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Approach-To-Critical With The Idaho State University Sub-Critical-Assembly

Using The Modified Source Method

by

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To the Graduate Faculty:

The members of the committee appointed to examine the thesis of Andrew Craig Layne find it satisfactory and recommend that it be accepted.

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DEDICATION

I dedicate this to my lovely wife Erika Lyn Layne. She made this thesis possible with her love, support, patience, and hard work.

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Abstract

An approach-to-critical experiment was performed using the Multiplication Source Approach (MSA) and a one-over-M plot. The final fuel plate predictions were 211, 212, and 203 fuel plates. The method makes inaccurate assumptions of equal detector efficiency and equal source importance between fuel plate configurations. The Modified Source Method (MSM) was performed to develop a correction factor that corrects the inaccurate assumptions in the MSA. The final fuel plate predictions from the MSM were 269, 254, and 242. An MCNP Kcode calculation was performed and determined 344 fuel pates are needed to reach criticality. The fuel plate predictions from the MSM method are closer to the actual number of fuel plates needed to reach criticality compared to the MSA. This is due to the inaccurate assumptions made in the MSA. The MSM results are also expected to align regardless of detector location, however, this did not occur. Further research is needed to determine the cause the misaligned MSM results.

1. Introduction

The Idaho State University (ISU) subcritical assembly (SCA) is a subcritical reactor used for teaching and research. Several approach-to-critical experiments are possible with the SCA. Senior reactor lab classes repeat several experiments every year. One of the experiments is a one-over-M approach-to-critical using the MSA. A one-over-M plot is created that is used to predict how many fuel plates are required for the assembly to reach criticality. The method requires inaccurate assumptions and will lead to inconsistencies between predictions. Another approach-to-critical experiment is the MSM. The MSM uses a computer model to develop a correction factor for each point on the MSA one-over-M plot.

The purpose of this thesis is to use the MSM in an approach-to-critical experiment. A one-over-M plot will be created using the MSA method. The MSA one-over-M plot will be used to develop the MSM one-over-M plot. The correction factor will be calculated using Monte Carlo Neutron Particle code (MCNP). An MCNP model of the ISU SCA was chosen because an MCNP model of the SCA has never been created. The MCNP model will be used to determine a correction factor for each point on the one-over-M plot. The number of fuel plates required for the reactor to reach criticality will be reported for both the MSA method and MSM. The MCNP model will also be used to simulate a MSA experiment. The ISU SCA does not have enough fuel plates to reach criticality and simulating the experiment in MCNP will allow more fuel plates to be used. An MCNP kcode calculation will also be used to predict the number of fuel plates needed to reach criticality.

2. Idaho State Subcritical Assembly Descriptions

The ISU SCA is a low enriched uranium water moderated and reflected assembly. The assembly was first designed and assembled by Dr. F. J. Jankowsky at Rutgers University in 1961. ISU obtained the SCA in 1972.[1] Prinya Jotikashthira, a graduate student at ISU, added a water handling system, shut-down system, working platform, and core lifting device to improve the SCA. The assembly includes 150 fuel plates assembled inside an aluminum tank filled with water. The tank sits on a grid of graphite blocks. The SCA has also been modeled by several computer codes including DISNEL and KENO V.A.[2]

2.1. Core Tank[1]

The core tank is made of a rolled aluminum sheet welded together. The tank has a height of 36”. The inside and outside diameters are 35.5” and 36”. The bottom of the tank and the lid of the tank have a thickness of 0.375”. The tank has two support brackets made of aluminum running down the inside of the tank that holds the fuel plate spacing assembly in place. On the bottom of the tank there is a honeycomb of aluminum piping 4” high. The aluminum honeycomb structure also supports the weight of the fuel plate spacing assembly. The core tank holds 142 gallons of water. The core tank sits on top of the graphite thermal column. During operation, the tank contains the fuel plate spacing grid, fuel plates, water, and a neutron source.

2.2. Graphite Blocks[1]

There are a total of ninety-five graphite blocks in the thermal column. The thermal column weighs approximately 4500 lbs. Each block is 4” wide, 4” high, and 122” long. Twelve blocks are stacked in each row with a total of eight rows. Each row is rotated 90^0

from the rows below and above. One graphite block has been removed from the third row from the top to allow a neutron detector to be inserted in its location. The thermal column provides a reflective surface at the bottom of the tank.

2.3. Fuel Plates[1]

There are 150 fuel plates. The fuel used is a uranium/aluminum mixture clad with aluminum. The uranium is enriched to 19.83 wt%. The plates are 0.08” thick 3” wide and 26” high. The fuel bearing portion of each pate is 0.04” thick 2.75” wide 23” high. There are a total of 7614.79 grams of uranium and 1510.27 grams of uranium-235 in all of the plates. Assuming equal distribution of uranium throughout all plates, each plate would have 50.77 grams of uranium and 10.07 grams of uranium-235.

2.4. Center Tube[1]

A tube placed in the center of the fuel plate spacing assembly is used to house a neutron source for experiments. The tube has an outer diameter of 0.98”, an inside diameter of 0.91”, and a height of 30.16”. The tube has an aluminum plate mounted on its side which has the same dimensions as a fuel plate. This allows the center tube to be positioned in the fuel plate spacing assembly. The plate sits flush with the bottom of the tube. The tube has a slit approximately 12.5” from the bottom where a small plate can be inserted or removed. This is to allow a neutron source to either be at the bottom of the assembly or near the middle of the assembly.

2.5. Fuel Plate Spacing Assembly[1]

The fuel plates are positioned in the core tank with a fuel plate spacing assembly. There are three different fuel plate spacing assemblies: a single, double, and triple plate assembly. The double fuel plate spacing assembly was used in this experiment. The assembly has 5 rows and 21 slots in each row. Each slot holds two plates. The spacing between each slot is 0.525”. There is no spacing between rows, and the rows are offset by 0.35”.

2.6. Water Tanks[1]

Three 53 gallon water tanks are used to store the distilled water used for the assembly. A water pumping system is used to pump approximately 143 gallons of water into the core tank. Operating water level of the SCA is set at 33”. Any water above this height will flow back into the water tanks. Water can also be drained from the core tank from the bottom.

2.7. Detectors

One He-3 and two BF_3 detectors were used in this experiment. The detector referred to as detector A has a sensitive height of 6” and a diameter of 0.5”. The gas is He-3 and has a pressure of 150 psia. The detector was placed approximately 1.5” from the outside surface of the tank and the bottom of the detector is 23” above the bottom of the tank. The detector referred to as detector B has a sensitive height of 36” and a diameter of 2”. The gas is BF_3 and has a pressure of 30 psia. The detector was placed in the thermal column third row from the top near the middle, where one graphite block has been removed. The detector referred to as detector C is the same model as detector B. The detector was placed approximately 2.875” from the outside surface of the tank and the bottom of the detector is 3.5” above the bottom of the tank. The equipment used in the setup for the detectors is shown in figure 2.1. The equipment used includes a Pre Amp, Amp, and Counter for each neutron detector.

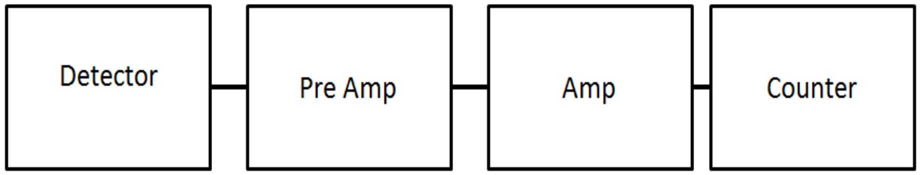

Figure 2-1 Block diagram of detector equipment

2.8. Neutron Source

The neutron source used in the experiment is a californium-252 source from Frontier Technology Corp. Californium-252 has a half-life of 2.646 years. There are two decay modes, spontaneous fission (3.092%) and alpha decay (96.908%). The average neutrons emitted per fission is 3.768. The source activity on June 24th 1993 was 112.3 mCi.

3. Theory

3.1. Multiplication factor K[3]

The state of a nuclear reactor is determined by how the neutron population in the core is changing. A nuclear chain reaction is maintained by the neutrons emitted from the fissioning fuel. A neutron can cause a fission reaction, be absorbed in the core material, or leak out of the core. If neutrons are being created faster than they can be absorbed or leak out, then the neutron population will increase. If neutrons are being absorbed or leaking out faster than they can be created, then the neutron population will decrease. If the neutrons are being absorbed or leaking out at the same rate as they are being created, then the neutron population will remain constant. The ratio of neutrons between generations is defined by a multiplication factor k which is defined as

$$k = \frac{Number\ of\ neutrons\ in\ one\ generation}{Number\ of\ neutrons\ in\ previous\ generation} \tag{3.1}$$

where k is the multiplication factor. When a reactor is found to have a k equal to one, the reactor is defined as critical. If k is greater than one, then the reactor is defined as super critical. If k is less than one, then the reactor is defined as sub critical. Like the name implies the ISU SCA is a subcritical assembly and cannot sustain a nuclear chain reaction without a neutron source.

3.2. Subcritical Multiplication[4]

Subcritical multiplication is the effect of fissile material increasing the effective source strength when k is less than one. A neutron source will emit neutrons at a constant rate. When a source is inserted in an SCA the source will emit S neutrons in the first generation. In the second generation, the source will emit S neutrons and the reactor assembly will emit k*S neutrons that came from the first generation. During the third generation, the source will emit S neutrons from the source and the reactor will emit S*k from the second generation and $S*k^2$ from the first generation. After many generations the neutron population will be

$$N = S + Sk + Sk^2 + Sk^3 + \cdots Sk^m \tag{3.2}$$

where N is the neutron population after many generations, S is the neutron source, and k is the multiplication factor of the SCA. The sum of the geometric series converges to

$$N = \frac{S}{1-k} \tag{3.3}$$

Provided K<1. It can be seen that as k approaches one, the neutron population in the reactor will approach infinity. As long as k is less than one, than the neutron population will reach a steady state.

3.3. Subcritical Multiplication Factor[4]

The neutron population in an SCA with a neutron source can be defined by the subcritical multiplication factor and the effective source strength. The neutron population is

$$N = S * M \tag{3.4}$$

where S is the effective source strength and M is the subcritical multiplication factor. The subcritical multiplication factor can also be defined by

$$M = \frac{neutron\ flux\ from\ source\ and\ fissions}{neutron\ flux\ from\ source} \tag{3.5}$$

Combining equation 3.3 and 3.4 the subcritical multiplication factor can also be defined by

$$M = \frac{1}{1-k} \tag{3.6}$$

where k is less than one. Previous experiments have been done to determine the ISU SCA subcritical multiplication factor. The subcritical multiplication factor has then been used to determine the number of fuel plates required to reach criticality.

3.4. Multiplication Source Approach[5]

The Multiplication Source Approach uses subcritical multiplication to determine the multiplication factor k of an assembly. Detectors located in or around the reactor give a count rate of neutrons from the reactor. The count rate measured by the detectors relates to the multiplication factor by

$$T = \frac{\epsilon S}{1-k} \tag{3.7}$$

where T is the detector count rate and ϵ is the detector efficiency due to detector sensitivity and geometry of the core. The effective source strength is difficult to determine without having high error and the detector efficiency is impossible to determine. Two different count rates can be taken and compared to find a ratio so that similar variables can be eliminated. The ratio will be

$$\frac{T_1}{T_2} = \frac{\epsilon_1 S_1 (1-k_2)}{\epsilon_2 S_2 (1-k_1)} \tag{3.8}$$

By assuming equal efficiencies of the detector and equal effective source strengths between each state, the relation will become

$$\frac{T_1}{T_2} = \frac{1-k_2}{1-k_1} \tag{3.9}$$

If state one is taken when there is no fuel in the reactor then k_1 will be equal to 0 and the count ratio will become

$$\frac{T_1}{T_2} = 1 - k_2 \tag{3.10}$$

were T_1 will always be the count rate with no fuel in the SCA. This relationship is known as the MSA method.

The MSA method gives inaccurate results when comparing predictions from different detectors. The efficiency of the detectors will change as more material is added to the core. Material added to the core will change the geometry of the core and thus change the amount of neutrons that the detector will count. The effective source strength will also change from state to state. The effectiveness of the source strength can change as the adjoint flux shifts due to core changes. This effect can be minimized by placing the neutron source at the center of the assembly. The assumption that the efficiency of the detector and effective source strengths are equal at each state remains inaccurate. The MSA can still be used, but detector location must be selected carefully.

3.5. Modified Source Method[5]

The MSM provides a correction factor to the MSA that will correct the inaccurate assumptions of the MSA. The MSA assumes that the detector efficiency and effective source strength remains the same as fuel is added to the SCA. The error in the MSA can vary as detector location is changed. As a result an MSA using different detector locations may produce different predictions. The MSM will correct the inaccurate assumptions and all detectors used in the experiment will produce the same prediction.

3.5.1. Correction factor

The MSM correction has previously been derived in *Application of the Modified Source Multiplication Technique to Subcritical Reactivity Worth Measurements in Thermal and Fast Reactor Systems*. The derivation is summarized below. The MSM correction factor is derived from the detector efficiency and neutron effective source strength. The detector efficiency is defined as

$$\epsilon = \frac{<\sigma_D \emptyset>}{F\emptyset} \tag{3.11}$$

Where $<\sigma_D \emptyset>$ is the reaction rate in the detector, σ_D is the detector cross section, F is the neutron production term, $\emptyset$ is the neutron flux, and $< >$ is used to integrate over time, space and energy. The source importance is defined as

$$source\ importance = \frac{<\emptyset^{+}S>}{<\emptyset^{+}F\emptyset>} \tag{3.12}$$

where S is the neutron source, and $\emptyset^{+}$ is the adjoint weighted neutron flux. Multiplying the source importance by the neutron production term gives the weighted source importance as

$$weighted\ source\ importance = \frac{<\emptyset^{+}S>}{<\emptyset^{+}F\emptyset>} < F\emptyset > \tag{3.13}$$

Combining the weighted source importance and detector efficiency gives

$$\in S_{w} = \frac{<\emptyset^{+}S>}{<\emptyset^{+}F\emptyset>} < \sigma_{D}\emptyset > \tag{3.14}$$

Solving equation 3.14 will give the correction factor for the MSM.

3.5.2. Computational Correction Factor

The traditional method to determine the MSM correction factor uses deterministic methods, but the correction factor can also be determined using computational methods. The homogeneous adjoint transport equation is defined as

$$A\emptyset^{+} = \frac{1}{k}F\emptyset^{+} \tag{3.15}$$

where A is the neutron loss term. Multiplying by $\emptyset$ and solving for k gives

$$k = \frac{F\emptyset^{+}\emptyset}{A\emptyset^{+}\emptyset} \tag{3.16}$$

Reactivity is defined as

$$\rho = \frac{k-1}{k} \tag{3.17}$$

Inserting the expression for k as defined in 3.16 into equation 3.17 reactivity can be defined as

$$\rho = \frac{\frac{F\emptyset^{+}\emptyset}{A\emptyset^{+}\emptyset}-1}{\frac{F\emptyset^{+}\emptyset}{A\emptyset^{+}\emptyset}} \tag{3.18 a}$$

Simplifying the reactivity can be defined as

$$\rho = \frac{F\emptyset^{+}\emptyset - A\emptyset^{+}\emptyset}{F^{+}\emptyset^{+}\emptyset} \tag{3.18 b}$$

The inhomogeneous transport equation is

$$S = (A - F)\emptyset \tag{3.19}$$

Combining equations 3.18 and 3.19, reactivity can be defined as

$$\rho = \frac{-\emptyset^{+}S}{F\emptyset^{+}\emptyset} \tag{3.20}$$

The count rate of the detector is defined as

$$T =< \sigma_D \emptyset > \tag{3.21}$$

The detector efficiency and neutron effective source strength can be defined by substituting equations 3.20 and 3.21 into equation 3.14

$$\in S_w = \rho T \tag{3.22}$$

The reactivity and count rate can be determined by a Monte Carlo computer code such as MCNP. By applying the correction factor where state 1 is taken with no fuel in the assembly the MSM correction factor is

$$MSM\ correction\ Factor = \frac{\rho_{2calc} T_{2calc}}{T_{1calc}} \tag{3.23}$$

Multiplying the MSM correction factor by the MSA count ratio the MSM will be

$$\frac{\rho_{2calc} T_{2calc}}{T_{1calc}} \frac{T_1}{T_2} = 1 - k_2 \tag{3.24}$$

where ρ_{2calc}, T_{1calc}, and T_{2calc} are from a computer model and T_1 and T_2 are from experimental measurements. The reactivity or ρ_{2calc} can be calculated in MCNP using a Kcode tally. The count rates T_{1calc} and T_{2calc} can be calculated in MCNP by modeling a cell in the same location as the sensitive length of the detectors filled with He-3 or BF_3 and modeling a neutron source in the same location as the experiment neutron source. An f4 cell tally count modified to count the absorption of thermal neutrons will give the track length absorption flux which can be used as the count rate.

3.6. One-Over-M Plot[4]

A one-over-M plot is a way to predict when a reactor or assembly will reach criticality. It can be seen from equation 3.5 that as k approaches 1, M will approach infinity. It is difficult to determine exactly when M reaches or begins to approach infinity. By taking the inverse of equation 3.5 one-over-M is defined as

$$\frac{1}{M} = 1 - k \tag{3.25}$$

It can be seen that as k approaches 1, one-over-M will approach zero. One-over-M is determined by using equation 3.10. State 1 in equation 3.10 will always be the first count rate taken with no fuel plates in the SCA. High accuracy is needed for the first count as it will impact all points on a one-over M plot. The count ratios in the one-over-M plot determine the y-axis values and the x-axis values are determined by the change in fuel or the number of fuel plates in the assembly.

3.6.1. Linear Estimate

An estimate of when a reactor or assembly will go critical is made by estimating when the plot will intercept the x axis. One way of predicting when the plot will intercept the x-axis is by assuming a linear trend between two data points. The number of plates to go critical can be estimated by the following equation

$$N_c = \frac{(N_2 - N_1)}{(C_1 - C_2)} C_1 + N_1 \tag{3.26}$$

where N_c is the number of plates to go critical, N_x is the number of fuel plates at point x, and C_x is the count ratio at point x. The equation must always be done with two adjacent points to insure maximum accuracy.

3.6.2. K Factor Trend[6]

The trend of a one-over-M plot can be shown by using the five factor formula. The five factor formula is defined as

$$k = \eta \text{Ƥ} \epsilon f P_{nl} \tag{3.27}$$

where η is the thermal fission factor which is the number of fission neutrons produced per neutron absorbed in the fuel, Ƥ is the resonance escape probability which is the fraction of neutrons that slow down to the thermal energy range, ϵ is the fast fission factor which is the total number of fission neutrons over the number of fission neutrons from thermal neutrons, f is the thermal utilization factor which is the probability that if a thermal neutron is absorbed it is absorbed by the fuel, and P_{nl} is the non-leakage probability which is the probability that a neutron will not leak out of the reactor. The non-leakage probability is the only factor that will change as fuel plates are added to the assembly. By plotting 1-k vs the change in P_{nl} a one-over-M plot will show the actual trend of a one-over-M plot.

The non-leakage probability can be estimated by the following relation

$$P_{nl} = \frac{e^{-\tau B^2}}{1+B^2L^2} \tag{3.28}$$

where B^2 is the geometric buckling of the reactor, L^2 is the thermal diffusion area, and τ is the fermi age. The geometric buckling is the only term that will change as fuel plates are added and is defined for a finite cylinder as

$$B^2 = (\frac{2.405}{R})^2 + (\frac{\pi}{H})^2 \tag{3.29}$$

where R is the radius of the assembly and H is the height of the assembly. The height of the assembly is held constant at the height of the fuel bearing portion of the fuel plates. The radius of the assembly can be related to the number of fuel plates by the following equation

$$R = \sqrt{\frac{\#plates * 10.05\,\frac{g}{plate} * .6022\frac{a\,cm^2}{mol\,b}}{5.64E-5\,\frac{a\,U235}{cm\,b} * 235\frac{g}{mol} * \pi * H\,cm}} \tag{3.30}$$

where the atom density of the uranium 235 is the atom density assuming equal distribution of materials over the area of the assembly.

A one-over-M plot was created using the five factor formula. The non-leakage probability was estimated using equations 3.28, 3.29, and 3.30. The other variables in the five factor formula were taken from the ISU Nuclear Reactor Lab Manual. Details of the calculations can be found in Appendix A. The plot is shown in figure 3.1. The plot also shows how the linear trend estimate is applied.

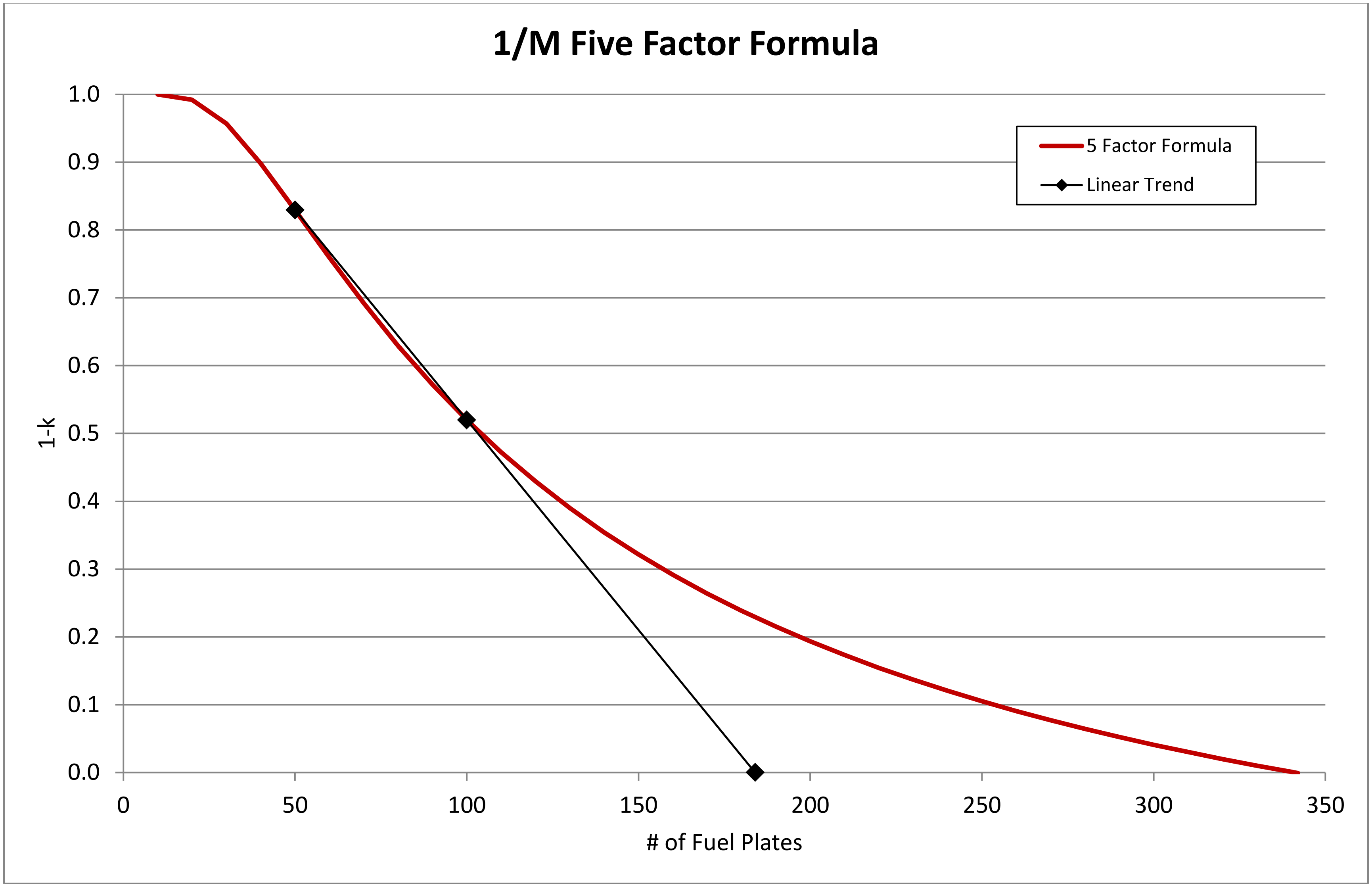

Figure 3-1 One-over-M plot trend with linear trend estimate

It can be seen that the plot does not follow a linear trend. Using a linear estimate to predict when the assembly will reach criticality will result in a conservative fuel plate prediction as seen in figure 3.1. As the assembly approaches criticality, fuel plate predictions will increase in accuracy. Fuel plate predictions will continue to increase as fuel is added to the core due to the increase in accuracy as criticality is approached. It is expected that the final prediction of fuel plates needed to reach criticality by both the MSA and MSM will be an underestimate. The MSM fuel plate prediction will be higher and closer to the actual number of fuel plates needed to reach criticality due to the MSM correction factor.

