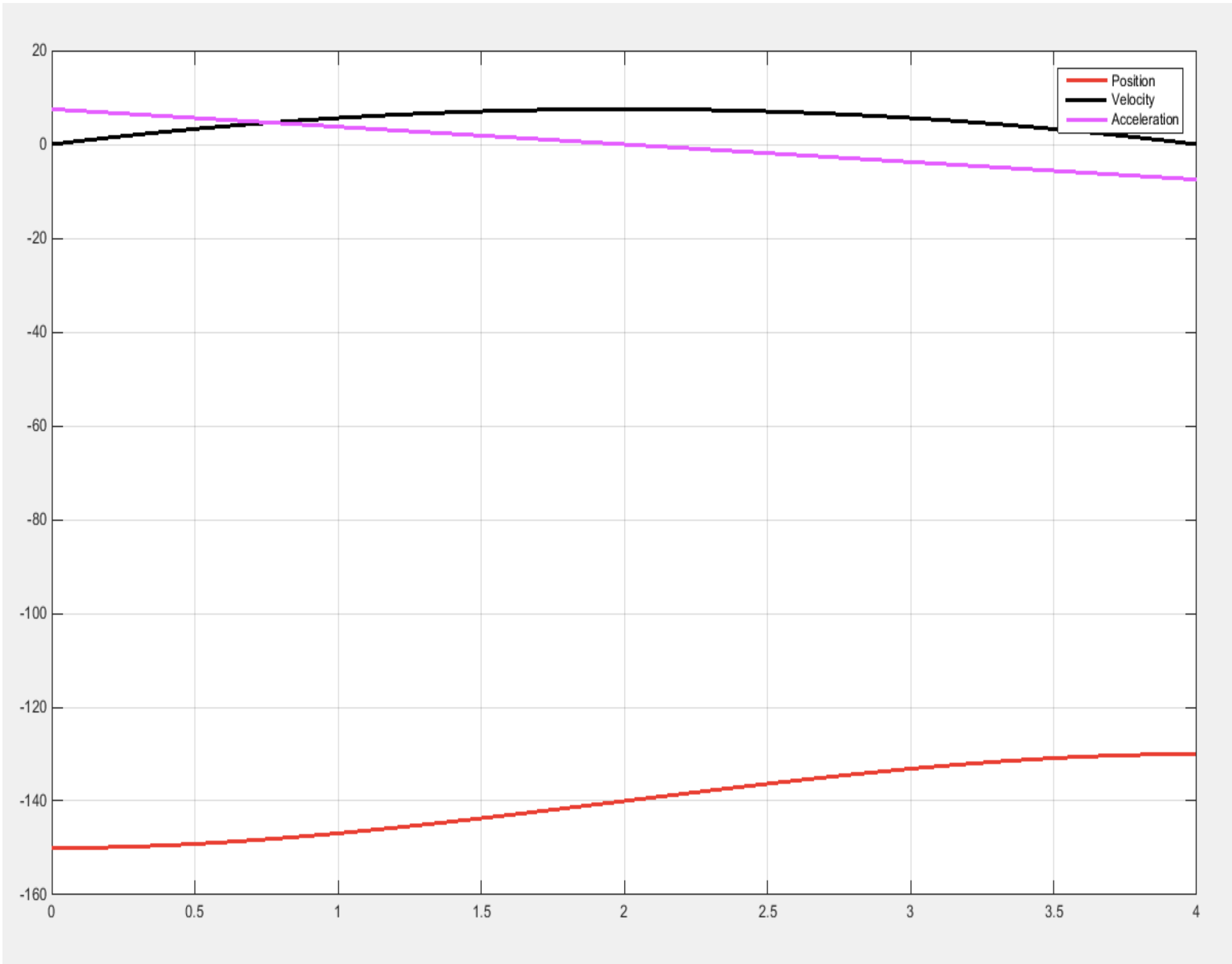

Figure 4.4: Position, Velocity and Acceleration for Finger 2 Joint 3.

4.5: Cubic Polynomials for Finger 3:

Cubic polynomial for finger 3 joint 1 is same as finger 1 joint 1. However, finger 3 joint 2 is not same. Thus, cubic polynomial for finger 3 joint 2 is given below.

Initial position is 20 degrees and final position is -127 degrees. The four parameter is found as below:

C_0=20; C_1 =0; C_2 =27 and C_3=-4.5. Thus the position, velocity and acceleration is given below:

Thus,

Position $\theta_{J32}(t) = -127 + 275t^2 - 4.5t^3$.