The one-over-M plot created using the five factor formula can only be used to show the trend that a one-over-M plot will follow. The actual plot shown assumes a homogenous core that is a finite cylinder. The ISU SCA is not a homogenous core and does not have a finite cylinder geometry, but the assumptions will still show the trend of the plot.

3.6.3. Approach-to-critical one-over-M plot - Multiplication Source Approach

A one-over-M approach-to-critical using the MSA experiment procedure is as follows:

1. Fill the assembly tank with water and insert the fuel plate spacing assembly into the tank with no fuel plates in the fuel plate spacing assembly.
2. Place neutron detectors around the assembly and do not move them again.
3. Place a neutron source in the center tube. Program the detectors placed around the assembly to count the neutrons. This count will always be T_1 from equation 3.10. A long count time is needed to lower the error in counting statistics. Remove the neutron source after the detectors have finished counting neutrons.

4. Plot the first point for each detector used on the one-over-M plot. Use the neutron count from step 3 for both T_1 and T_2 in equation 3.10. This will give a normalized value of 1.
5. Remove the fuel plate spacing assembly from the tank and insert only a few fuel plates. Place the fuel plate spacing assembly back into the tank in the same location that it was removed from. Placing the fuel plate spacing assembly in a different location will contribute to uncertainty in experimental results.
6. Place the neutron source back into the assembly in the same location. Program the neutron detectors to count neutrons. This neutron count will be T_2 from equation 3.10. After the neutron detectors are done counting remove the neutron source.
7. Use equation 3.10 to plot one point for each detector on a one-over-M plot.
8. Use equation 3.26 to predict the number of fuel plates needed to reach criticality for each detector.
9. Remove the fuel plate spacing assembly from the tank and add fuel plates. Add no more than half of the difference between the predicted number of fuel plates and the current number of fuel plates to insure that the assembly does not reach criticality. Place the fuel plate spacing assembly back into the tank.
10. Repeat steps 6 through 10 until there are no more fuel plates to add or until the fuel plate prediction is barely under what is already in the assembly.

3.6.4. Approach-to-critical one-over-M plot - Multiplication Source Method

A one-over-M plot created using the MSA approach can be adjusted using the MSM to increase the accuracy of the results. The MSA method will vary greatly between different detectors. Detector location can drastically change results. The MSA method can also under predict or over predict fuel plates needed depending on detector location. The MSM correction factor will correct these errors for each data point on the one-over-M plot. The one-over-M plot will still under predict the number of fuel plates needed to go critical due to the linear trend estimate, but will give accurate estimates of k at each point and each detector used will also give more consistent results regardless of detector location.

The MSM one-over-M plot is created using the following procedure:

1. Create an MCNP model with the geometry of the ISU SCA with no fuel plates in the assembly.
2. Add the neutron detectors to the model including the He-3 and BF_3 as the fill gas.
3. Place an f4 tally, track length flux, on each neutron detector and modify it to track the absorption of neutrons by He-3 or BF_3.
4. Add the neutron source to the model.
5. Run the model and note the reaction rate from each detector.
6. Create a separate model for each fuel plate configuration from the MSA.
7. Run all models and note the track length flux from each detector.
8. Create a model for each fuel plate configuration and remove the neutron source from the model.
9. Add a kcode tally for each fuel plate configuration and run the model.
10. Use equation 3.24 to determine the data points for the MSM one-over-M plot.
11. Use equation 3.26 to make new fuel plate predictions.

3.6.5. Approach-to-critical one-over-M plot - Pre Calculated Correction Factors

The MSM method can be applied using pre calculated correction factors. This will allow ISU lab classes to repeat the experiment without requiring students to use MCNP. The experiment can be performed as follows.

1. Fill the assembly tank with water and insert the fuel plate spacing assembly into the tank with no fuel plates in the fuel plate spacing assembly.
2. Place neutron detectors around the assembly and do not move them again.
3. Place a neutron source in the center tube. Program the detectors placed around the assembly to count the neutrons. This count will always be T_1 from equation 3.10. A long count time is needed to lower the error in counting statistics. Remove the neutron source after the detectors have finished counting neutrons.
4. Plot the first point for each detector used on the one-over-M plot. Use the neutron count from step 3 for both T_1 and T_2 in equation 3.10. This will give a normalized value of 1.
5. Remove the fuel plate spacing assembly from the tank and insert the same amount of fuel plates that was used to determine the first MSM correction factor. The fuel plates must also be in the same location that the fuel plates were in when determining the MSM correction factor. Place the fuel plate spacing assembly back into the tank in the same location that it was removed from. Placing the fuel plate spacing assembly in a different location will contribute to uncertainty in experimental results.
6. Place the neutron source back into the assembly in the same location. Program the neutron detectors to count neutrons. This neutron count will be T_2 from

equation 3.10. After the neutron detectors are done counting remove the neutron source.

7. Use equation 3.10 to plot one point for each detector on a one-over-M plot for the MSA.
8. Use equation 3.24 to plot one point for each detector on a one-over-Mplot for the MSM.
9. Use equation 3.26 to predict the number of fuel plates needed to reach criticality for each detector for the MSM and MSA plots.
10. Remove the fuel plate spacing assembly from the tank and add fuel plates. The number of fuel plates to add will need to match the number of fuel plates used to determine the MSM correction factor. Fuel plates should also be placed in the same location. Place the fuel plate spacing assembly back into the tank.
11. Repeat steps 6 through 10 until there are no more fuel plates to add.

4. MCNP Model

The ISU SCA was modeled in MCNP to calculate neutron count rates and multiplication factors.

4.1. Fuel Plate Spacing Assembly

MCNP is a computer code that uses random numbers to predict particle energy, path length, path direction, and particle interactions. One particle can require upwards of 150,000 random numbers to track its path. Complex geometry requires even more random numbers. Using more random numbers leads to more calculations which causes long computer run times. The ISU SCA was originally modeled in MCNP with the fuel spacing grid. Computer run times were taking longer than practical. To speed up run times the fuel spacing grid was removed from the model. The fuel spacing grid is made of aluminum which has a low absorption cross section compared to other materials in the assembly. The volume of the fuel spacing grid assembly is also small compared to the geometry of the core. The aluminum frame will not have a significant effect on the results of the model. The results of the 18 fuel plate configuration with and without the fuel spacing assembly are shown in table 4.1 below to show that it has little effect. The two aluminum support brackets and the aluminum honeycomb bracket were not modeled in the assembly as a result.

Table 4-1 Fuel plate spacing assembly

Model	MCNP tally track length flux Model (n/cm^2/source particle)	Error
Assembly	3.49E-08	8.20%
No Assembly	3.51E-08	7.93%

4.2. Fuel Composition

It is known that each fuel plate contains 10.07 grams of uranium-235 and 50.77 grams of uranium-238. The fuel bearing section of the fuel plates also contains aluminum and the cladding is pure aluminum. It is unknown whether the uranium is in the form of UO_2 mixed with aluminum or just uranium mixed with aluminum. The results presented in this report assume a UO_2 composition of the fuel. Other computer models made of the SCA have assumed a UO_2 composition of the fuel as well. [2] Atom density calculations for the fuel plates can be found in appendix B.

4.3. Model Construction[7]

The ISU SCA was modeled in MCNP. The input file used for the 150 plate configuration can be found in appendix C. The model was configured to allow future users to easily adjust the configuration of fuel plates.

4.3.1. Graphite Blocks

A Cell in MCNP defines the space that an object is in. There is little space between each graphite block. To simplify the input file all of the graphite blocks were modeled as one cell. Modeling the graphite as one cell has no significant effect on the results. One block was removed to allow a neutron detector to be placed in its location. The area between the neutron detector and graphite block was modeled as a void.

4.3.2. Fuel Plates

A lattice in MCNP allows a structure to be repeated in a pattern. A universe can be used to fill the lattice and defines the objects within each area of the lattice. The fuel plates were modeled using a rectangular lattice and two different universes. Each universe contains two fuel plates and water. The location of the fuel plates in each universe changes to allow the rows to be offset. The location of fuel plates in the fuel plate spacing assembly can easily be changed to allow different configurations.

4.3.3. Detectors

The three detectors used were modeled in MCNP. Each detector is only modeled using the sensitive area of the detector. Detector A was filled with He-3 with a pressure of 1.03 Mpa. Detectors B and C were filled with BF_3 with a pressure of .21 Mpa. Atom density Calculations for the detectors can be found in appendix D. An f4 cell track length flux tally was placed on each cell and modified to count the absorption of thermal neutrons by the detector. The fm modifier is

$$fm = N \int_0^{14Mev} \sigma_a \emptyset dE$$

were N is the atom density of He-3 in the detector, σ_a is the neutron absorption cross section, and $\emptyset$ is the flux. The fm modifier will track the absorption of neutrons by He-3 integrated over the energy of the neutrons.

4.3.4. Neutron source

The neutron source in the MCNP model was modeled as a point source in the central tube. The source was modeled using a watt fission spectrum.

4.3.5. Pictures

The MCNP code with 150 fuel plates is shown in figures 4.1 - 4.4. The only difference between MCNP model geometries is the number of fuel plates in the assembly.

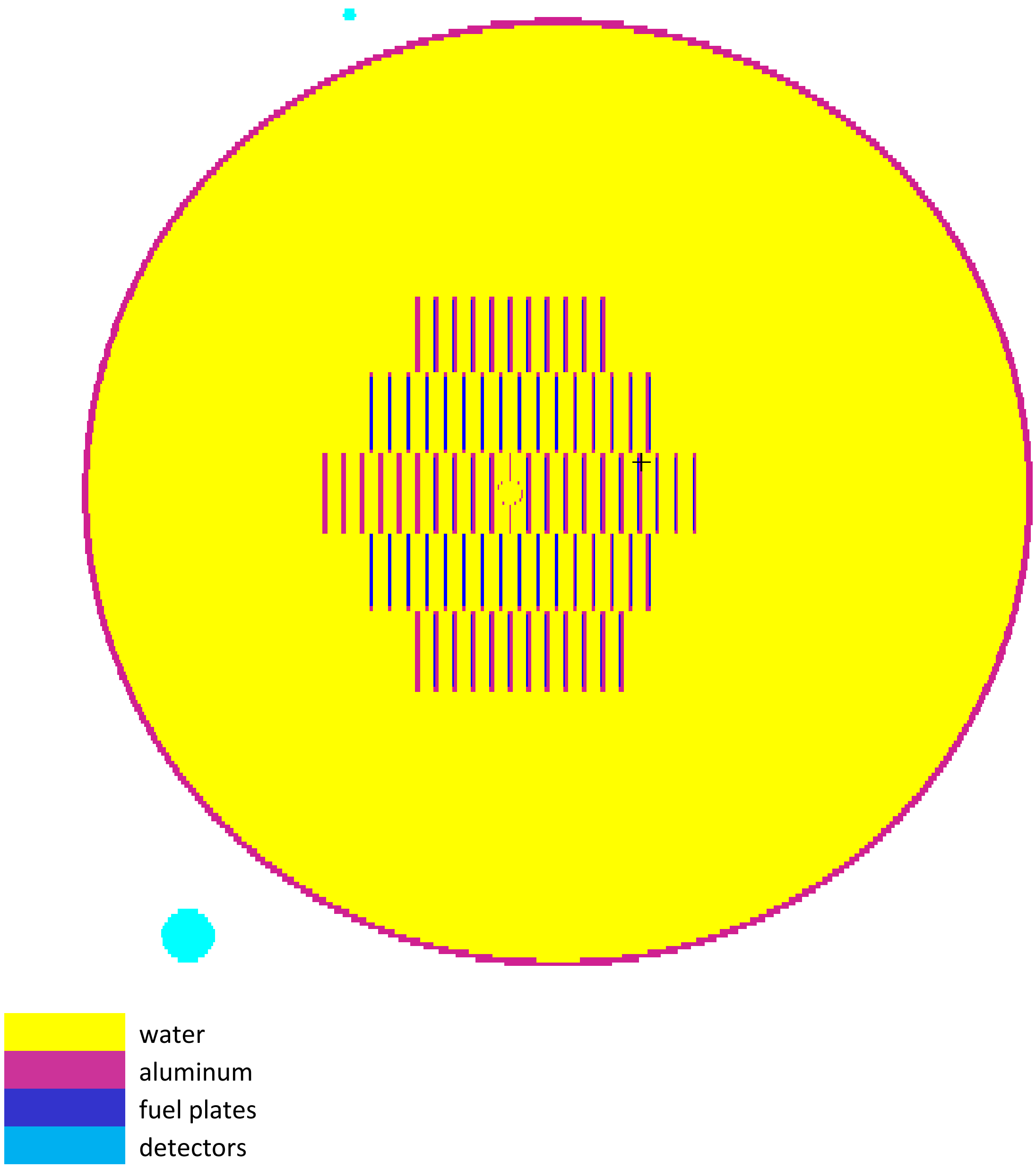

Figure 4-1 Plan view of MCNP model with detectors A & C

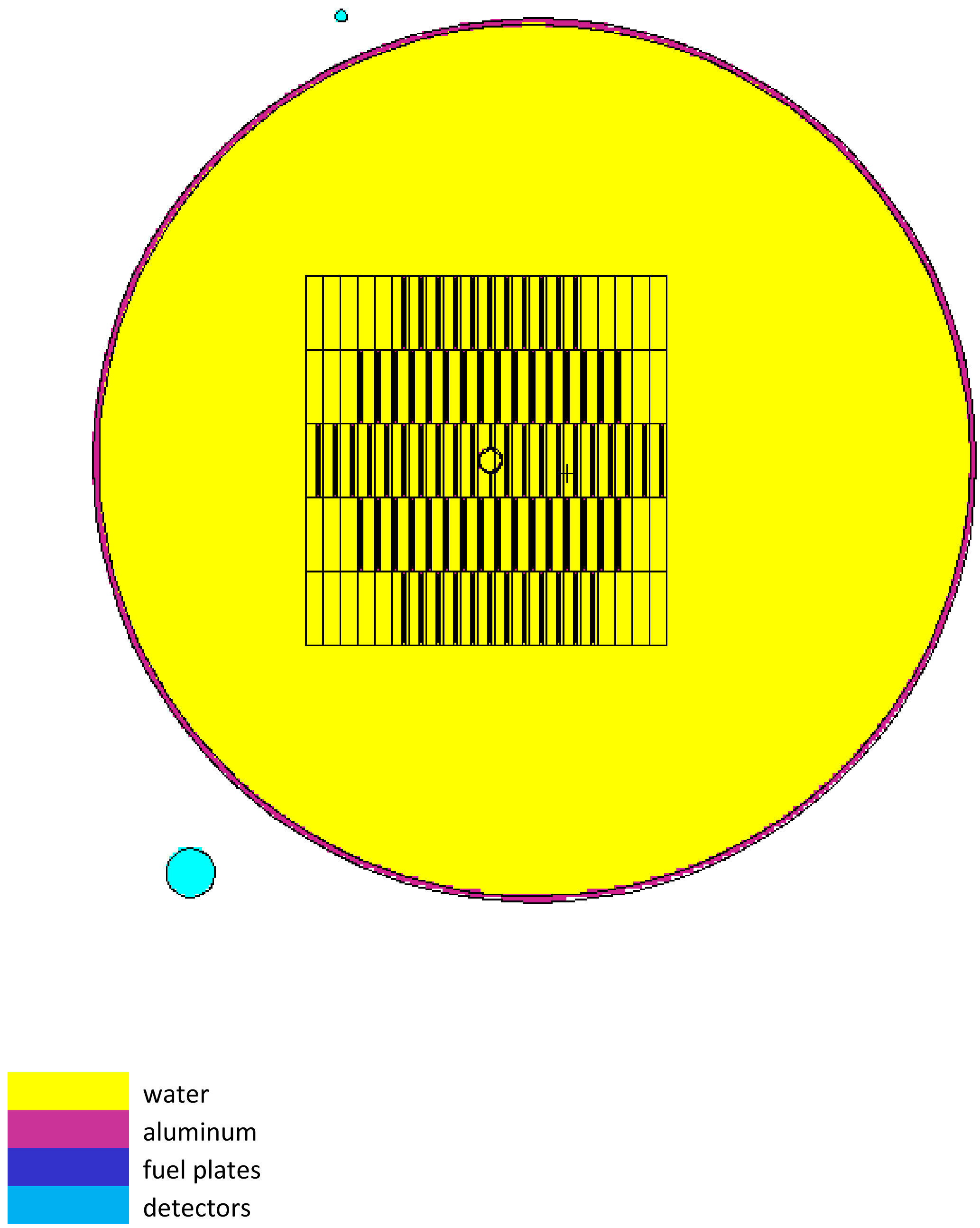

Figure 4-2 Plan view of MCNP model with detectors A & C and lattice grid

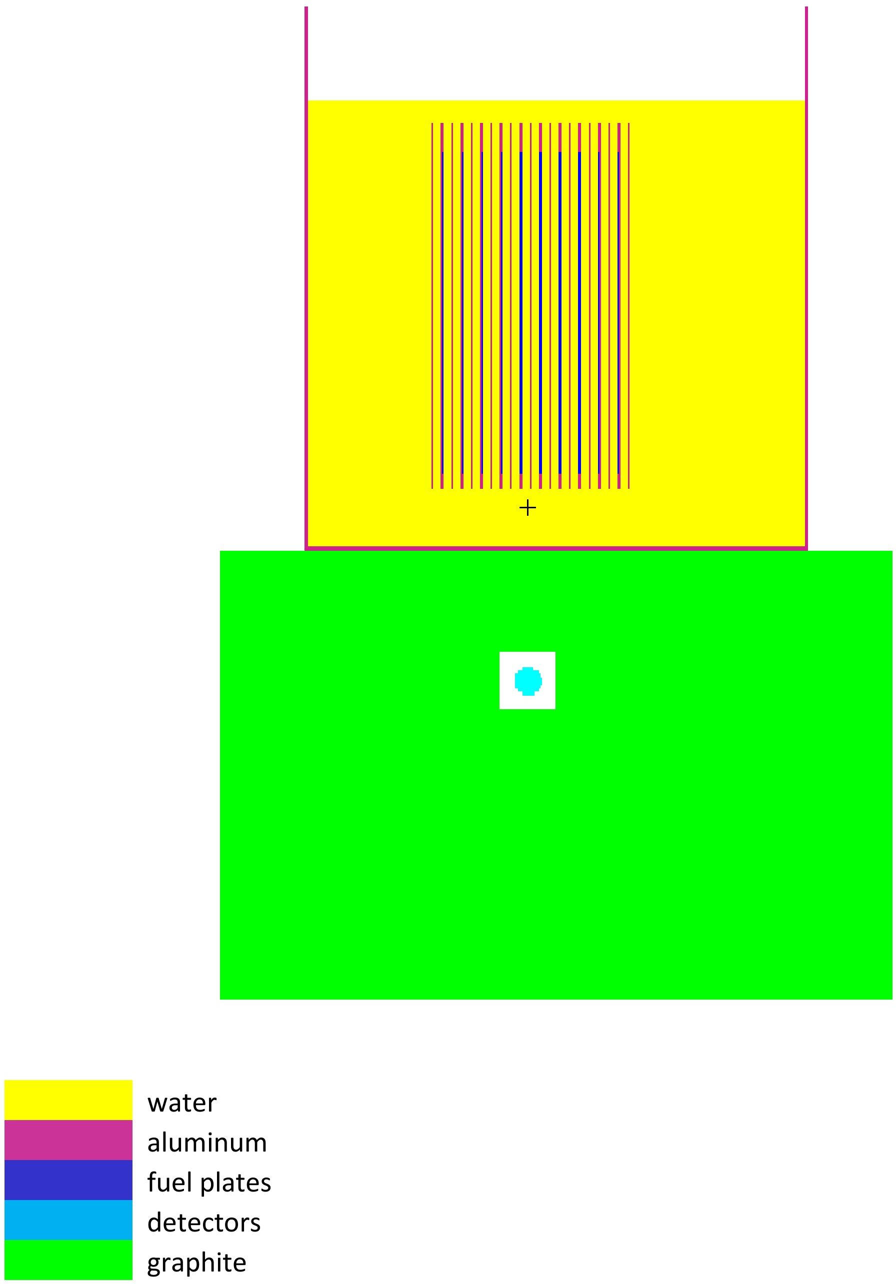

Figure 4-3 Elevation view of MCNP model (XZ plane) and detector B

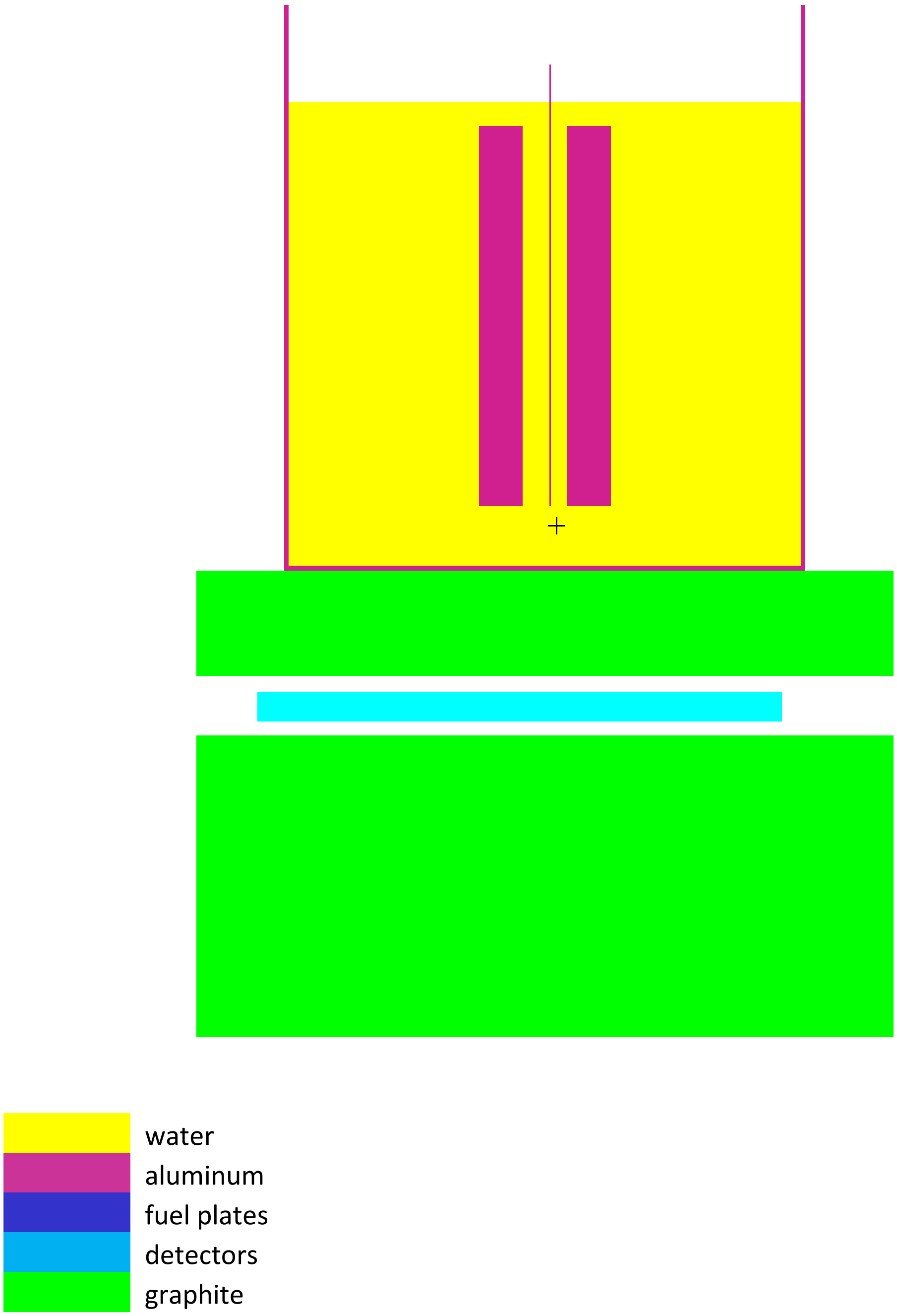

Figure 4-4 Elevation view of MCNP model (YZ plane) and detector B

5. MSA Experimental Results

An approach-to-critical experiment was performed using the MSA method. A count rate for each detector was taken with no fuel plates in the assembly, one for each detector. Next 18 fuel plates were added to the assembly and a count rate was taken. The count rates were used to plot a point on a one-over-M plot for each detector. Equation 3.10 was used to find the y axis value and the x axis value is the number of fuel plates, 18. An estimate of number of fuel plates needed to go critical was made for each detector using equation 3.24. The lowest estimate was used in determining how many fuel plates to add to the assembly. The process of adding fuel plates, taking a neutron count reading, plotting a point on a one-over-M plot, and making a prediction of the number of fuel plates needed to go critical was repeated until no fuel plates were left. The fuel plate configurations from the experiment had 0, 18, 26, 40, 52, 80, 104, 136, and 150 plates. A picture of each fuel plate configuration can be found in appendix E.

5.1. Experimental Detector Counts

Detector count readings were taken for each detector at each fuel plate configuration. The results are shown in table 5.1. The time period with 0 plates in the assembly was a 7000 second period and all other readings were taken over a 400 second period. If count readings were taken over the same period then detector neutron count readings could be used to find a count ratio. Since the configuration with 0 plates was taken over a 7000 second period, the count readings need to be normalized by finding the count rate. The count rate was determined by dividing the detector count by the time period. The detector neutron count rates for each detector and each fuel plate configuration are shown in table 5.2.

Table 5-1 Experimental neutron detector counts (counts)

plates	Detector A	Detector B	Detector C
0	2893	21803	2934
18	322	3321	264
26	457	4157	371
40	590	8030	581
52	712	10081	776
80	1578	16715	1224
104	2478	26889	1792
136	4196	46564	4158
150	5161	57141	5249

Table 5-2 Experimental detector count rates (counts/sec)

plates	Detector A	Detector B	Detector C
0	0.413	3.115	0.419
18	0.805	8.303	0.660
26	1.143	10.393	0.928
40	1.475	20.075	1.453
52	1.780	25.203	1.940
80	3.945	41.788	3.060
104	6.195	67.223	4.480
136	10.490	116.410	10.395
150	12.903	142.853	13.123

5.2. MSA One-Over-M plot

The count rates were used to create a one-over-M plot. Equation 3.10 was used to determine the y-axis value for the plot. State 1 was always taken to be the 0 plate configuration because the k multiplication factor is known to be 0. The x-axis value for the plot is the number of fuel plates in the assembly. The count ratios for each detector for each fuel plate configuration are shown in table 5.3. The one-over-M plot for each detector is shown in Appendix F.1 through F.3. Appendix F.4 gives an example of how extrapolation or the linear trend estimate is used. All detectors are also plotted on the same one-over-M plot in figure 5.1 with a zoomed in view in Appendix F.5.

Table 5-3 Count Ratios used for 1/M plot (T_1/T_2)

plates	Detector A	Detector B	Detector C
0	1.0000	1.0000	1.0000
18	0.5134	0.3752	0.6351
26	0.3617	0.2997	0.4519
40	0.2802	0.1552	0.2886
52	0.2322	0.1236	0.2161
80	0.1048	0.0745	0.1370
104	0.0667	0.0463	0.0936
136	0.0394	0.0268	0.0403
150	0.0320	0.0218	0.0319

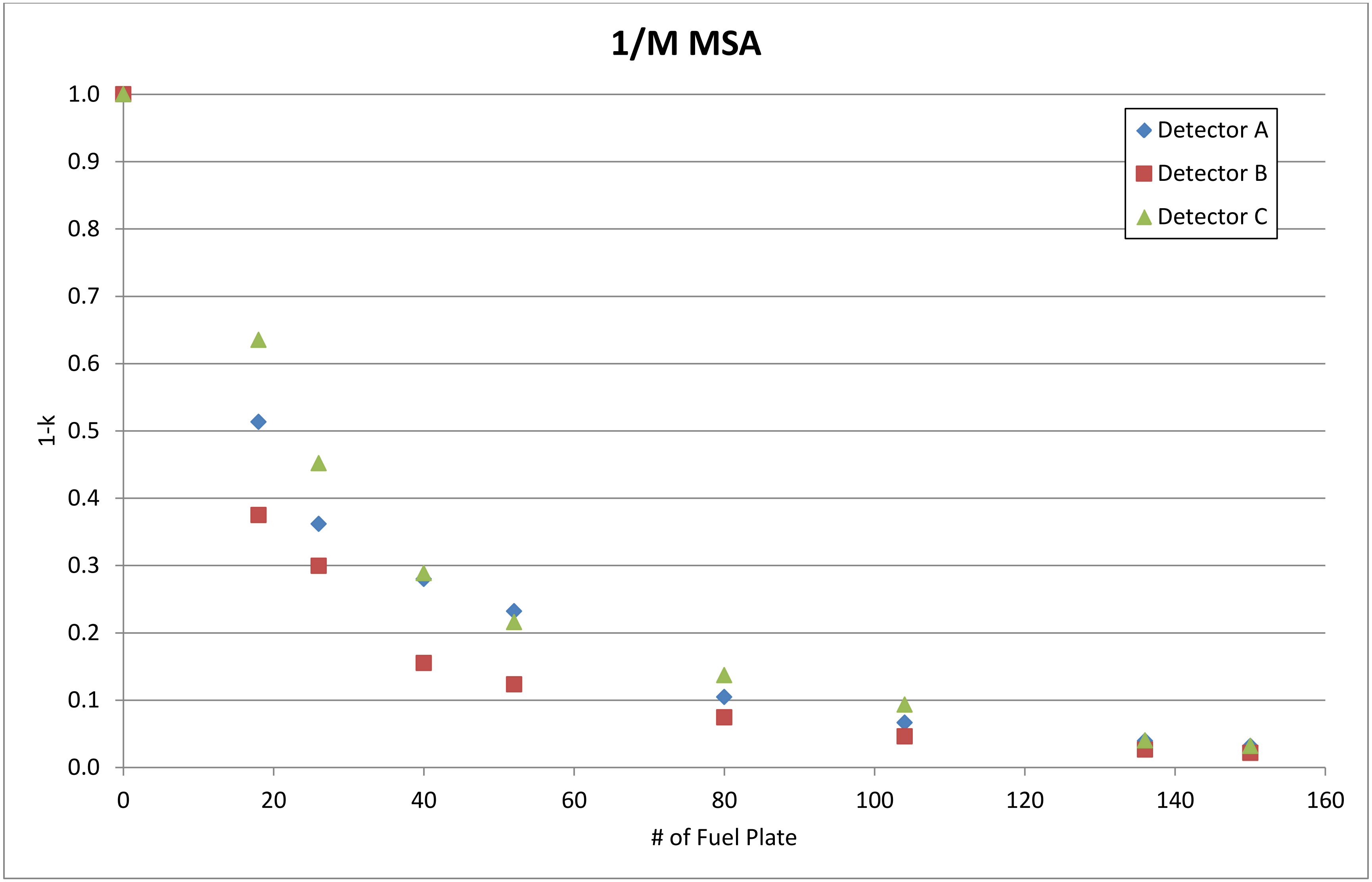

Figure 5-1 One-over-M plot of all three detectors

5.3. K Estimate

An estimate of the multiplication factor k was made from the count ratios used in the one-over-M plot. An estimate was made for each detector at each fuel plate configuration using equation 3.10. Since the count rate with no fuel plates in the SCA is used for all count ratios it has the greatest impact on the results. The results are shown in table 5.4.

Table 5-4 K estimates from count ratios

plates	Detector A	Detector B	Detector C	Average	Standard Deviation
0	0	0	0	0	0
18	0.4866	0.6248	0.3649	0.4921	0.1062
26	0.6383	0.7003	0.5481	0.6289	0.0625
40	0.7198	0.8448	0.7114	0.7587	0.0610
52	0.7678	0.8764	0.7839	0.8094	0.0478
80	0.8952	0.9255	0.8630	0.8946	0.0255
104	0.9333	0.9537	0.9064	0.9311	0.0193
136	0.9606	0.9732	0.9597	0.9645	0.0062
150	0.9680	0.9782	0.9681	0.9714	0.0048

5.4. Fuel Plate Predictions

The number of fuel plates needed to reach criticality was predicted for each fuel plate configuration and each detector. Equation 3.26 was used in determining the number of fuel plates needed and the results are shown in table 5.5. Two data points were used to determine each prediction, the first being the data point at which the prediction is being made and the second is the data point immediately preceding the first. The predictions made were used to determine how many fuel plates to add. The number of fuel plates added should be half of the difference between the lowest prediction and the current number of fuel plates in the SCA to insure that the assembly does not go critical.

Table 5-5 Prediction of the number of fuel plates needed reach criticality MSA

plates	Detector A	Detector B	Detector C	Average	Standard Deviation
0	N/A	N/A	N/A	N/A	N/A
18	37	29	49	38	10
26	45	58	46	50	7
40	88	55	65	69	17
52	110	99	88	99	11
80	103	123	129	118	13
104	146	143	156	148	6
136	182	180	160	174	12
150	211	212	203	209	5

6. MCNP Results

The calculations made in MCNP were used to calculate a correction factor to be applied to the MSA. Two calculations were run for each fuel plate configuration. One calculation was used to determine the multiplication factor k. The other was used to determine the calculated neutron detector count rates. The results were used to plot the corrected one-over-M plots and make new fuel plate predictions.

6.1. MCNP Model Tally Counts

MCNP calculations were done to determine calculated neutron detector counts. Each detector sensitive area was model and filled with He-3. An f4 track length flux tally was placed on each cell and modified to track the absorption reaction rate of neutrons by He-3. The reaction rate of each detector at each fuel plate configuration is shown in table 6.1. The results are in neutrons absorbed per cm^2 per source particle. The percent error for each detector and fuel plate configuration is shown in table 6.2. Ten statistical tests are performed by MCNP6 to provide reliability that the results are correct. The tests include tally mean, relative error, variance of the various, figure of merit, and tally PDF. The tests do not guarantee accuracy and further understanding of the input code is needed to verify results. All results passed all ten statistical tests. Relative error is recommended to be below 0.1. All output results were below 0.05 relative error.

Table 6-1 MCNP tally track length flux (n/cm^2/source particle)

plates	Detector A	Detector B	Detector C
0	3.87E-08	1.39E-08	2.26E-09
18	9.70E-08	4.59E-08	4.69E-09
26	1.34E-07	6.45E-08	5.85E-09
40	1.90E-07	1.25E-07	9.35E-09
52	2.38E-07	1.87E-07	1.24E-08
80	4.53E-07	3.16E-07	1.95E-08
104	7.10E-07	5.22E-07	3.21E-08
136	1.23E-06	9.14E-07	5.37E-08
150	1.58E-06	1.12E-06	6.69E-08

Table 6-2 MCNP tally error (%)

plates	Detector A	Detector B	Detector C
0	6.78%	3.34%	1.48%
18	7.66%	1.27%	2.46%
26	6.86%	0.60%	1.22%
40	5.70%	0.42%	0.99%
52	5.57%	0.37%	0.94%
80	4.78%	0.40%	1.05%
104	4.13%	0.36%	0.92%
136	6.82%	0.52%	1.37%
150	5.70%	0.44%	1.12%

6.2. K Code results

An MCNP kcode calculation was used to determine the multiplication factor k. A separate calculation was done for each fuel plate configuration. The kcode calculation used 10,000 neutrons per generation. There were 220 generations run and the first 20 generations were skipped to allow the fission source to converge. The results are shown in table 6.3. All results passed the source entropy convergence check. Neutron particles are born at a specified origin for the first generation. The next generation will use the previous generation to determine where neutrons are born. The source entropy convergence check is to insure that the location where neutrons are being born has converged.

Table 6-3 MCNP Kcode results

plates	k	standard deviation
0	N/A	N/A
18	0.35653	0.00035
26	0.42942	0.00038
40	0.55316	0.00040
52	0.61946	0.00046
80	0.70003	0.00045
104	0.76913	0.00046
136	0.83526	0.00047
150	0.85385	0.00045

6.3. MSM Correction Factor

An MSM correction factor was calculated for each detector at each fuel plate configuration. Equation 3.23 was used to calculate the correction factor. Calculated count rates used are from table 6.1 and calculated multiplication factor k variables are from table 6.2. The correction factor for each fuel plate configuration and detector are shown in table 6.4.

Table 6-4 MSM correction factor

plates	Detector A	Detector B	Detector C
18	1.612	2.130	1.333
26	1.980	2.654	1.476
40	2.187	4.013	1.846
52	2.338	5.132	2.092
80	3.506	6.825	2.590
104	4.232	8.686	3.274
136	5.242	10.848	3.909
150	5.965	11.821	4.320

6.4. MSM Correction Factor Error[8]

The MSM correction factor is a function of several variables and each has its own error. To calculate the overall error, the multivariate propagation of error formula can be used. The uncertainty in the MSM data points are calculated by

$$\sigma_U = \sqrt{(\frac{\partial U}{\partial X_1})^2\sigma_{X_1}^2 + (\frac{\partial U}{\partial X_2})^2\sigma_{X_2}^2 + \cdots + (\frac{\partial U}{\partial X_n})^2\sigma_{X_n}^2}$$

were U is equation 3.23, $\frac{\partial U}{\partial X_1}$ is the partial derivative with respect to the variable X_1, and σ_{X_1} is the uncertainty of variable X_1. The uncertainty calculations can be found in Appendix G. The uncertainty of the MSM correction for each detector and each fuel plate configuration is shown in table 6.5.

Table 6-5 MSM correction factor uncertainty (σ)

plates	Detector A	Detector B	Detector C
18	0.09	0.05	0.05
26	0.08	0.05	0.04
40	0.06	0.04	0.03
52	0.06	0.04	0.02
80	0.04	0.03	0.02
104	0.03	0.02	0.02
136	0.02	0.02	0.01
150	0.02	0.02	0.01

6.5. MSM One-Over-M Plot

MSM one-over-M plots were created using the MSM correction factor and MSA count ratios. Data points were found by multiplying the MSA count ratios by the MSM correction factor. The MSM count ratios calculated are shown in table 6.6. MSM one-over-M plots were created for each neutron detector. Appendix F.1 through F.3 show one-over-M plots for each detector with both the MSA and the MSM data points to show

how the MSM corrects the data points. Figure 6.1 shows all neutron detectors plotted on the same MSM one-over-M plot to show how the data points align by applying the MSM correction factor. The figure also has error bars showing the error within two standard deviations from calculating the MSM correction factor.

All MSM detector data points should align regardless of detector location. It can be seen that five data points from detector B do not align with the other two detectors. This may be explained from either a faulty detector used when performing the MSA experiment or from the detector geometry being modeled incorrectly in MCNP. Detector B is located in the graphite which may also be causing errors in the MCNP model. Figure 6.2 shows all data points from the MSA and all data points from the MSM on the same plot. F.6 through F.8 show a comparison of two detectors for each plot, showing MSA and MSM results.

Table 6-6 Count ratios for 1/M plot MSM

plates	Detector A	Detector B	Detector C
0	1.0000	1.0000	1.0000
18	0.8274	0.7990	0.8463
26	0.7161	0.7956	0.6669
40	0.6128	0.6226	0.5328
52	0.5428	0.6343	0.4519
80	0.3673	0.5087	0.3547
104	0.2823	0.4025	0.3063
136	0.2065	0.2903	0.1576
150	0.1911	0.2578	0.1380

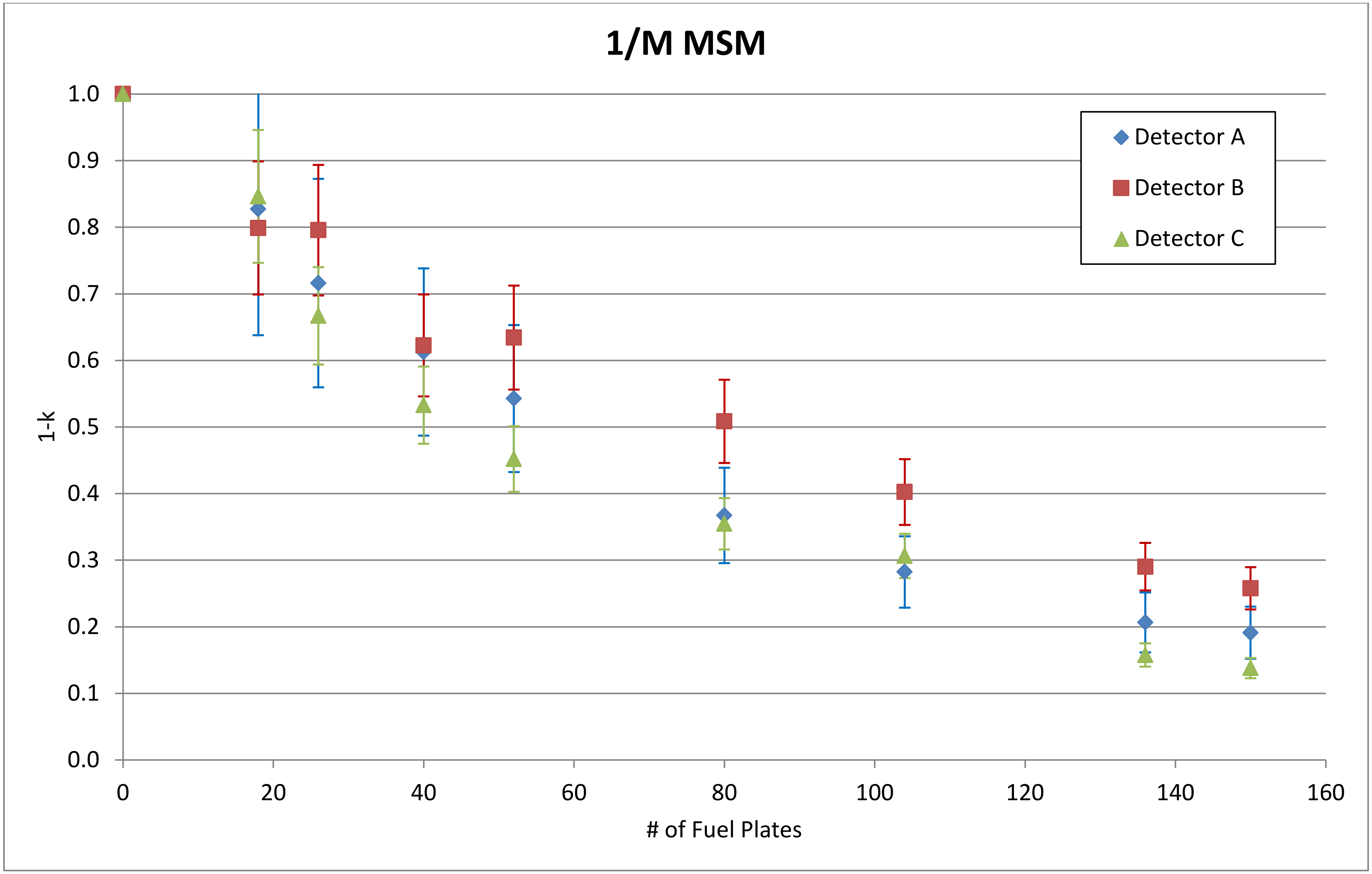

Figure 6-1 MSM one-over-M plot of all detectors

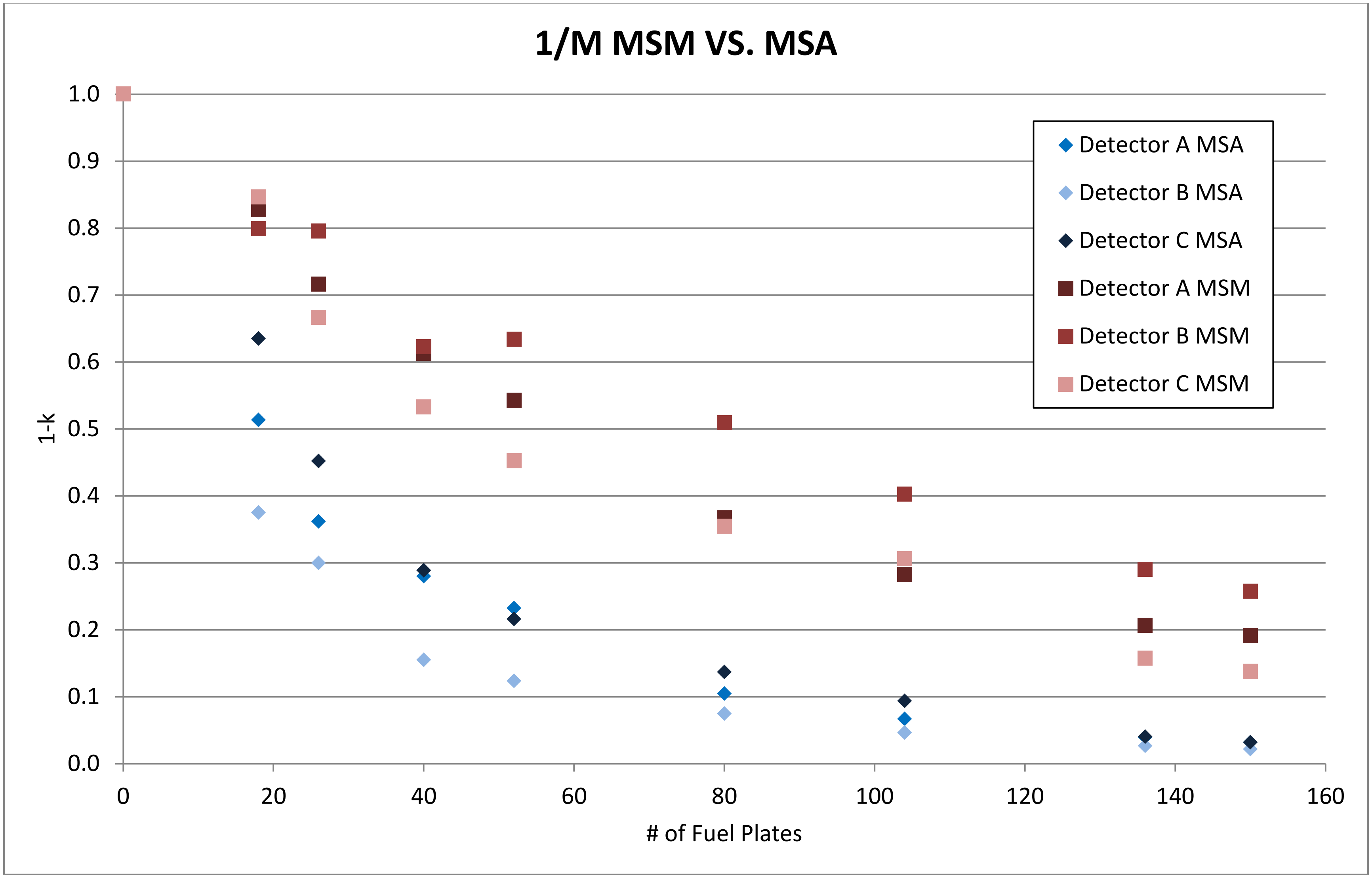

Figure 6-2 MSA and MSM One-over-M plot for all detectors

6.6. K Estimates from One-Over-M Plot

An estimate of the multiplication factor k was made for each fuel plate configuration using the three neutron detectors. Equation 3.24 was used to calculate the multiplication factor k. The results are shown in table 6.7 below. The MSM improved the MSA predictions of the multiplication factor k. This can be seen by comparing the MSA multiplication k predictions in table 5.7 to the MSM multiplication factor k predictions in table 6.7.

Table 6-7 k estimate from count ratios MSM

plates	Detector A	Detector B	Detector C	Average	Standard Deviation
18	0.1726	0.2010	0.1537	0.1758	0.0195
26	0.2839	0.2044	0.3331	0.2738	0.0530
40	0.3872	0.3774	0.4672	0.4106	0.0402
52	0.4572	0.3657	0.5481	0.4570	0.0744
80	0.6327	0.4913	0.6453	0.5898	0.0698
104	0.7177	0.5975	0.6937	0.6696	0.0519
136	0.7935	0.7097	0.8424	0.7819	0.0548
150	0.8089	0.7422	0.8620	0.8044	0.0490

6.7. Fuel Plate Predictions

The number of fuel plates needed to reach criticality was estimated from the MSM data using a linear extrapolation. Estimates were made between each step in fuel plate configurations by using equation 3.26. The results are shown in table 6.8. Applying the MSM correction factor increased predictions by an average of 46 fuel plates. The MSM fuel plate predictions are still an under prediction, but the accuracy increased by applying the MSM correction factor. The accuracy also increases as more fuel plates are added into the SCA for both the MSA and MSM. Detector B predicted a large number of fuel plates needed to go critical by using the count ratios from the 18 and 26 fuel plate configurations.

Detector B also predicted a negative number of fuel plates needed to go critical by using the count ratios from the 40 and 52 fuel plate configurations. The difference in the number of fuel plates between each configuration is small enough that it only leads to a small change in the count ratios. A small error can have a large impact on the predictions. The calculated averages for the 26 and 52 fuel plate configurations were ignored due to this error.