Velocity $\dot{\theta}_{J32}(t) = 54t - 4.5t^2$

Acceleration is $\ddot{\theta}_{J32}(+) = 54 - 18t$

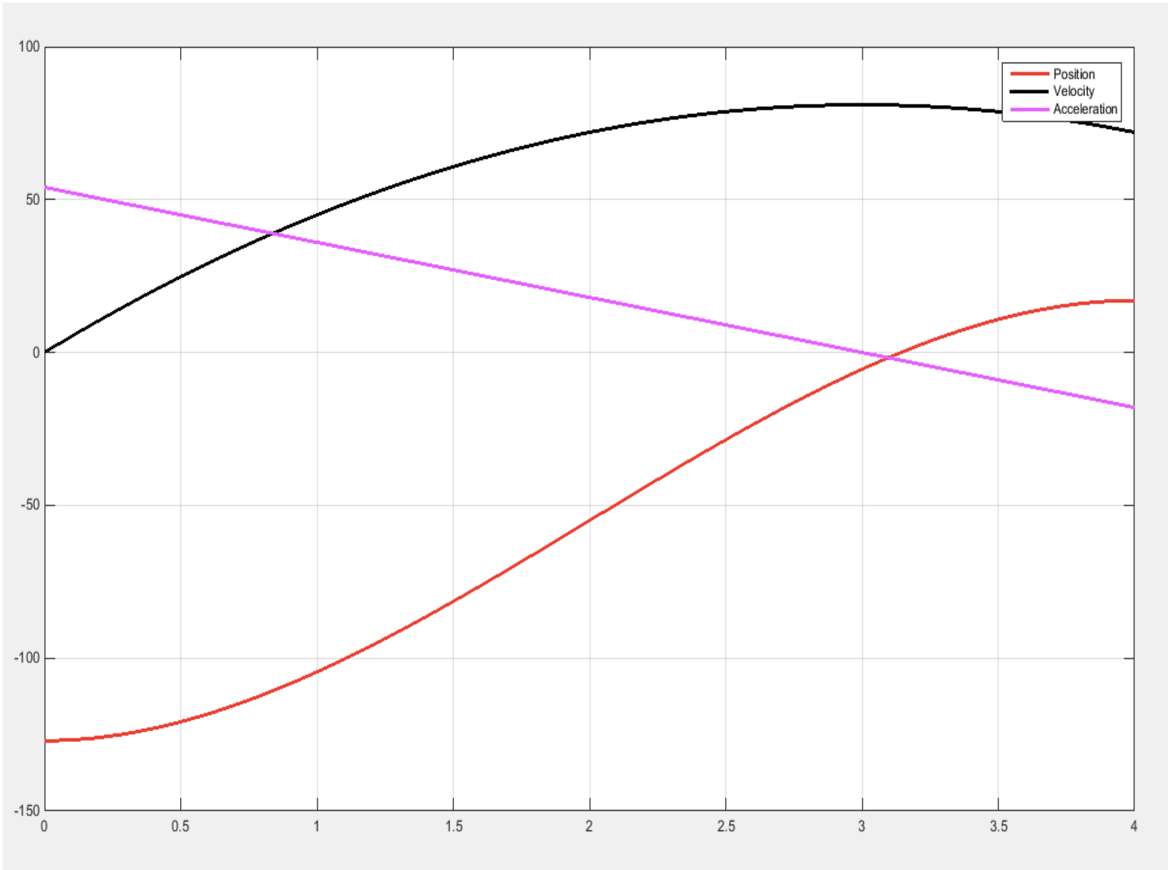

Figure 4.5: Position, Velocity and Acceleration for Finger 3 Joint 2.

Chapter 5

Conclusions and Future Work

5.1 Conclusion:

In this thesis I have worked on Barrett Hand BH 282. Barrett Hand is three finger robotic hand. I have found the forward kinematics and inverse kinematics of this Barrett hand. Then I have found the Jacobian of Barrette hand. By using Jacobian, the liner and angular velocity can be found. By using forward and inverse kinematic the position can be found. After that I have designed a PID controller. By using PID controller, we can control the Barrett hand position in different way. We can tune PID controller and we can see the position of the hand. From those position we can determine the overshoot, rise time, setting time and peak time. From the graph we can also tell, how we can determine the position better and accurate. At last, I have calculated cubic polynomials for Barrett hand. Cubic polynomial technique is really important to find the position, velocity and acceleration. By using cubic polynomial technique, we can find position, velocity and acceleration. From the graph we can also create relationship between position, velocity and acceleration.

5.2 Future Work:

In the PID controller I have considered the static force. Also, I have ignored the value of inductance in the motor. However, for the future work anyone can consider dynamics system and find the transfer function by using Lagrangian. In that case, the motor transfer function is not needed. Finding transfer function by using Lagrangian is difficult process. Moreover, after finding the transfer function, there are some way to control the hand. As for example, force control or position control.