Table 6-8 Fuel plate predictions to go critical from linear trend MSM

plates	Detector A	Detector B	Detector C	Average	Standard Deviation
18	104	90	117	104	14
26	77	1898	56	N/A	N/A
40	123	90	96	103	18
52	145	-603	119	N/A	N/A
80	139	193	182	171	29
104	184	195	256	212	39
136	223	219	170	204	30
150	323	261	248	277	40

6.8. Initial count rate comparison

The MSA and MSM experiment performed used the 0 fuel plate configuration as the initial count rate. The multiplication factor k in this configuration is known to be 0. This allows the k_1 term to be eliminated from the equations used. However, the application of the MSM is not usually performed with the initial count rate at a configuration with a multiplication factor k of 0. MSM one-over-M plots were created using the 18 fuel plate configuration as the initial count rate and compared to the 0 fuel plate configuration as the initial count rate. The initial k_1 can no longer be eliminated and Equation 3.24 will become

$$\frac{\rho_{2calc} T_{2calc}}{\rho_{1calc}\, T_{1calc}} \frac{T_1}{T_2}(1-k_1) = 1-k_2 \qquad (7.1)$$

where state 1 was taken with 18 fuel plates in the SCA. MSM one-over-M plots were created using Equation 7.1 and 3.24 for each detector and are shown in figures 6.3 through 6.5. The plots also show the MCNP kcode calculation (1-k). It can be seen that the results improved when using an initial count rate with a k multiplication factor value greater than zero.

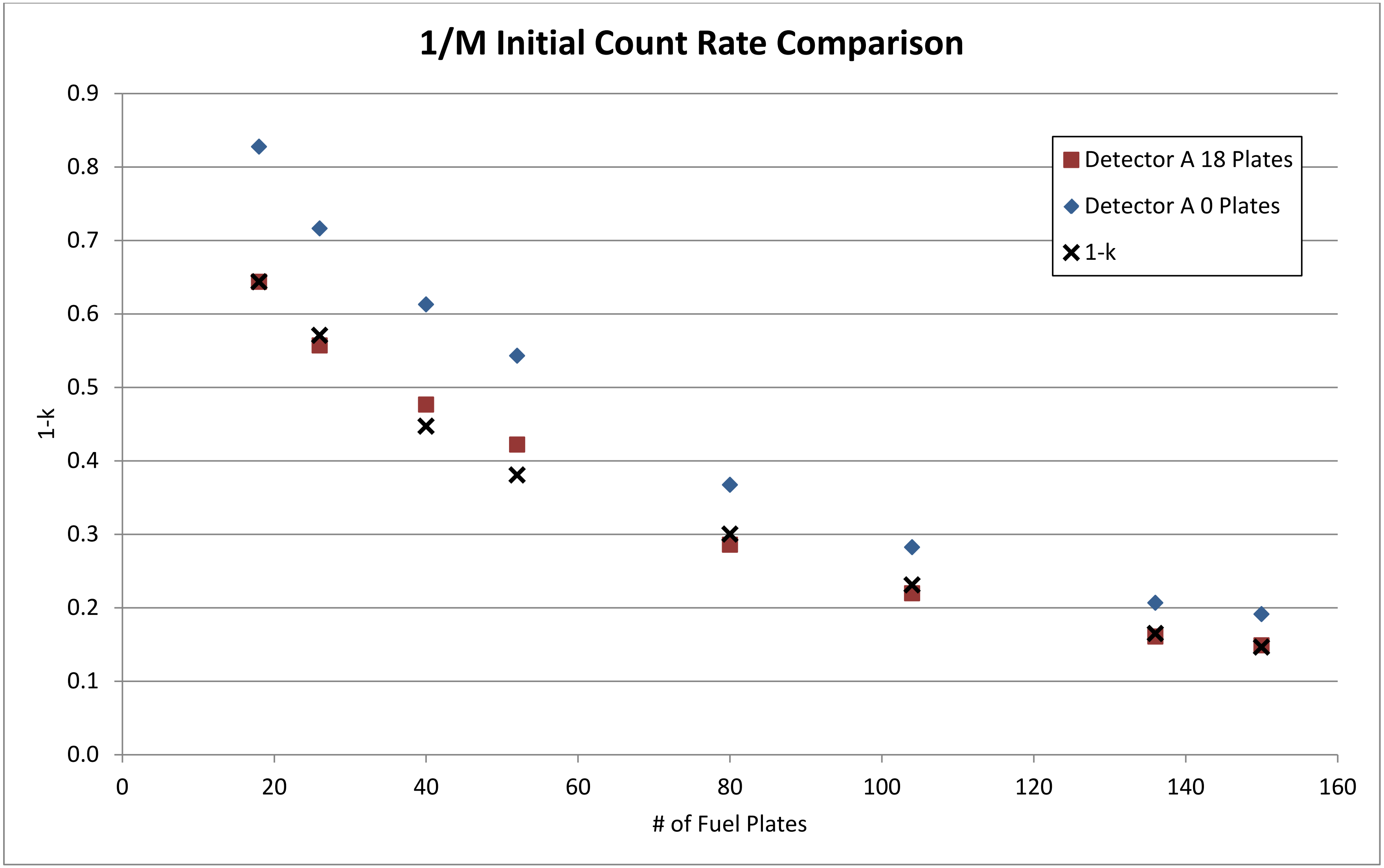

Figure 6-3 MSM one-over-M plot initial count rate comparison

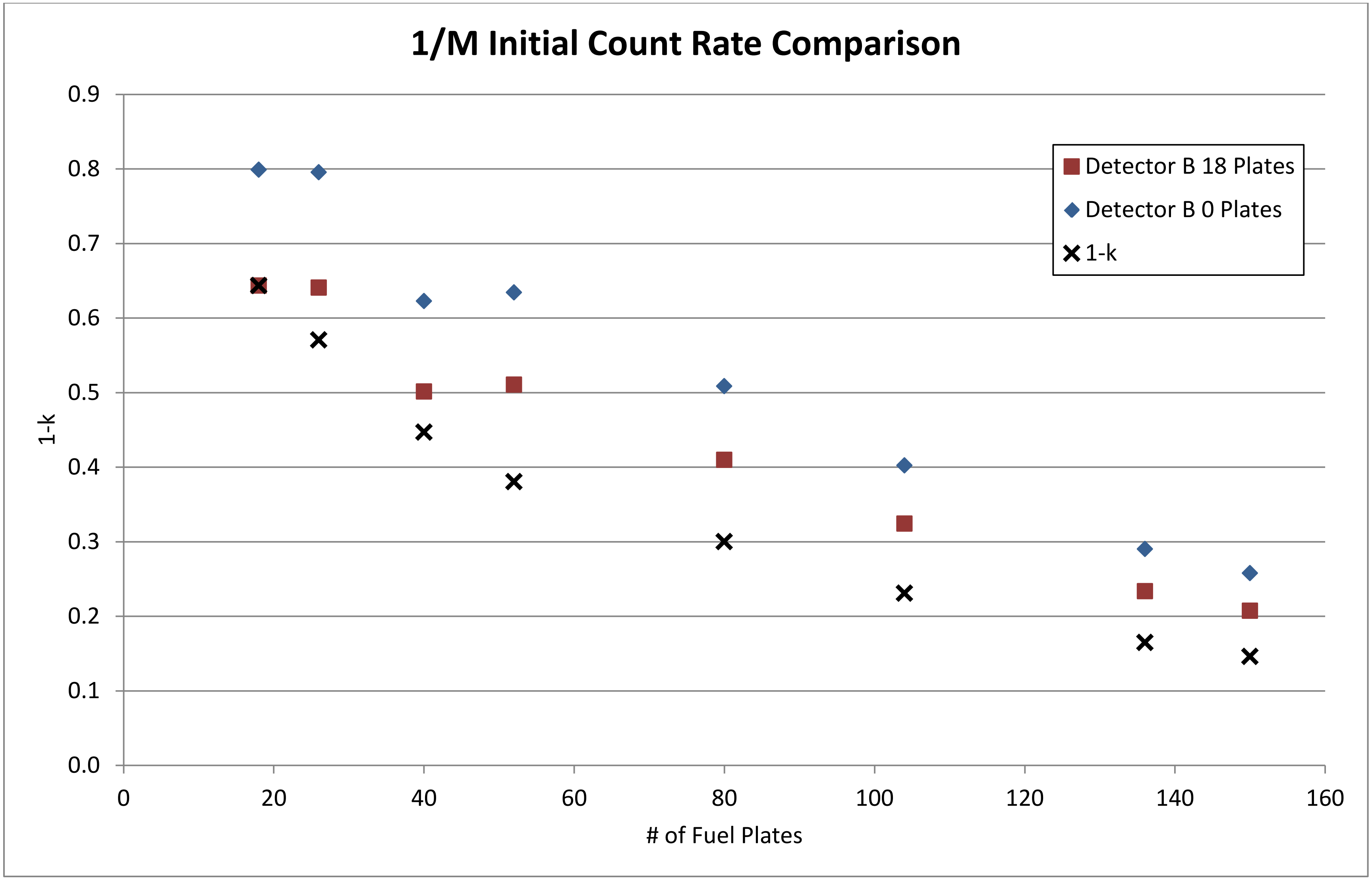

Figure 6-4 MSM one-over-M plot initial count rate comparison

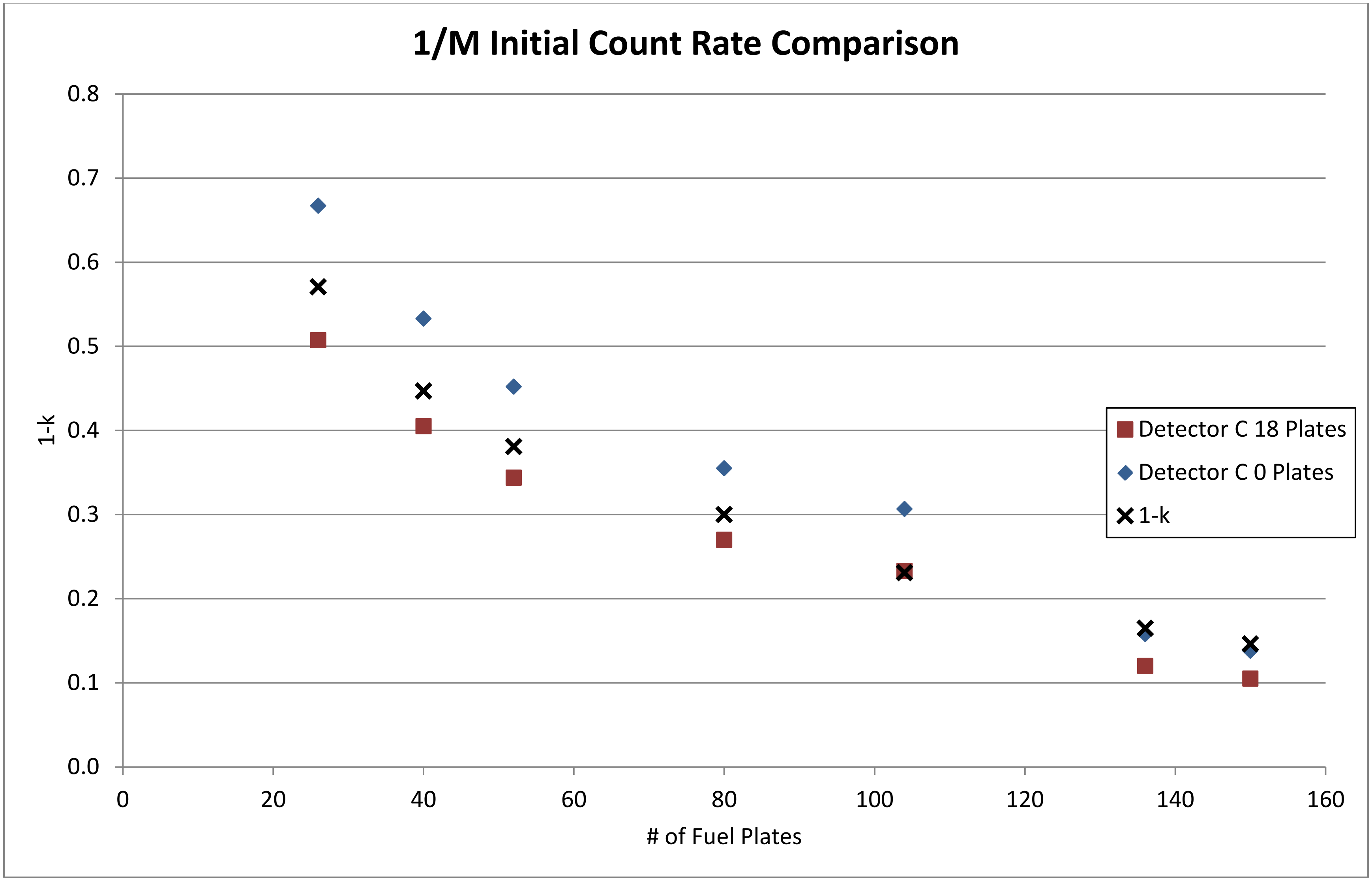

Figure 6-5 MSM one-over-M plot initial count rate comparison

6.9. Simulated MSA Using MCNP

The MSA experiment was stopped because there were no fuel plates left to be added to the assembly. The MSA method can be simulated in MCNP to continue the experiment. The same MCNP code used to calculate the MSM correction factor can be used to simulate the MSA approach-to-critical experiment. The procedure of the experiment is the same as the MSA except the neutron detector counts are obtained from the MCNP model calculations.

The experiment was continued with 208 fuel plates in the MCNP model, which was the last fuel plate prediction made by the MSA experiment. The other fuel plate configurations were 240, 280, 310, 326, and 336. The ISU SCA fuel plate grid assembly is limited to holding 208 fuel plates. To allow more than 208 fuel plates to be added into the MCNP model the rows and columns of fuel plates were extended as needed.

6.9.1. Simulated MSA MCNP Tally Counts

The track length flux tally was used as the neutron detector tally count. The tally track length flux obtained from each detector at each fuel plate configuration is shown in table 6.9.

Table 6-9 MCNP tally track length flux (n/cm^2/source particle)

plates	Detector A	Detector B	Detector C
0	3.87E-08	1.39E-08	2.26E-09
150	1.58E-06	1.12E-06	6.69E-08
208	3.32E-06	2.29E-06	1.79E-07
240	6.68E-06	3.63E-06	2.97E-07
280	1.65E-05	7.36E-06	6.78E-07
310	3.91E-05	1.60E-05	1.70E-06
326	9.66E-05	3.32E-05	4.08E-06
336	2.52E-04	8.11E-05	1.04E-05

6.9.2. Simulated MSA One-Over-M Plot

The track length flux tallies obtained were used to plot a one-over-M plot. Count ratios were calculated using equation 3.10. The count ratios are shown in table 6.10. The one-over-M plot is shown in figure 6.6 and 6.7.

Table 6-10 Count ratios for simulated MSA 1/M plot

plates	Detector A	Detector B	Detector C
0	1.00000	1.00000	1.00000
150	0.02450	0.01236	0.03383
208	0.01165	0.00606	0.01262
240	0.00579	0.00382	0.00762
280	0.00234	0.00189	0.00334
310	0.00099	0.00087	0.00133
326	0.00040	0.00042	0.00055

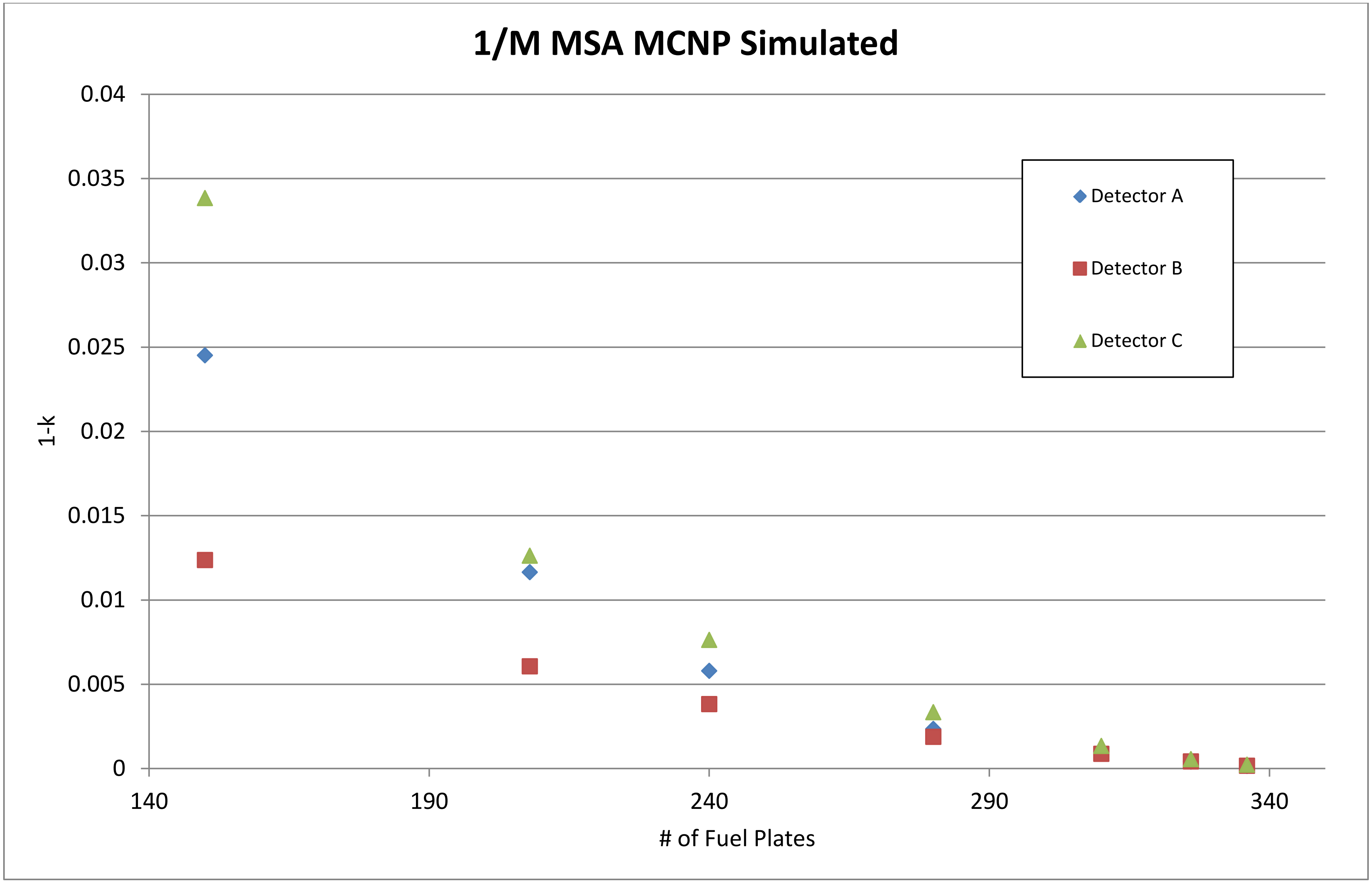

Figure 6-6 Table 6 11 MSA one-over-M plot simulated in MCNP

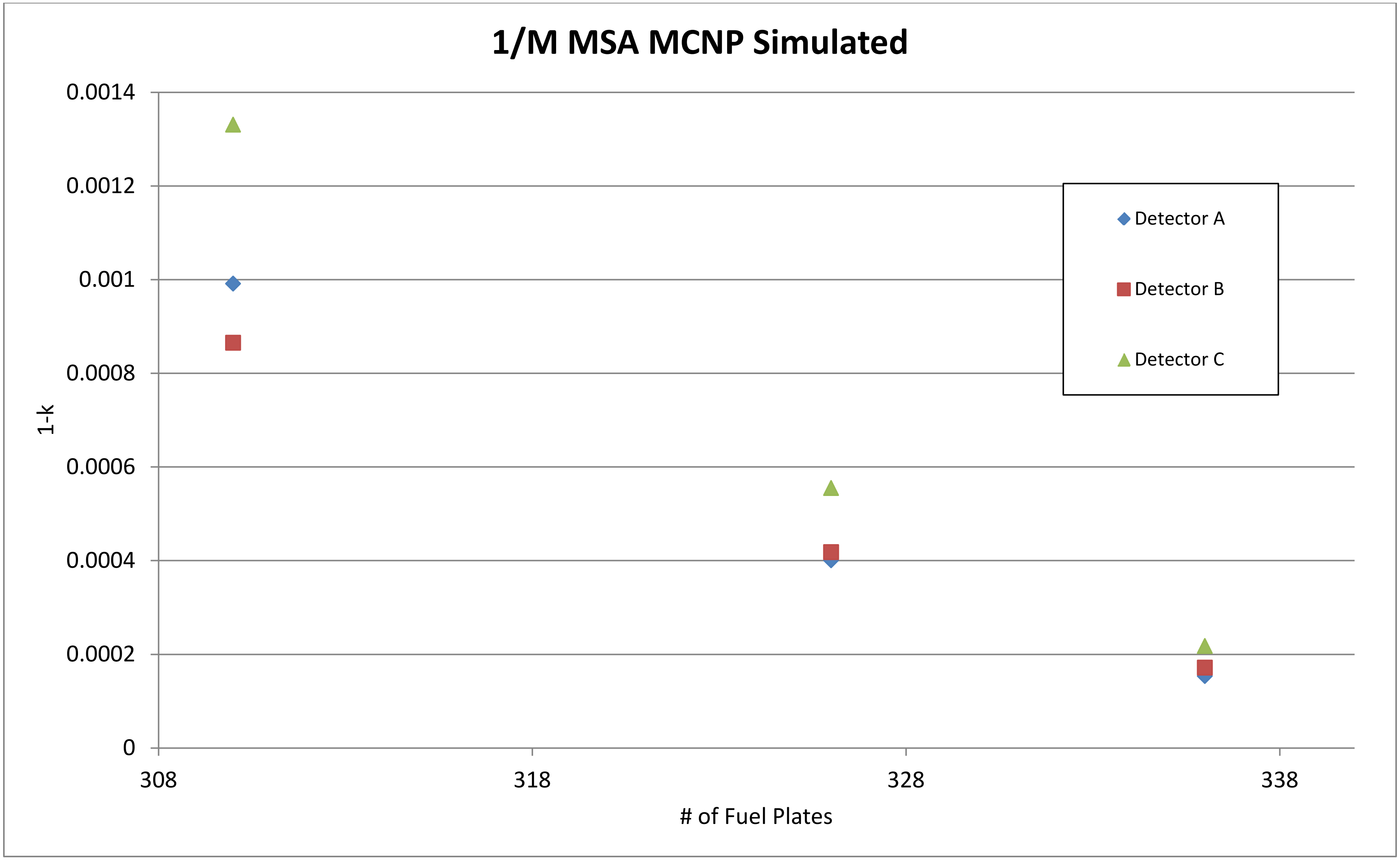

Figure 6-7 MSA one-over-M plot simulated in MCNP (280 plates to 336 plates)

6.9.3. Simulated MSA K Estimate

An estimate of the multiplication factor K was made for each neutron detector at each fuel plate configuration. Equation 3.10 was used to calculate multiplication factor K. The results are shown in table 6.11.

Table 6-11 k estimate from simulated MSA count ratios

plates	Detector A	Detector B	Detector C	Average	Standard Deviation
150	0.9755	0.9876	0.9662	0.9764	0.0088
208	0.9884	0.9939	0.9874	0.9899	0.0029
240	0.9942	0.9962	0.9924	0.9943	0.0016
280	0.9977	0.9981	0.9967	0.9975	0.0006
310	0.9990	0.9991	0.9987	0.9989	0.0002
326	0.9996	0.9996	0.9994	0.9995	0.0001
336	0.9998	0.9998	0.9998	0.9998	0.0000

6.9.4. Simulated MSA Fuel Plate Predictions

The number of fuel plates needed to reach criticality was predicted for each neutron detector at each fuel plate configuration. The prediction was calculated using equation 3.26. The predictions are shown in table 6.12.

Table 6-12 Fuel plate predictions simulated MSA

plates	Detector A	Detector B	Detector C	Average	Standard Deviation
150	200	211	207	206	6
208	261	264	243	256	11
240	272	294	289	285	12
280	307	319	311	312	6
310	332	335	330	332	3
326	337	341	337	338	2
336	342	343	342	343	0

6.10. Kcode Fuel Plate Predictions

The number of fuel plates needed to reach criticality was predicted using a kcode calculation in MCNP. The ISU SCA was modeled using 334, 336, 338, 340, 342, 344, 346, 348, 350, 352, and 354 fuel plates. Various configurations were used for each number of fuel plates. All fuel plate configurations can be found in Appendix H. The results of the kcode calculations are shown in table 6.13. Figure 6.8 shows the multiplication factor k plotted against the number of fuel plates. The plot also shows error bars with a 95% confidence interval. It can be seen that the multiplication factor k varies slightly due to fuel plate configuration. The lowest number of fuel plates that reached criticality was configuration 3 with 344 fuel plates.