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Appendix A:

Forward Kinematics for Finger 1:

```
P11 = Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]
Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]

Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]
Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]

P21 = -Sin[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]
-Cos[θj11 + θj12 + ϕ2 + ϕ3] Sin[θj41]

-Cos[θj11 + θj12 + ϕ2 + ϕ3] Sin[θj41]
-Cos[θj11 + θj12 + ϕ2 + ϕ3] Sin[θj41]

P31 = Sin[θj11 + θj12 + ϕ2 + ϕ3]
Sin[θj11 + θj12 + ϕ2 + ϕ3]

Sin[θj11 + θj12 + ϕ2 + ϕ3]
Sin[θj11 + θj12 + ϕ2 + ϕ3]

P41 = 0
0

P12 = Sin[θj41]
Sin[θj41]

Sin[θj41]
Sin[θj41]

P22 = Cos[θj41]
Cos[θj41]

Cos[θj41]
Cos[θj41]

P32 = 0
0

P42 = 0
0

P13 = -Cos[θj41] Sin[θj11 + θj12 + ϕ2 + ϕ3]
-Cos[θj41] Sin[θj11 + θj12 + ϕ2 + ϕ3]
```

Forward Kinematics for Finger 2:

```
P11 = Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]
Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]

Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]
Cos[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]

P21 = Sin[θj41] Cos[θj11 + θj12 + ϕ2 + ϕ3]
Cos[θj11 + θj12 + ϕ2 + ϕ3] Sin[θj41]

-Cos[θj11 + θj12 + ϕ2 + ϕ3] Sin[θj41]
-Cos[θj11 + θj12 + ϕ2 + ϕ3] Sin[θj41]

P31 = Sin[θj11 + θj12 + ϕ2 + ϕ3]
Sin[θj11 + θj12 + ϕ2 + ϕ3]

Sin[θj11 + θj12 + ϕ2 + ϕ3]
Sin[θj11 + θj12 + ϕ2 + ϕ3]

P41 = 0
0

P12 = -Sin[θj41]
-Sin[θj41]

Sin[θj41]
Sin[θj41]

P22 = Cos[θj41]
Cos[θj41]

Cos[θj41]
Cos[θj41]

P32 = 0
0

P42 = 0
0

P13 = -Cos[θj41] Sin[θj11 + θj12 + ϕ2 + ϕ3]
-Cos[θj41] Sin[θj11 + θj12 + ϕ2 + ϕ3]
```

Appendix B

Jacobian for Finger 1:

Joint 1:

```
T01 = {{Cos[θ41], -Sin[θ41], 0}, {-Sin[θ41], -Cos[θ41], 0}, {0, 0, -1}}

{{Cos[θ41], -Sin[θ41], 0}, {-Sin[θ41], -Cos[θ41], 0}, {0, 0, -1}}

wT0 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

MatrixForm[wT0]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
V11 = {0, 0, 1}

{0, 0, 1}
```

Finding v1w=wT0*T01*V11

```
V1w = wT0.T01.V11

{0, 0, -1}

V1w // MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

Points in world frame, so P1w=wT0*P10

```
P1w = {0, -25, 0}

{0, -25, 0}

Q1w = Cross[V1w, P1w]

{-25, 0, 0}
```

Now J41={V11,cross product of P1w and V1w}

Jacobian for Finger 2:

Joint 1:

```
T01 = {{Cos[θ41], -Sin[θ41], 0}, {Sin[θ41], Cos[θ41], 0}, {0, 0, 1}}
```

```
{{Cos[θ41], -Sin[θ41], 0}, {Sin[θ41], Cos[θ41], 0}, {0, 0, 1}}
```

```
wT0 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
MatrixForm[wT0]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
V11 = {0, 0, 1}
```

```
{0, 0, 1}
```

Finding v1w=wT0*T01*V11

```
V1w = wT0.T01.V11
```

```
{0, 0, 1}
```

```
V1w // MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Points in world frame, so P1w=wT0*P10

```
P1w = {0, 25, 0}
```

```
{0, 25, 0}
```

```
Q1w = Cross[V1w, P1w]
```

```
{-25, 0, 0}
```

Now J41={V11,cross product of P1w and V1w}

Jacobian for Finger 3:

Joint 1:

```
T01 = {{Cos[Pi], -Sin[Pi], 0}, {0, -1, 0}, {0, 0, 1}}
```

```
{{-1, 0, 0}, {0, -1, 0}, {0, 0, 1}}
```

```
wT0 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
MatrixForm[wT0]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
V11 = {0, 0, 1}
```

```
{0, 0, 1}
```

Finding v1w=wT0*T01*V11

```
V1w = wT0.T01.V11
```

```
{0, 0, 1}
```

```
V1w // MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Points in world frame, so P1w=wT0*P10

```
P1w = {0, 0, 0}
```

```
{0, 0, 0}
```

```
Q1w = Cross[V1w, P1w]
```

```
{0, 0, 0}
```

Now pi={V11,cross product of P1w and V1w}