Table 6-13 MCNP Kcode results

plates	Configuration 1		Configuration 2		Configuration 3		Configuration 4	
	k	standard deviation	k	standard deviation	k	standard deviation	k	standard deviation
334	0.99263	0.00039	0.99460	0.00041	0.99611	0.00042	0.99714	0.00040
336	0.99253	0.00041	0.99575	0.00046	0.99910	0.00043	0.99628	0.00040
338	0.99436	0.00042	0.99623	0.00041	0.99790	0.00045	0.99802	0.00047
340	0.99551	0.00041	0.99745	0.00042	0.99872	0.00044	0.99851	0.00043
342	0.99636	0.00046	0.99810	0.00043	0.99915	0.00044	0.99896	0.00040
344	0.99732	0.00044	0.99939	0.00044	1.00088	0.00040	1.00074	0.00043
346	1.00027	0.00043	0.99960	0.00044	1.00036	0.00045	1.00093	0.00043
348	0.99863	0.00044	1.00120	0.00046	1.00120	0.00046	1.00350	0.00045
350	0.99912	0.00050	1.00063	0.00042	1.00356	0.00043	1.00312	0.00045
352	0.99878	0.00043	1.00244	0.00046	1.00339	0.00042	1.00390	0.00042
354	1.00030	0.00042	1.00289	0.00041	1.00331	0.00044	1.00469	0.00045

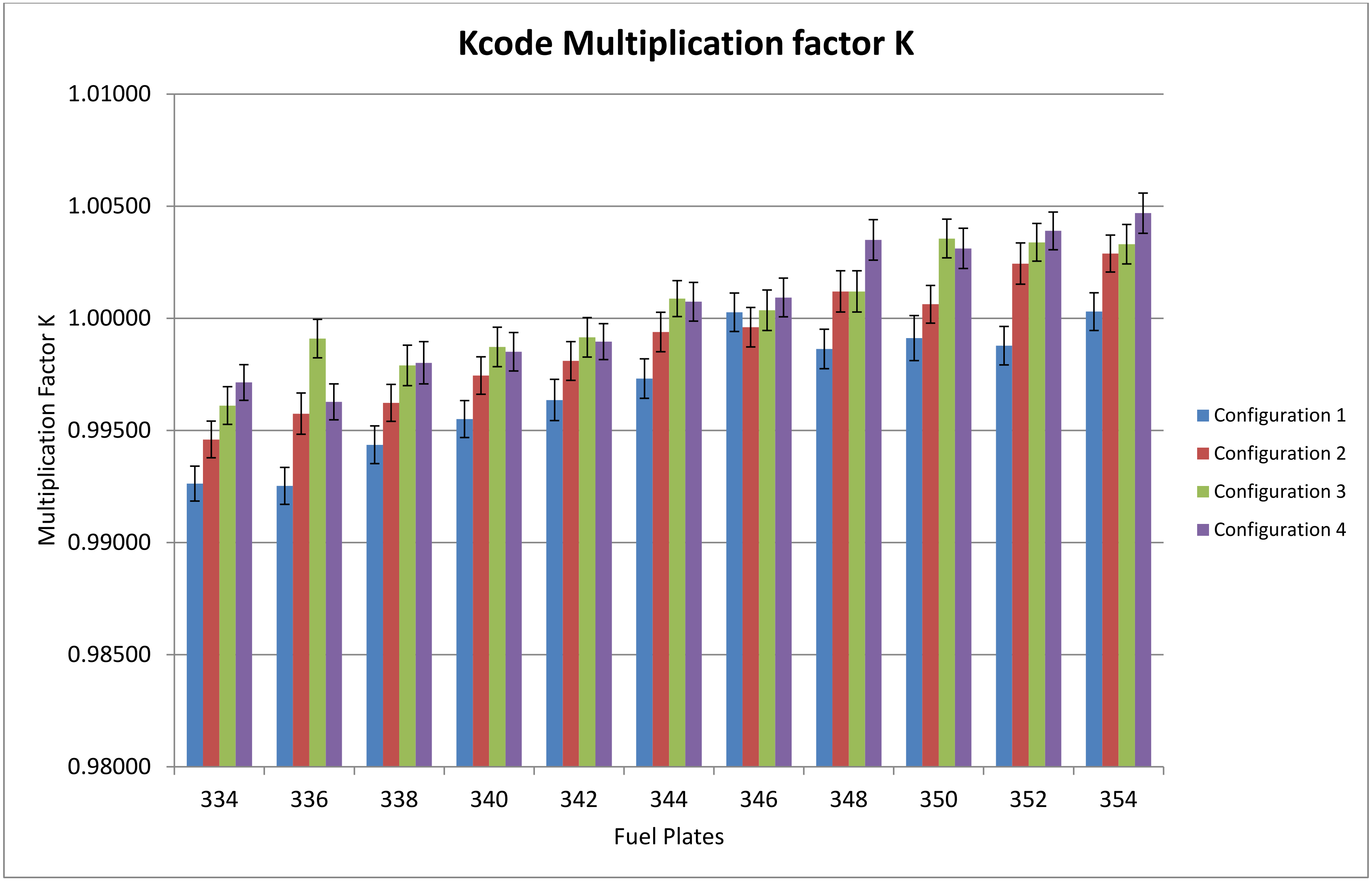

Figure 6-8 Multiplication factor k for various fuel plate configurations

6.11. Dose Rate[9]

The neutron source used in the experiment is a californium-252 source from Frontier Technology Corp. Californium has a half-life of 2.646 years. There are two decay modes, spontaneous fission (3.092%) and alpha decay (96.908%). The average neutrons emitted per fission is 3.768. The source is originally dated on the 24th of June 1993. The source was calibrated on the 19th of July 2005. The experiment was done on March 19th, 2014. The source activity on the day the experiment was performed can be estimated using the previous activities by the following relation

$$\alpha(t) = \alpha_0 e^{-\frac{\ln(2)}{T_{1/2}}t} \tag{6.1}$$

were α_0 is the initial activity, t is the time that has passed, $\alpha(t)$ is the activity after time t, and $T_{1/2}$ is the half-life of californium-252. The neutrons emitted per second was estimated using the following equation

$$neutron\ source\ rate = \alpha \frac{decays}{s} * 3.9\% \ \frac{SF}{decay} * 3.768 \frac{n}{SF} \tag{6.2}$$

where α is the activity of californium, 3.9% is the percent of decays that result in spontaneous fission, and 3.768 is the number of neutrons emitted per fission. The estimation of neutrons emitted per second is shown in table 6.14. The neutron source rate is only a rough estimate.

Table 6-14 Neutron source data

Date	24-Jun-93	19-Jul-05	19-Mar-14
mass (µg)	209.4	N/A	N/A
activity (mCi)	112.3	4.773	0.999
neutrons emitted (n/s)	4.81E+08	2.06E+07	4.30E+06

The neutron source rate was used with the MCNP code to determine the dose rate. The maximum dose rate will occur with the most fuel plates in the assembly. A tmesh tally was used with a dose modification to calculate the dose rate around the ISU SCA with all 150 fuel plates in the assembly. Figures 6.9 through 6.11 show the MCNP calculated dose rate. The dose rate at 30 cm and 100 cm from the edge of the SCA water tank is 0.4 mrm and 0.14 mrm.

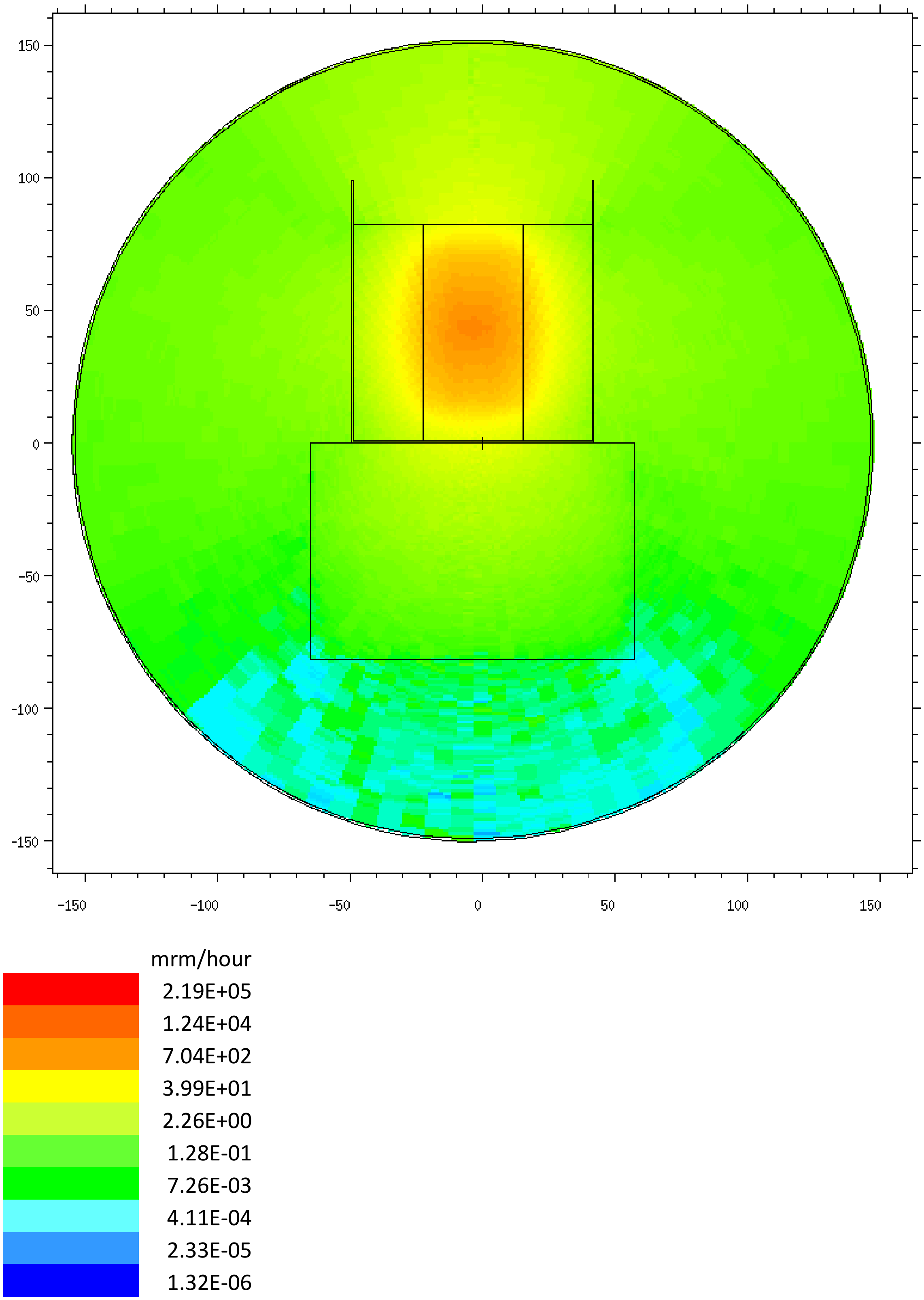

Figure 6-9 MCNP calculated dose rate elevation view

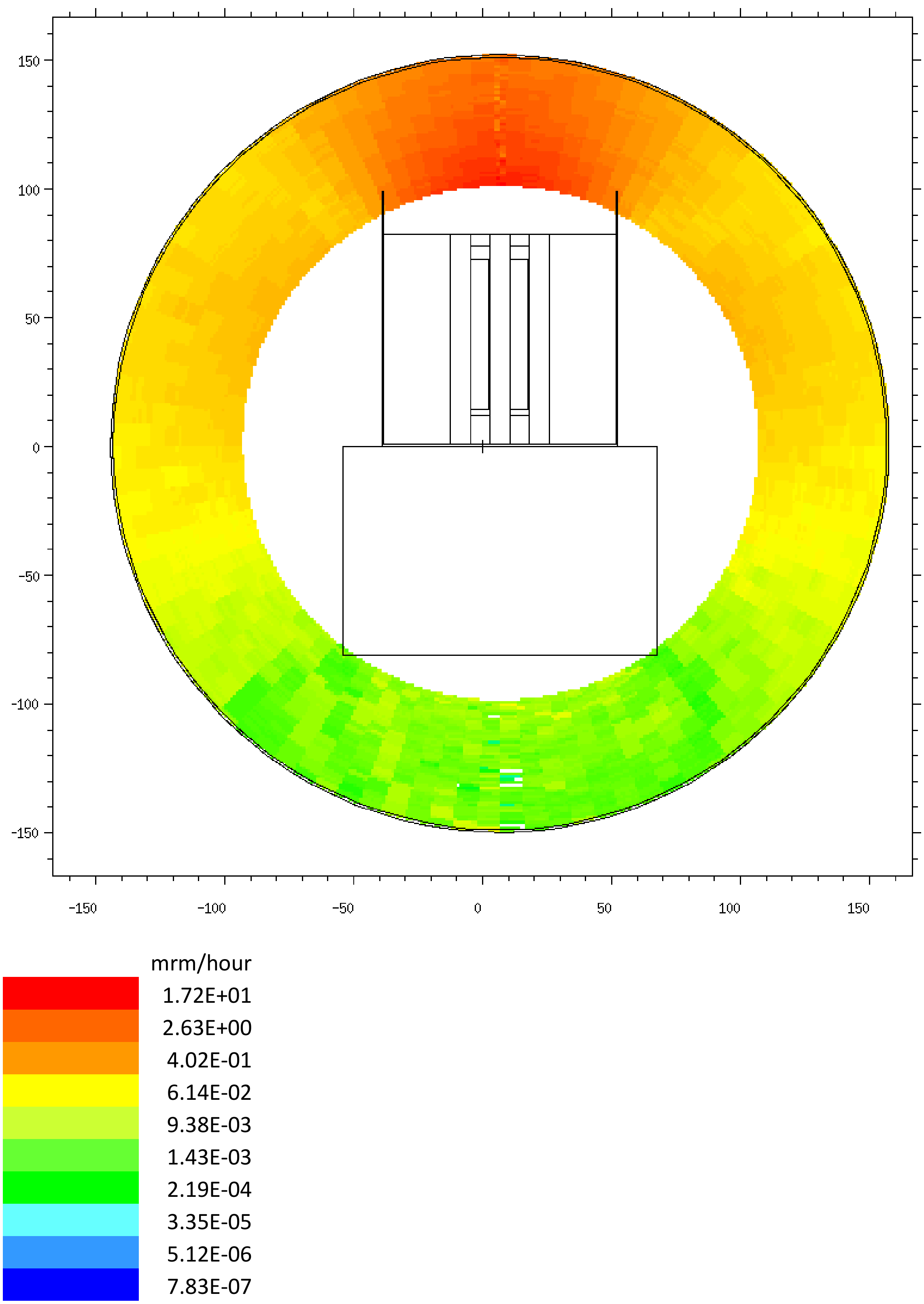

Figure 6-10 MCNP calculated dose rate elevation view

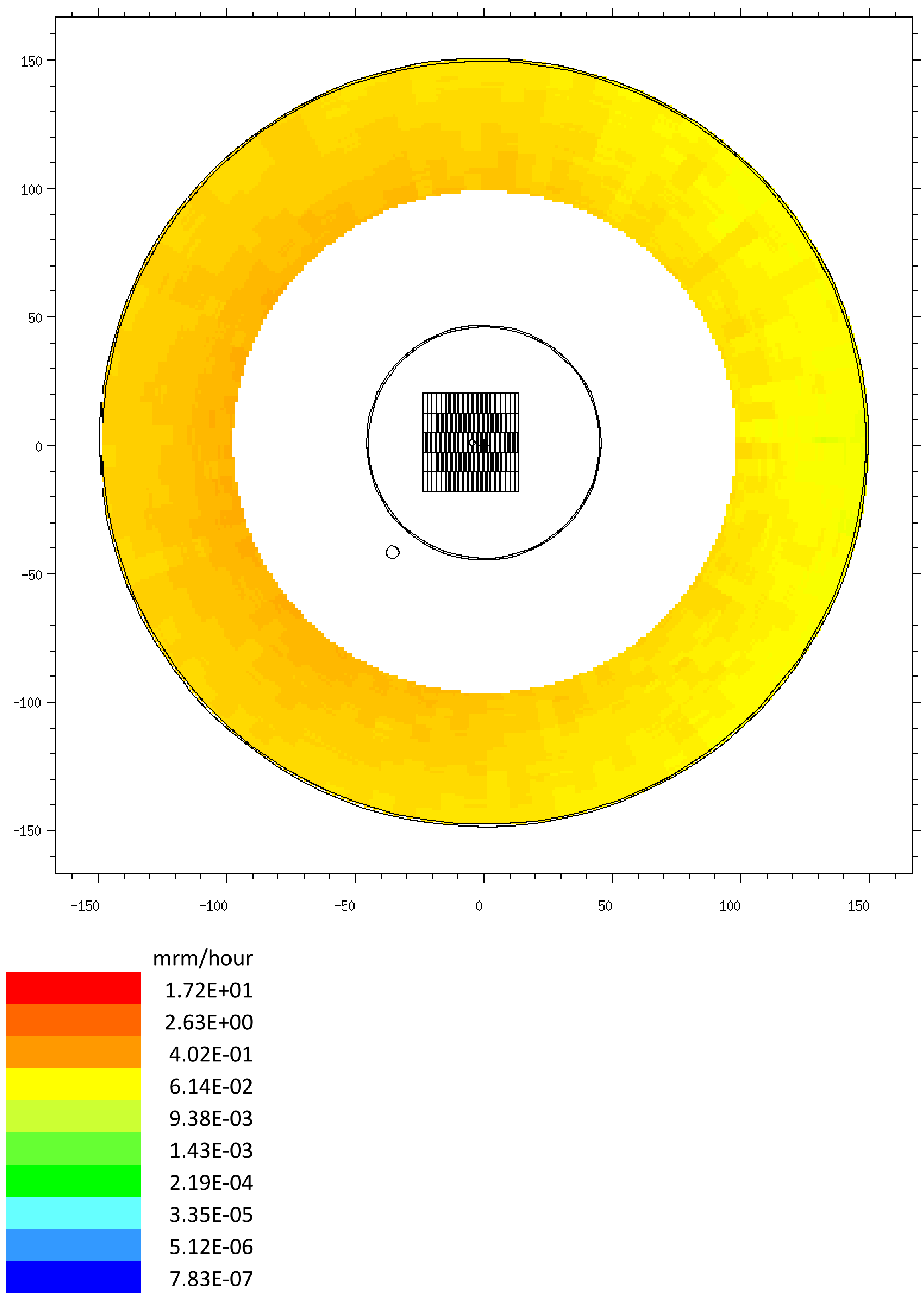

Figure 6-11 MCNP calculated dose rate plan view

6.12. Fmesh Tally

MCNP was used to plot the neutron flux in the ISU SCA with 150 plates in the assembly. An fmesh tally was used in the MCNP code to plot the neutron flux. Figures 6.12 and 6.13 show the elevation and plan view of the neutron flux.

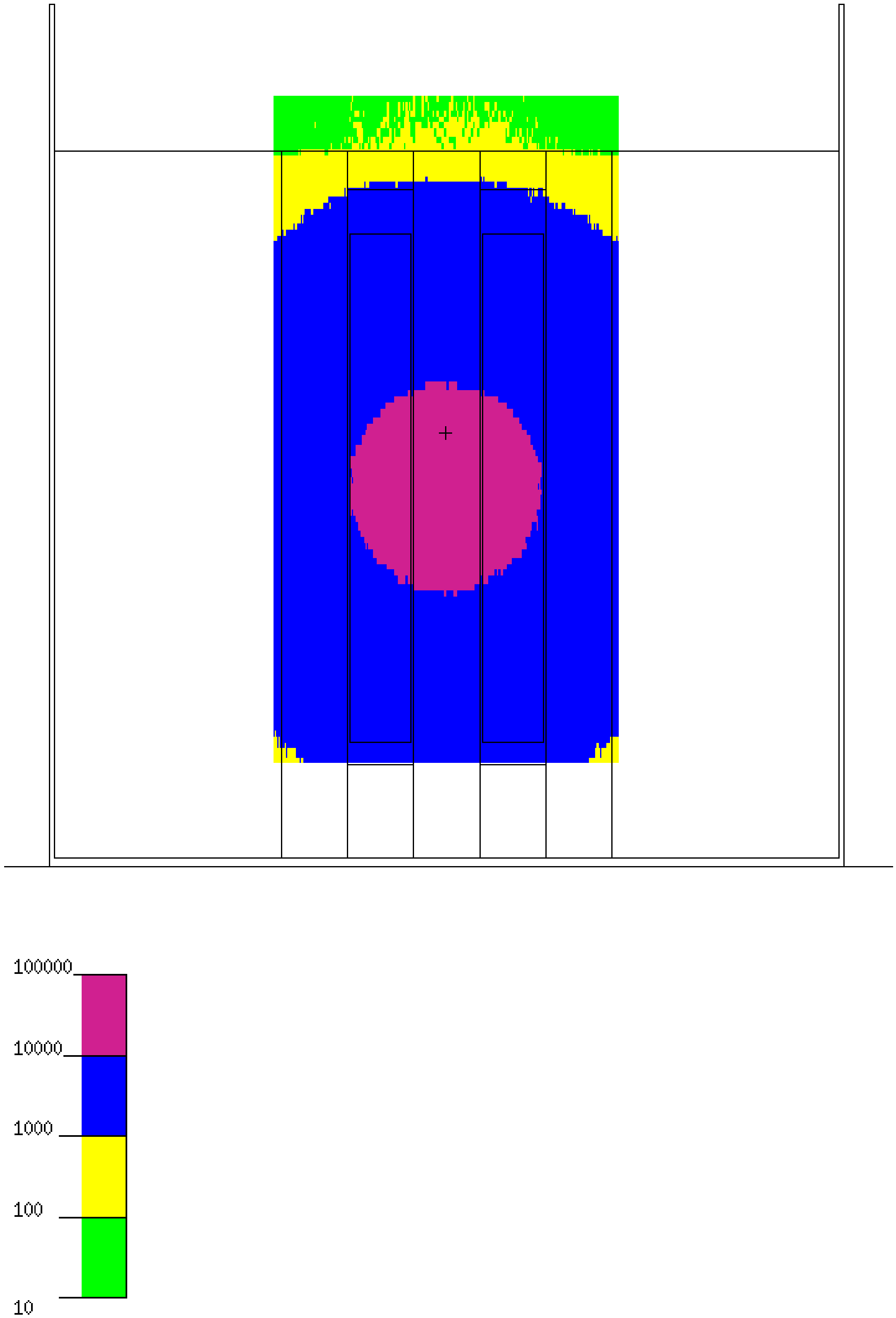

Figure 6-12 MCNP calculated neutron flux elevation view

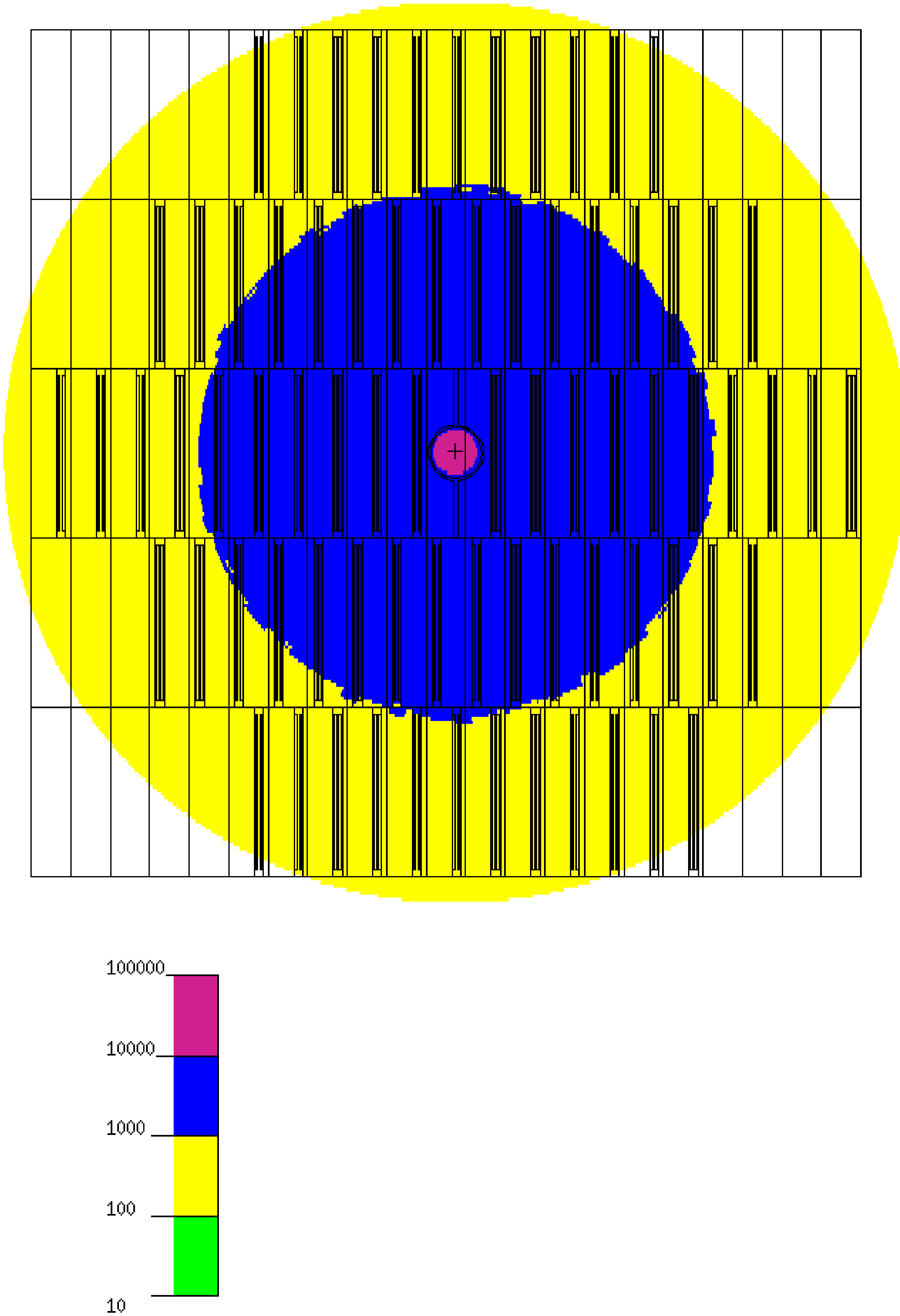

Figure 6-13 MCNP calculated neutron flux top view

7. Further Research

Upgrades on the ISU SCA have been planned to increase the multiplication factor. Research can be done using the MCNP model in supporting the upgrades. MCNP can be used to predict how the upgrades will change the multiplication factor. Once the upgrades have been completed the MSM can be redone on the ISU SCA.

The MCNP model of the ISU SCA can also be used to determine the optimal detector location when performing the MSA experiment. Results for the MSA will vary by detector location. Simulating the MSA in MCNP with several different detector locations will show how the location affects the results and results can be compared to determine the best location.

The MSM in an approach-to-critical experiment has been shown to improve fuel plate predictions. Further research can be done by supporting the application of the MSM in other SCAs or in a reactor.

The results from the detector located in the graphite did not align when applying the MSM correction factor. Further research can be done to determine what the error was. The first step would be to switch the detector with detector C and compare the results. This would verify that the detector is working properly. ISU has ordered new neutron detectors. The experiment can also be repeated with the new detectors.

Deterministic methods can be used to determine the MSM correction factor. The results can be compared to this experiment. Results can be used to further validate this experiment or help determine why detector B was not aligned with the other detectors.

The ISU SCA multiplication factor k can vary slightly by changing the fuel plate configuration. Several MCNP models can be created with various fuel plate configurations to determine the optimal fuel plate configuration with 150 fuel plates.

8. Conclusion

An approach to critical experiment was performed on the ISU SCA using both the MSA and MSM to determine the number of fuel plates needed to reach criticality. The MSA experiment was performed first. MSM correction factors were calculated to improve fuel plate predictions determined by the MSA method. Correction factors were calculated from an MCNP model of the ISU SCA. A simulated MSA, five factor formula calculation, and MCNP kcode was also used to predict the number of fuel plates needed to reach criticality. Fuel plate predictions from each experiment are shown in table 8.1.

Table 8-1 Final fuel plate predictions

Method	Detector A	Detector B	Detector C
MSA (150 fuel plates)	211	212	203
MSM (150 fuel plates)	269	254	242
Simulated (336 fuel plates)	342	343	342
Kcode (calculated)		344	
5 Factor (calculated)		342	

It can be seen that the MSM fuel plate predictions are closer to the MCNP kcode calculated prediction when compared to the MSA fuel plate predictions. This is due to the assumptions made in the MSA which assumes equal efficiency of neutron detectors and equal effective source strength between different fuel plate configurations. The MSM correction factor corrects the inaccurate assumptions, which increases the accuracy and aligns the data points on the one-over-M plot. The MSM is still an under prediction of fuel plates needed to reach criticality. This is due to the linear trend assumption. The one-over-M plot does not actually follow a linear trend, but the linear trend is still useful in predicting the number of fuel plates needed to reach criticality.

The MSM correction factors were expected to align the data points on the one-over-M plot. Results from detector B were not aligned for some fuel plate configurations and further research is needed to determine the cause. The experiment can be repeated with new detectors or deterministic methods can be used to validate results.

The MSA, MSM, simulated MSA, Kcode, and five factor formula were all used to estimate the multiplication factor k when all 150 fuel plates are in the ISU SCA. Multiplication factor k predictions are shown in table 8.2. The Kcode calculation is the most accurate prediction of the multiplication factor k. The MSA and simulated MSA predictions are higher than the Kcode calculation due to the inaccurate assumptions of the MSA. The MSM predictions are accurate, but vary due to uncertainty from experimental results. The five factor formula is far below the actual multiplication factor k. This is due to the assumption made when calculating the five factor formula, that the ISU SCA is a homogenous finite cylinder.

Table 8-2 k estimate with 150 fuel plates

Method	Detector A	Detector B	Detector C
MSA	0.9680	0.9782	0.9681
MSM	0.8117	0.7392	0.8613
Simulated	0.9751	0.9878	0.9663
Kcode	0.8539		
5 Factor	0.6784		

9. References

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Appendix A: **Five Factor Formula Calculations**

The radius of the assembly was chosen and number of fuel plates was estimated using the relation in equation 3.29. Equation 3.28 was used to estimate the geometric buckling. Equation 3.27 was used to estimate the non-leakage probability. The multiplication factor k was estimated using equation 3.26. The thermal utilization factor is calculated in appendix B. The thermal fission factor, resonance escape probability, fast fission factor, Fermi age are from the Idaho State University Nuclear Reactor Lab Manual. The diffusion area is given in Introduction to Nuclear Engineering.[10]

Table A- 1 Multiplication factor K

Factors for 5 Factor Formula		
f	0.66	Fraction
ϵ	1.01	Fraction
Ƥ	0.97	Fraction
η	2.18	n/abs
τ	42	cm2
L^2	2.74	cm2

Table A- 2 Data points for five factor formula one-over-M plot

plates	radius (cm)	geometric buckling	non-leakage probability	k	1-k
10	5.0	0.2356	3.07E-05	0.0000	1.0000
20	7.1	0.1192	5.04E-03	0.0078	0.9922
30	8.6	0.0805	2.79E-02	0.0431	0.9569
40	10.0	0.0611	6.59E-02	0.1018	0.8982
50	11.1	0.0494	1.10E-01	0.1705	0.8295
60	12.2	0.0417	1.56E-01	0.2407	0.7593
70	13.2	0.0361	2.00E-01	0.3080	0.6920
80	14.1	0.0320	2.40E-01	0.3705	0.6295
90	15.0	0.0287	2.77E-01	0.4279	0.5721
100	15.8	0.0262	3.11E-01	0.4801	0.5199
110	16.5	0.0240	3.42E-01	0.5276	0.4724
120	17.3	0.0223	3.70E-01	0.5707	0.4293
130	18.0	0.0208	3.95E-01	0.6099	0.3901
140	18.7	0.0195	4.18E-01	0.6457	0.3543
150	19.3	0.0184	4.40E-01	0.6784	0.3216
160	19.9	0.0174	4.59E-01	0.7084	0.2916
170	20.6	0.0166	4.77E-01	0.7360	0.2640
180	21.2	0.0158	4.93E-01	0.7614	0.2386
190	21.7	0.0151	5.08E-01	0.7849	0.2151
200	22.3	0.0145	5.23E-01	0.8066	0.1934
210	22.8	0.0140	5.36E-01	0.8268	0.1732
220	23.4	0.0135	5.48E-01	0.8456	0.1544
230	23.9	0.0130	5.59E-01	0.8631	0.1369
240	24.4	0.0126	5.70E-01	0.8795	0.1205
250	24.9	0.0122	5.80E-01	0.8949	0.1051
260	25.4	0.0118	5.89E-01	0.9093	0.0907
270	25.9	0.0115	5.98E-01	0.9229	0.0771
280	26.4	0.0112	6.06E-01	0.9356	0.0644
290	26.8	0.0109	6.14E-01	0.9477	0.0523
300	27.3	0.0106	6.21E-01	0.9591	0.0409
310	27.8	0.0104	6.28E-01	0.9699	0.0301
320	28.2	0.0102	6.35E-01	0.9801	0.0199
330	28.6	0.0099	6.41E-01	0.9898	0.0102
340	29.1	0.0097	6.47E-01	0.9990	0.0010
341	29.1	0.0097	6.48E-01	0.9999	0.0001
342	29.2	0.0097	6.48E-01	1.0008	-0.0008

Appendix B: **Fuel Plate Atom Density Calculations**

Volume of fuel bearing portion of 1 plate

$$2.75\ in * .04\ in * 23\ in * \frac{16.38706\ cm^3}{in^3} = 41.45927\ cm^3$$

Uranium 235 atom density

$$\frac{10.07\ \frac{g\ U235}{plate} * \frac{1\ plate}{41.459\ cm^3} * .602214\ \frac{atoms\ cm^2}{mol\ b}}{235.0439\ \frac{g\ U235}{mol}} = 6.22194\ E - 4\ \frac{atoms\ U235}{b\ cm}$$

Grams of Uranium 238 per plate

$$\frac{7614.79\ grams\ Uranium - 1510.27\ grams\ U235}{150\ plates} = 40.6968\ \frac{g\ U238}{plate}$$

Uranium 238 atom density

$$\frac{40.6968\ \frac{g\ U235}{plate} * \frac{1\ plate}{41.459\ cm^3} * .602214\ \frac{atoms\ cm^2}{mol\ b}}{238.0508\ \frac{g\ U235}{mol}} = 2.48325\ E - 3\ \frac{atoms\ U235}{b\ cm}$$

Oxygen atom density

$$\left(6.22194\ E - 4\ \frac{a\ U235}{b\ cm} + 2.48325\ E - 3\ \frac{a\ U235}{b\ cm}\right) * 2 = 6.21093\ E - 3\ \frac{atoms\ O}{b\ cm}$$

Mass of Oxygen per plate

$$\frac{6.21093\ E - 3\ \frac{atoms\ O}{b\ cm} * 15.9994\ \frac{g\ O}{mol} * \frac{41.459\ cm^3}{1\ plate}}{.602214\ \frac{atoms\ cm^2}{mol\ b}} = 6.84118\ \frac{g\ O}{plate}$$

Mass of UO_2

$$6.84118\ g\ O + 40.6968\ g\ U238 + 10.07 g\ U235 = 57.6065\ g\ UO_2$$

Volume of UO_2

$$\frac{57.6065\ g\ UO_2}{10.97\,\frac{g\ UO_2}{cm^3}} = 5.25127\ cm^3$$

Volume of aluminum in fuel bearing plate

$$41.459\ cm^3 1\ plate - 5.25127\ cm^3 UO_2\ 1\ plate = 36.208\ cm^3 Aluminum\ 1\ plate$$

Aluminum atom density

$$\frac{2.7\ \frac{g\ Al}{cm^3} * \frac{36.208\ cm^3 Al}{41.459\ cm^3\ 1\ plate} * .602214\,\frac{atom\ cm^2}{mol\ b}}{26.98154\ \frac{g\ Al}{mol}} = 5.26297E-2\ \frac{atoms\ Al}{b\ cm}$$

Cell atom density for 5 factor calculations - One cell has 2 fuel plates and water

Table B- 1 Volume percentages

cell	volume	%
fuel	82.919	0.094703
cladding	121.592	0.138873
water	671.050	0.766423
total	875.561	

Table B- 2 Atom densities

	isotope	atom density	atom cell density	σ_a	Σ_a
fuel	Al	5.29E-2	5.01E-3	0.23	1.15E-3
	U-235	5.96E-4	5.64E-5	687	3.88E-2
	U-238	2.38E-3	2.25E-4	2.73	6.15E-4
	O	5.95E-3	5.63E-4	0.00027	1.52E-7
water	O	3.34E-2	2.56E-2	0.00027	6.91E-6
	H	6.68E-2	5.12E-2	0.332	1.70E-2
cladding	Al	6.03E-2	8.37E-3	0.23	1.93E-3
				$\Sigma_{a\ fuel}$	3.94E-2
				$\Sigma_{a\ moderator}$	2.01E-2

Fuel utilization

$$f = \frac{\Sigma\text{a fuel}}{\Sigma\text{a fuel} + \Sigma\text{a moderator}}$$

$$\frac{3.94\text{E} - 2}{3.94\text{E} - 2 + 2.01\text{E} - 2} = .66$$

Appendix C: **MCNP 150 Fuel Plate Code**

```
Model of the Idaho State University Subcritical assembly
c
c ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~CELL CARDS~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
c *********************************CORE TANK**********************************
10 100 -2.7 -1 2 -5 4:-3 4 -1     imp:n=1                                    $ core tank
c
c
c *********************************THERMAL COLUMN*******************************
20 400 -1.6 (710:-711:712:-713) -4 6 -7 8 -9 10 imp:n=1                      $ graphite model as 1 block
c
c
c *********************************CENTER TUBE********************************
55 100 -2.7 140 -141 142 -143:141 142 -144 -145 146 -147 148 imp:n=1          $ center tube with aluminum plate
c
c
c *********************************Fuel Plates********************************
c rows 1 3 5
60 200 .0619460952 158 -159 -160 161 (-151 152:-153 154) u=1 imp:n=1           $fuel bearing
61 100 -2.7    #60 -147 148 -150 155 142 -144              u=1 imp:n=1          $cladding
62 300 -.992   (147:-148:150:-155:-142:144)
               -190 191 -192 193 194 -195                  u=1 imp:n=1          $water
c rows 2 4
70 200 .0619460952 158 -159 -160 161 (-163 164:-165 166) u=2 imp:n=1           $fuel bearing
71 100 -2.7    #70 -147 148 -162 167 142 -144              u=2 imp:n=1          $cladding
72 300 -.992   (147:-148:162:-167:-142:144)
               -190 191 -192 193 194 -195                  u=2 imp:n=1          $water
c ******************************WATER ADDED IN TANK***************************
c lattice of fuel and water
800 300 -.992 -170 171 -172 173 lat=1 u=3 fill= -10:10 -2:2 0:0
      3 3 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 3 3 3 3
     3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3
      1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 1 1 1 1 1
     3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3
      3 3 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 3 3 3 3 3 u=3 imp:n=1         $ lattice of fuel plates and water
801 0 3 -800 180 -181 182 -183 #55 fill=3            imp:n=1              $ box around lattice
802 300 -.992 -2 3 -800 #801 #55                     imp:n=1              $ water around lattice
c
c *********************************DETECTORS**********************************
700 700 -.00033598128     -700 701 -702              imp:n=1                    $ detector A
710 800 -.00005096692     714 -715 -716              imp:n=1                    $ detector B
720 800 -.00005096692     -720 721 -722              imp:n=1                    $ detector C
c
c
c ***********************************VOID AREA*******************************
900 0 #700 #710 #720 -900 #10 #20 #55 #801 #802 imp:n=1                        $ allow neutrons to travel
901 0 900 -901                              imp:n=1                         $ small cell to count neutron population
902 0 901                                   imp:n=0                         $ kills neutrons
c
c

c ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~SURFACE CARDS~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
c
c       orgin is at the bottom of water level radius 0
c ***********************************CORE TANK********************************
1       CZ 45.72                                                     $ outside diameter of core tank
2       CZ 45.085                                                    $ inside diameter of core tank
3       PZ 0                                                         $ bottom water level of tank set at orgin
4       PZ -.9525                                                    $ bottom of tank
5       PZ 98.1075                                                   $ top of tank
c
c
c *********************************THERMAL COLUMN*****************************
6       PZ -82.2325                                                  $bottom of thermal column
7       PX 60.96                                                     $ 12:00 side of thermal column
8       PX -60.96                                                    $ 6:00 side of thermal column
9       PY 60.96                                                     $ 3:00 side of thermal column
```

```
10       PY -60.96                                       $ 9:00 side of thermal column
c
c
c ***********************************CENTER TUBE*********************************
140      C/Z -4.6038 0 1.15                              $ tube id
141      C/Z -4.6038 0 1.25                              $ tube od
142      PZ 10.795                                       $ bottom plane for cyl and plate and fuel plates
143      PZ 87.395                                       $ upper plane for cylinder
144      PZ 76.835                                       $ upper plane for plate same for fuel plates
145      PX -4.5022                                      $ wall plane for plate
146      PX -4.7054                                      $ wall plane for plate
147      PY 3.80009                                      $ wall plane for plate same for fuel plates rounded 3.81
148      PY -3.80009                                     $ wall plane for plate same for fuel plates rounded 3.81
c
c
c ***********************************FUEL PLATES*********************************
c fuel for row 1, 3 and 5
150      PX -4.3815                                      $cladding
151      PX -4.4323                                      $fuel
152      PX -4.5339                                      $fuel
153      PX -4.6355                                      $fuel
154      PX -4.7371                                      $fuel
155      PX -4.7879                                      $cladding
158      PZ 13.335                                       $ lower plane for fuel bearing portion
159      PZ 71.755                                       $ upper plane for fuel bearing portion
160      PY 3.4925                                       $ wall plane for fuel bearing portion
161      PY -3.4925                                      $ wall plane for fuel bearing portion
c fuel for row 2 and 4
162      PX -5.2705                                      $cladding
163      PX -5.3213                                      $fuel
164      PX -5.4229                                      $fuel
165      PX -5.5245                                      $fuel
166      PX -5.6261                                      $fuel
167      PX -5.6769                                      $cladding
c lattice
170      PX -4.15925
171      PX -5.93725
172      PY 3.81
173      PY -3.81
c box around lattice
180      PX -23.7172
181      PX   13.62074
182      PY -19.049999999999999
183      PY 19.049999999999999
c
c
c lattice lines made a bit bigger to let tally be taken instead of extending to infinity fot cells 62 and 72
190      PX -4.15
191      PX -5.94
192      PY 3.82
193      PY -3.82
194      PZ -.1
195      PZ 81.3
c **************************************TALLY SURFACES***************************
300      C/Z      -4.6038 0 20.32
301      PZ       10.795       $142
302      PZ       76.835       $144
c
c
c **************************************DETECTORS********************************
c detector A
700      C/Z -20 46 .637                                 $ cylinder
701      PZ 60.48                                        $ bottom plane of detector
702      PZ 75.72                                        $ top plane of detector
c detector B
710      PZ -19.3675                                     $ top plane
711      PZ -29.5275                                     $ bottom plane
712      PX 0                                            $ positve x plane
713      PX -10.16                                       $ negative x plane
714      PY -50.1274
```

```
715     PY   41.3126                                    $ detector does not extend to end of block on one side
716     C/y -5.08 -24.4475 2.54
c y planes are 9 positive and 10 negative
c detector C
720     C/Z -35.56 -42.6928 2.54                        $ cylinder
721     PZ 10.795                                       $ bottom plane of detector
722     PZ 102.235                                      $ top plane of detector
c
c
c ***********************************WATER HEIGHT*********************************
800     PZ 81.28                                        $ water height
c
c
c *************************************VOID AREA**********************************
900     SO 150                                          $ sphere at orgin
901     SO 151                                          $ sphere at orgin
c
c

c ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~DATA CARDS~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
c
c ************************************SOURCE OR K CODE***************************
c COMMENT OUT SOURCE OR K CODE
sdef pos -4.6038 0 42.545 erg d1
sp1 -3 1.18 1.03419
c stop f14 .05
nps 1e5
c kcode 10000 .9 20 220
c ksrc -4.6038 0 42.545
c
c
c ***************************************TALLY COUNTS ****************************
c detector counts
F14:n 700
fm14 3.3598128261760-4 700 -2
E14   1.2e-9 5e-3 14
F24:n 710
fm24 5.0966919229191-5 800 -2
E24   5e-10 2.4e-6 14
F34:n 720
fm34 5.0966919229191-5 800 -2
E34   5e-10 2.4e-6 14
c ******************************************Fmesh tally****************************
c F11:n 300
c FS11 -301 302
c C11 0 1
c F21:n 301
c FS21 -300
c C21 0 1
c F31:n 302
c FS31 -300
c C31 0 1
c
c
c ***************************************dose ****************************
c tmesh
c SMESH1:n dose=4.3+6
c CORA1 0 199i 100
c CORB1 1 49i 180
c CORC1 1 99i 360
c endmd
c :n dose
c
c ******************************************Fmesh tally****************************
c fmesh4:n geom=cyl origin= -4.6038 0 10.795
c           imesh= 10 20.32     iints= 5 10
c           jmesh= 76.835       jints= 60
c           kmesh= 1            kints= 20
c           AXS= 0 0 1 VEC= 0 1 0
c            EMESH 5e-10 14
```

```
c
c ***************************************number generator***********************
RAND stride=300000
c
c ***************************************MATERIALS*****************************
m100    13027.80c   1                                      $for pure aluminum structures (fuel cladding/core
tank/ fuel assembly)
m200    92235.80c   .0006222194
        92238.80c   .0024832469
        13027.80c   .0526296961                            $uranium/aluminum fuel bearing plate
        08016.80c   .0062109327
m300    01001.80c   2
        08016.80c   1                                      $water
mt300   lwtr.20t
m400    06012.50c   1                                      $graphite
mt400   grph.60t
m500    92235.80c   .0006222194
        92238.80c   .0024832469
m600    13027.80c   .0526296961
        08016.80c   .0062109327
m700    02003.80c   1    $ He-3
m800    05010.80c   1
        09019.80c   3
```

Appendix D: **Detector Atom Density Calculations**

Detector A

Density of He3 at standard temperature and pressure = $.000165 \frac{g}{cm^3}$

Density

$$\frac{1030\ kpa}{101\ kpa} * .000165 \frac{g}{cm^3} = .001682673 \frac{g}{cm^3}$$

Atom density

$$\frac{.001682673 \frac{g}{cm^3} * .602214 \frac{atomcm^2}{mol\ b}}{3.016029319 \frac{gHe3}{mol}} = 3.359813E-4 \frac{a\ He3}{b\ cm}$$

Detectors B & C update to BF3

Density of BF_3 at standard temperature and pressure = $.00276 \frac{g}{cm^3}$

Density

$$\frac{210\ kpa}{101\ kpa} * .00276 \frac{g}{cm^3} = 0.005738614 \frac{g}{cm^3}$$

Atom density

$$\frac{0.005738614 \frac{g}{cm^3} * .602214 \frac{atoms\ cm^2}{mol\ b}}{67.8062096 \frac{g\ BF3}{mol}} = 5.0966919E-5 \frac{a\ \mathrm{BF3}}{b\ cm}$$

Appendix E: **Fuel Plate Configuration Pictures**

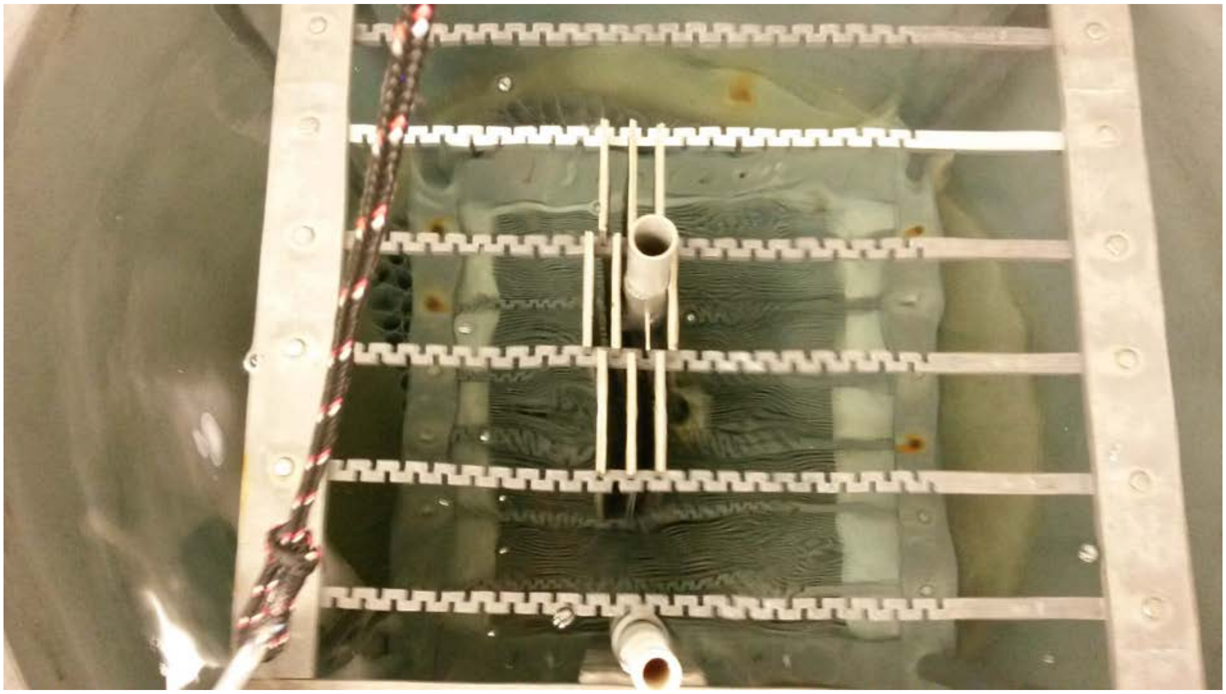


Figure E- 1 18 fuel plates

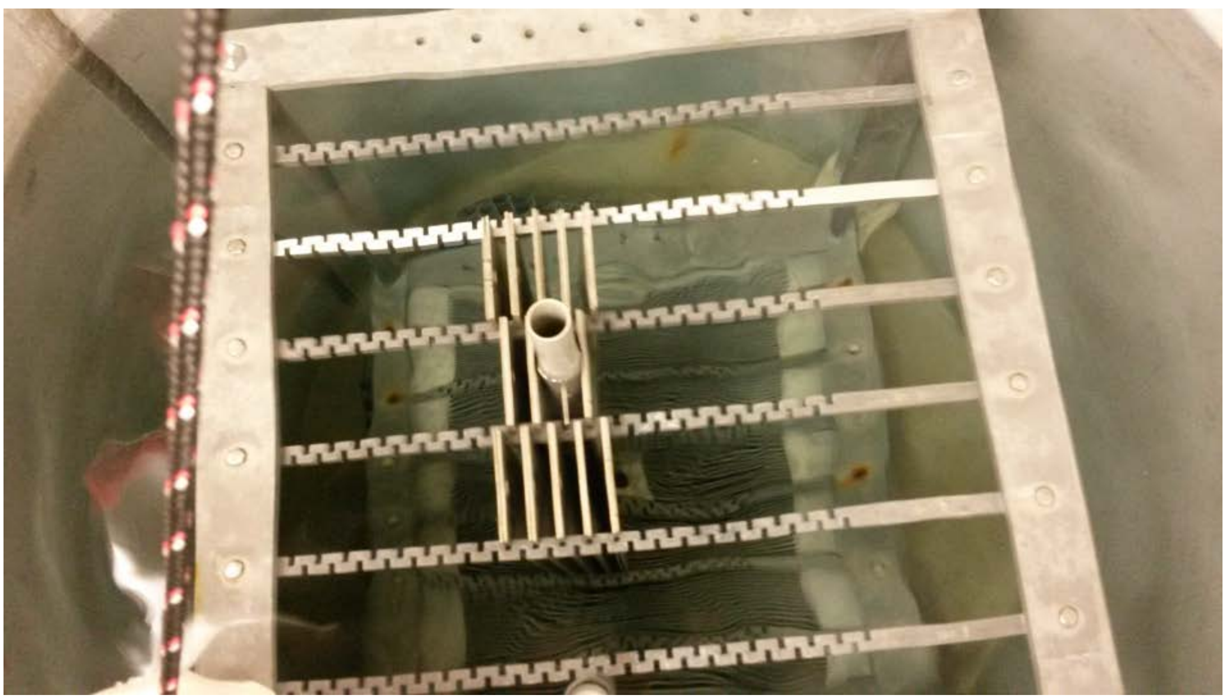


Figure E- 2 26 fuel plates

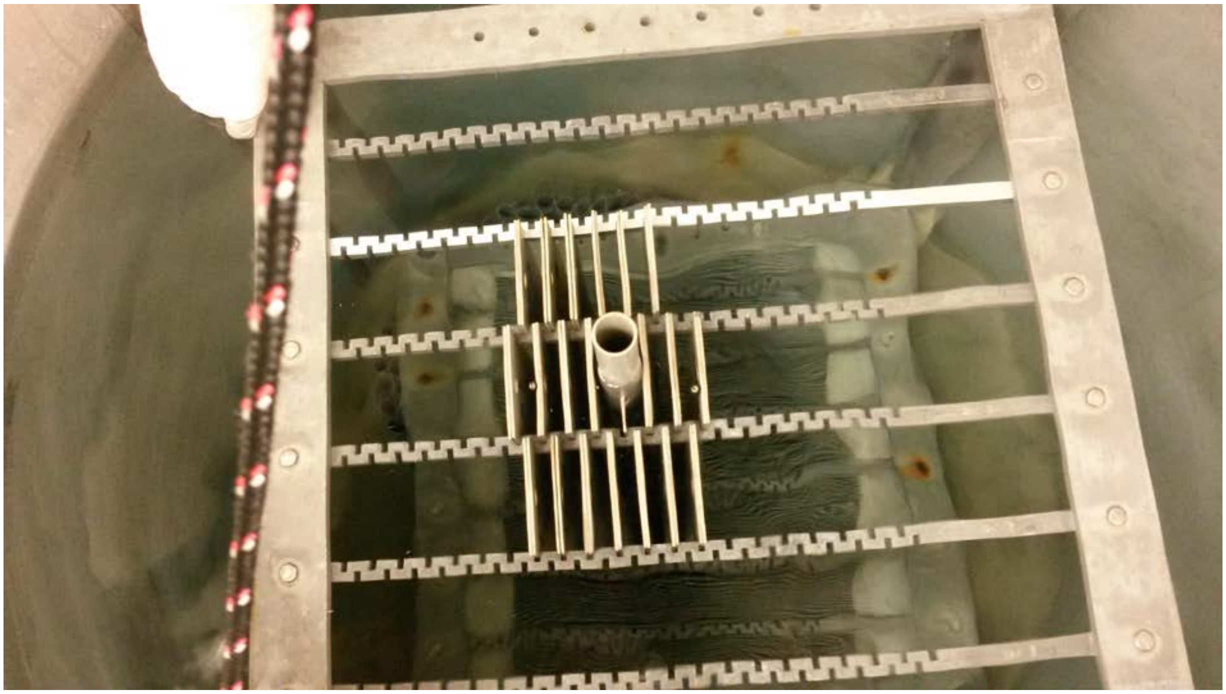


Figure E- 3 40 fuel plates

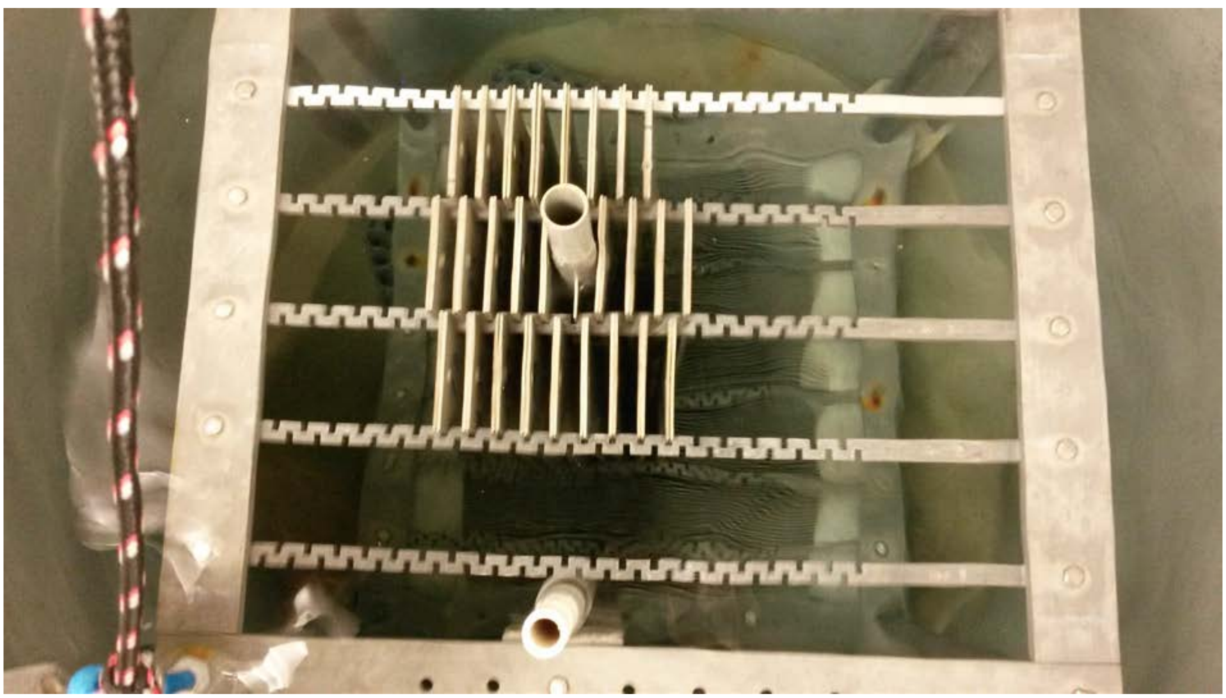


Figure E- 4 52 fuel plates

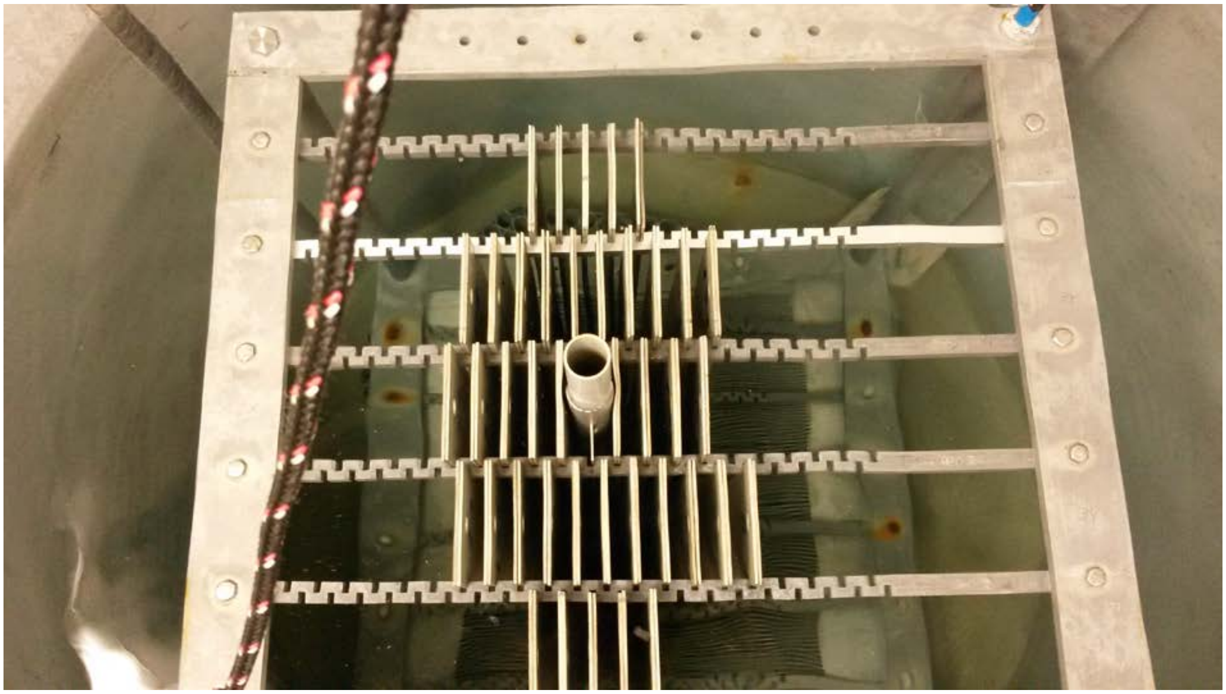


Figure E- 5 80 fuel plates


Figure E- 6 104 fuel plates

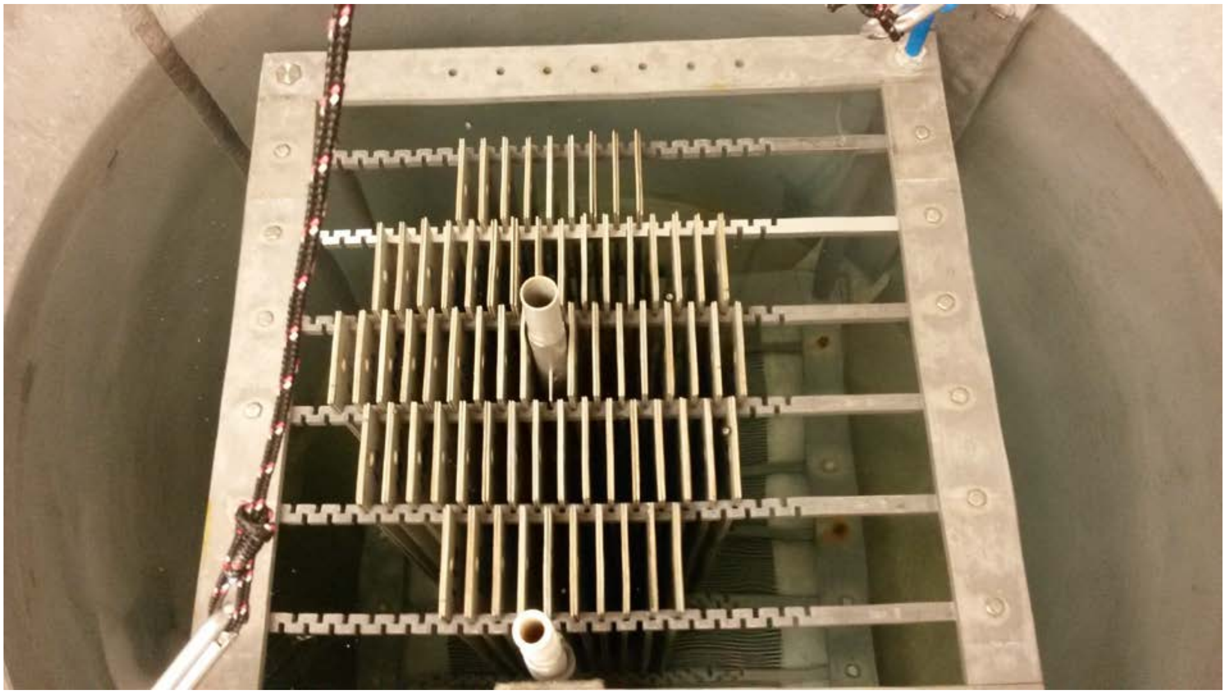


Figure E- 7 136 fuel plates

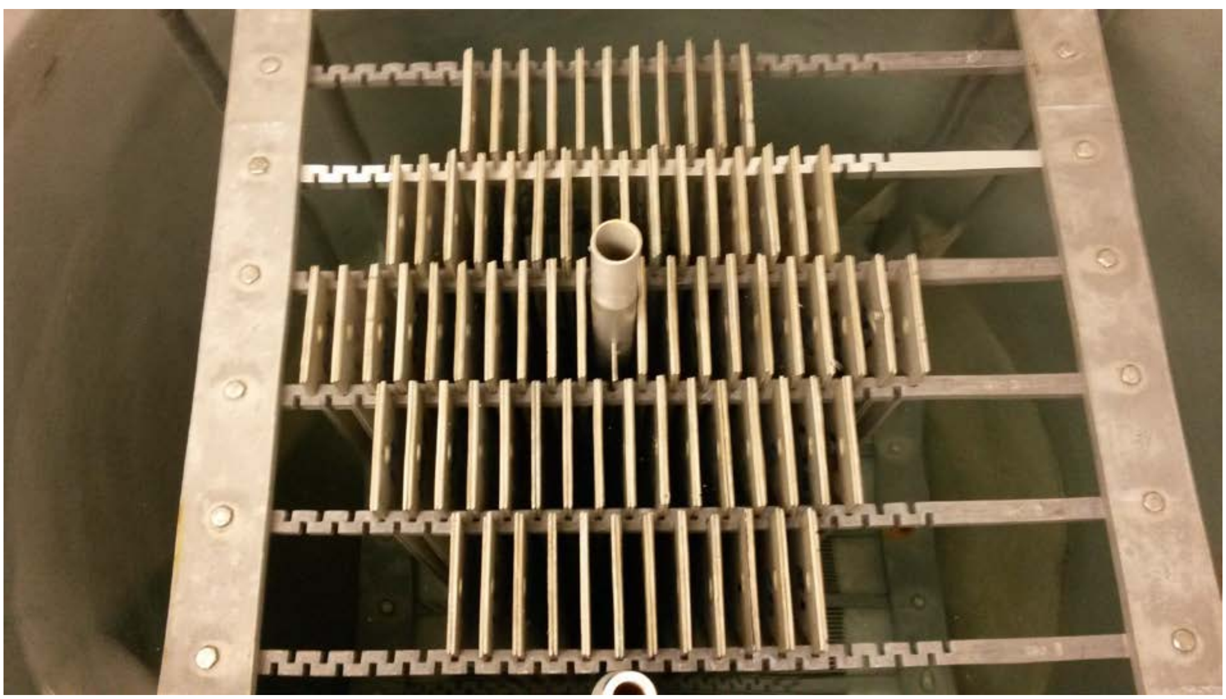


Figure E- 8 150 fuel plates

Appendix F: **One-Over-M Plots**

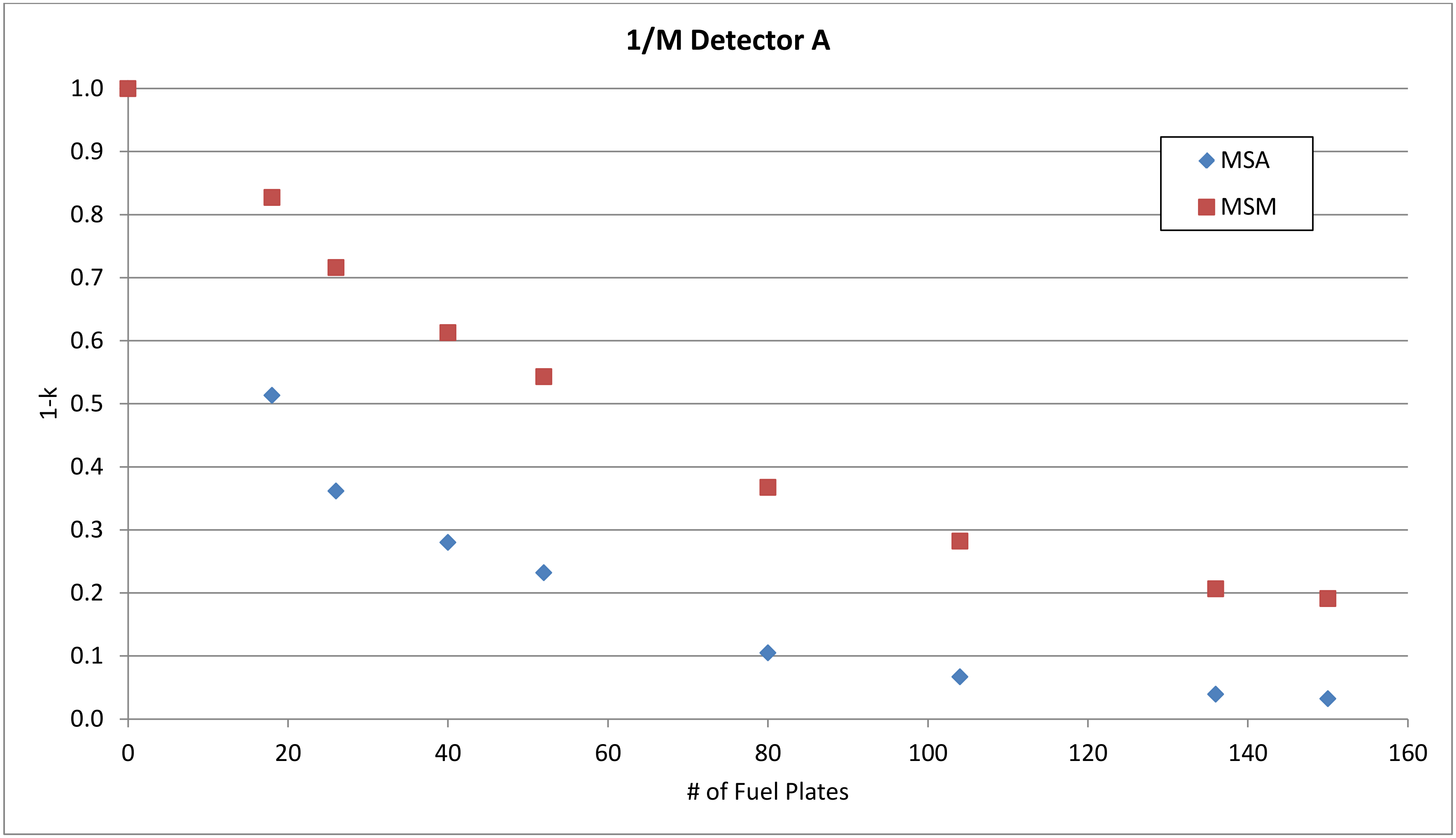


Figure F- 1 Detector A MSM and MSA one-over-M plot

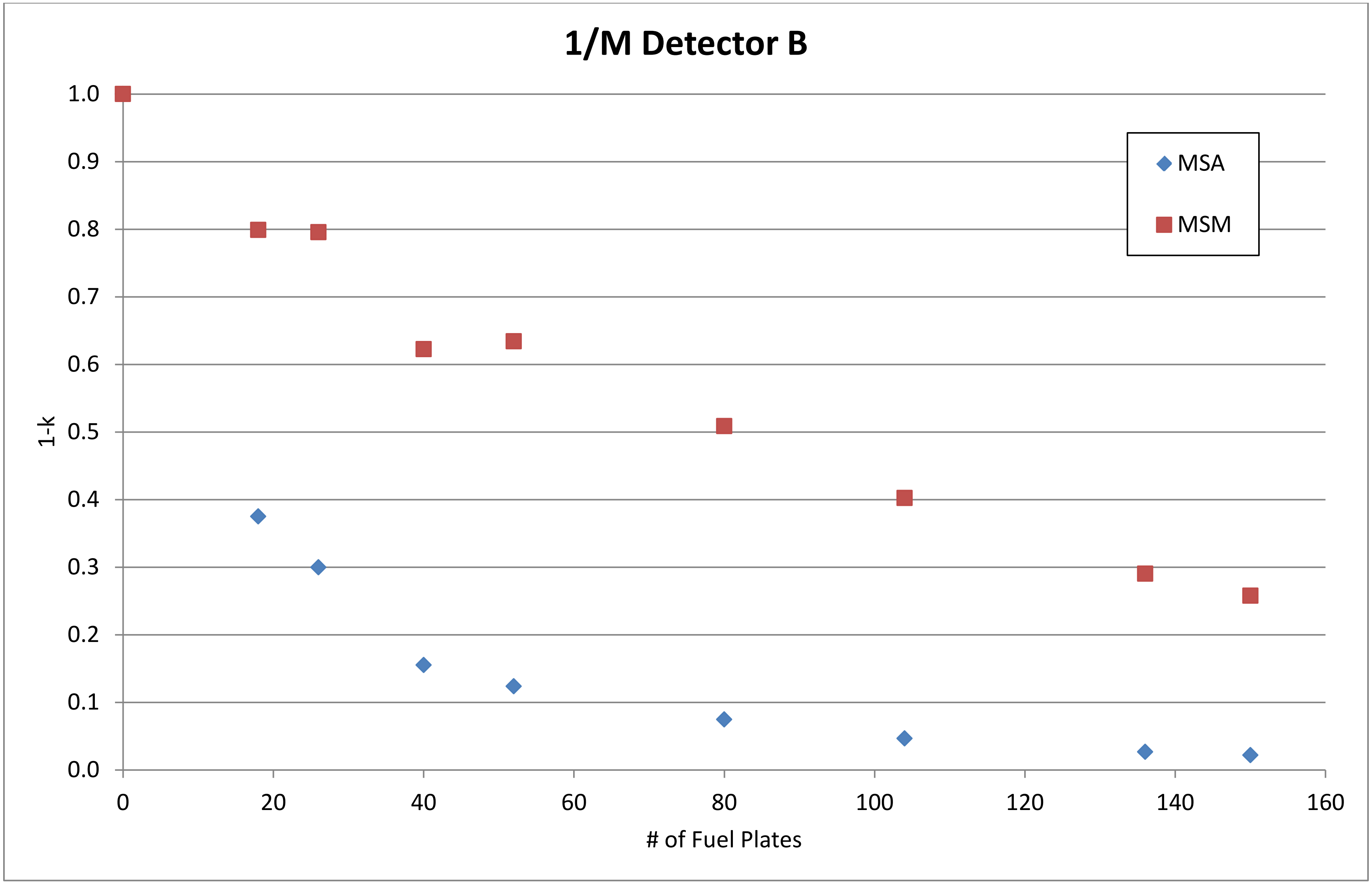


Figure F- 2 Detector B MSM and MSA one-over-M plot

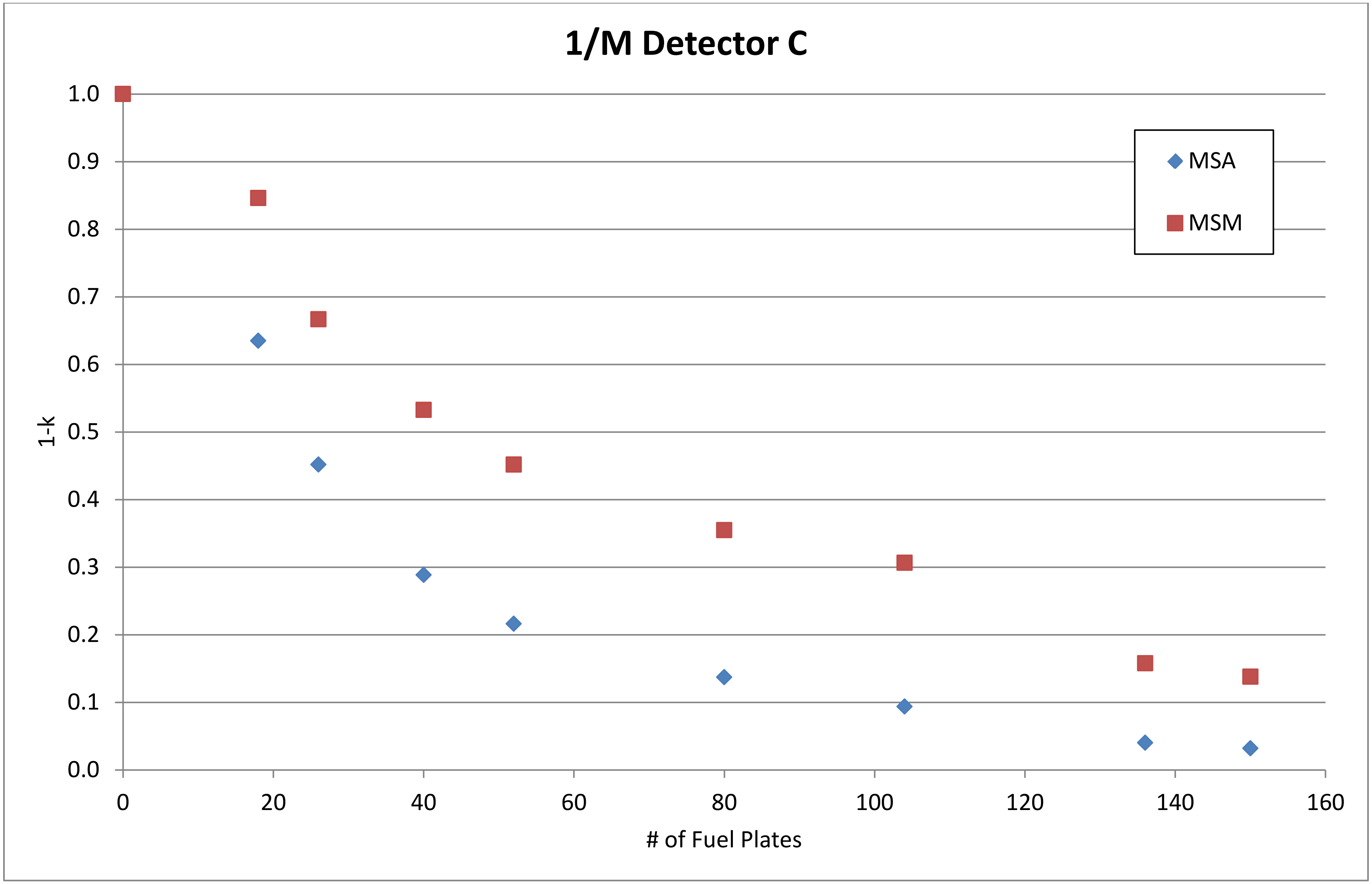


Figure F- 3 Detector C MSM and MSA one-over-M plot

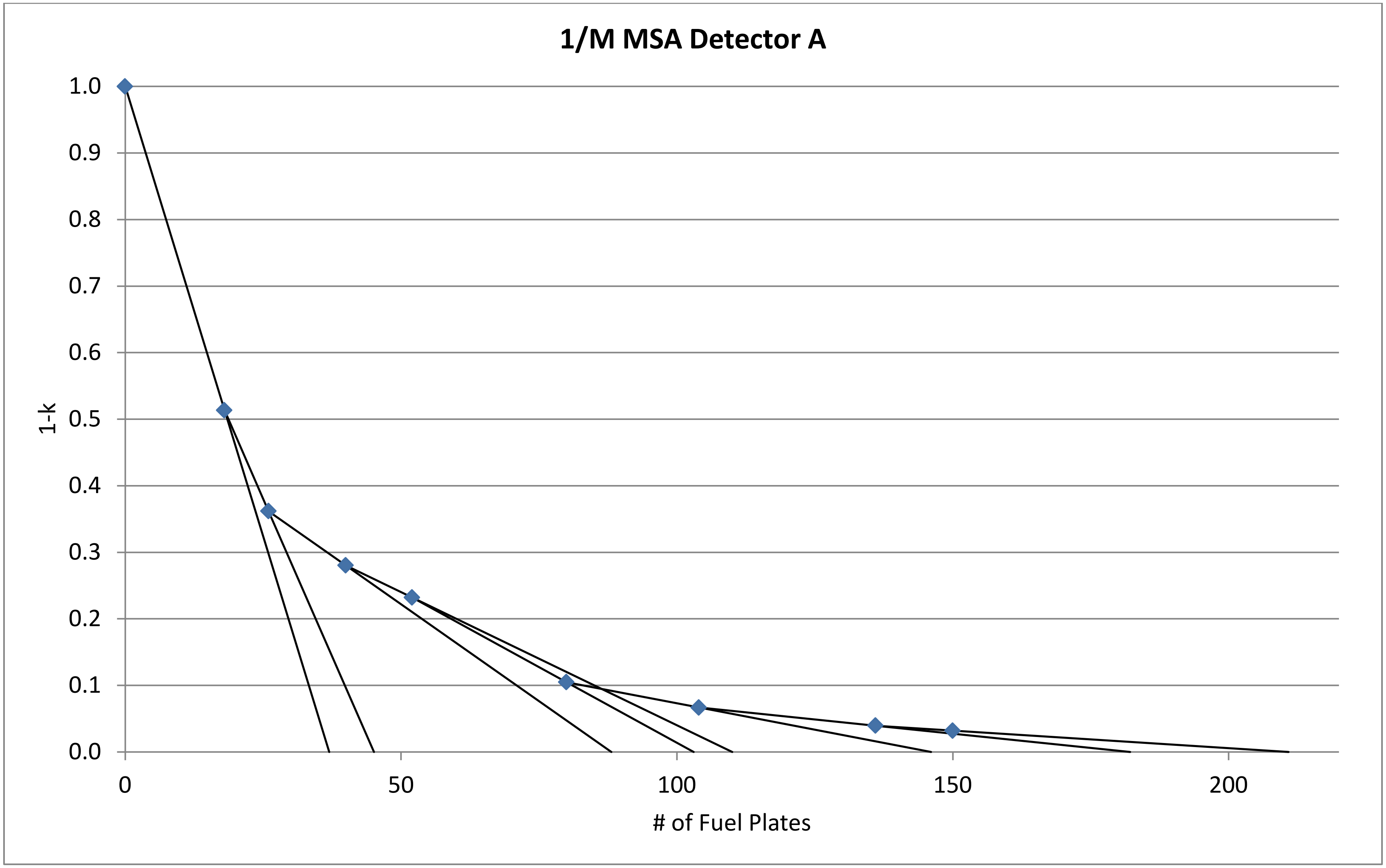


Figure F- 4 Detector A One-Over-M Plot with linear trend extrapolation lines

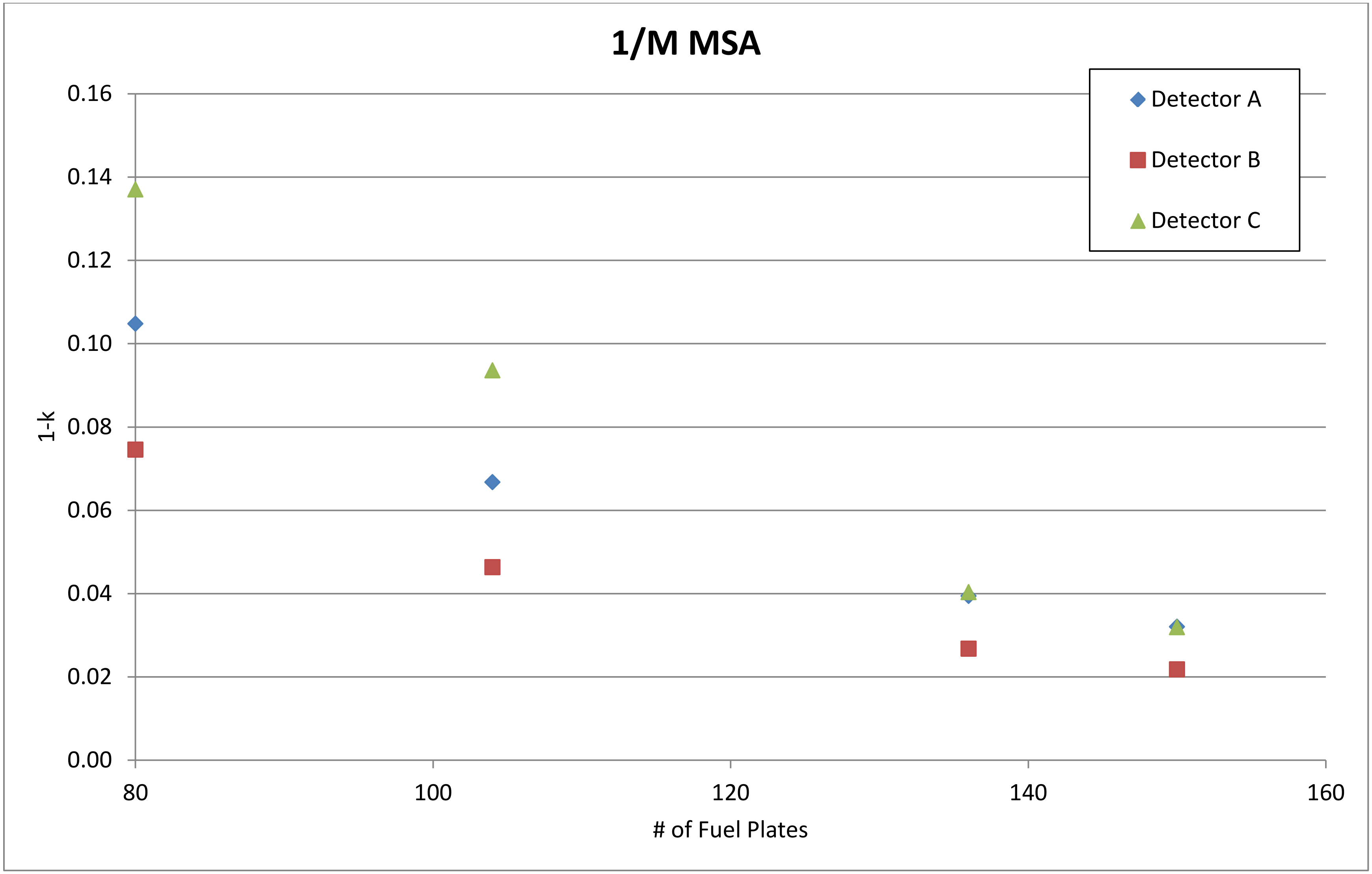


Figure F- 5 One-over-M plot of all three detectors (80 Plates to 150 plates)

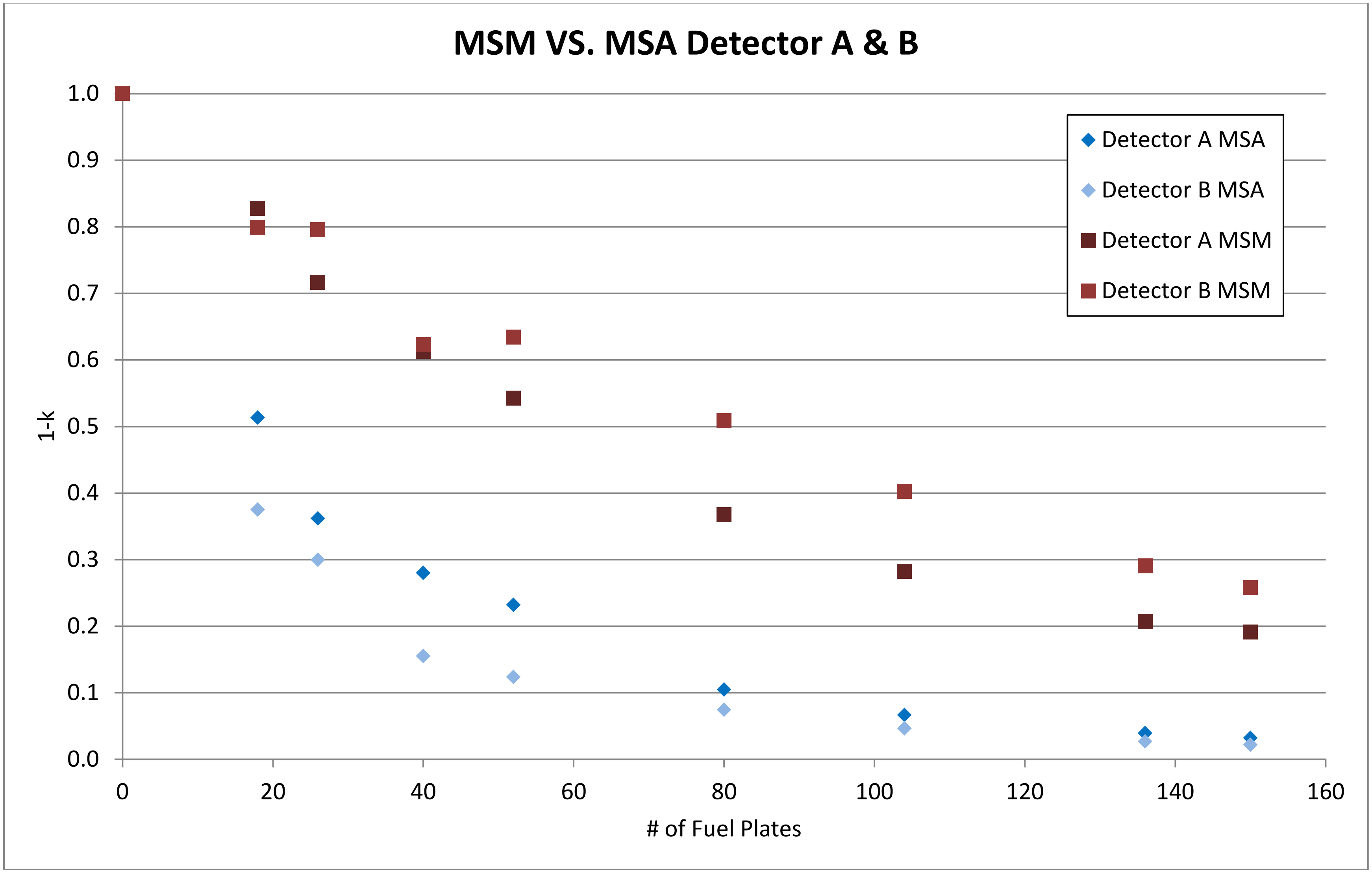


Figure F- 6 MSA and MSM One-over-M plot for detectors A & B

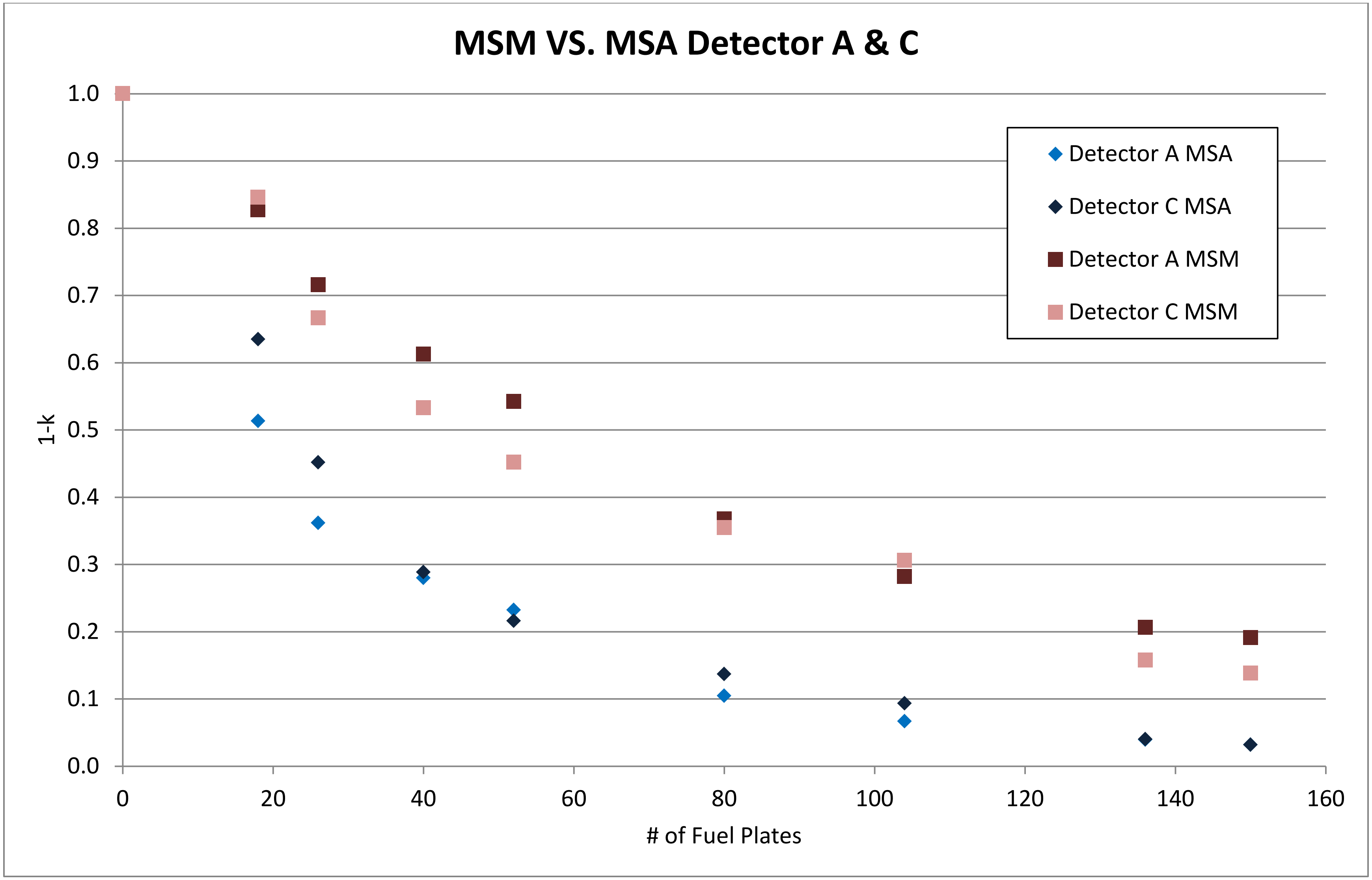


Figure F- 7 MSA and MSM One-over-M plot for detectors A & C

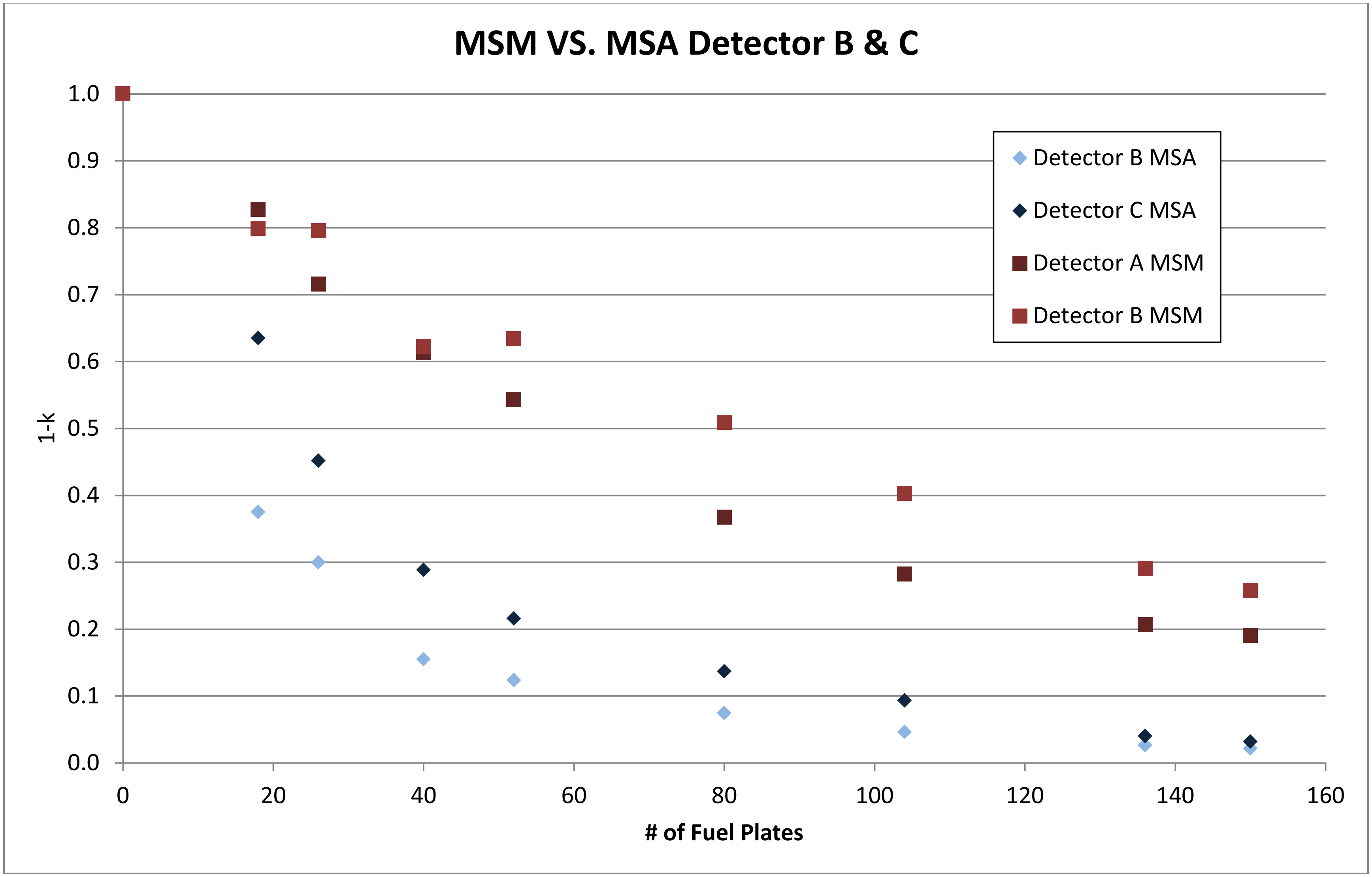


Figure F- 8 MSA and MSM One-over-M plot for detectors B & C

Appendix G: **MSM Correction Factor Uncertainty**

MSM correction factor uncertainty

$$\sigma_U = \sqrt{(\frac{\partial U}{\partial k_2})^2\sigma_{k_2}^2 + (\frac{\partial U}{\partial T_{2calc}})^2\sigma_{T_{2calc}}^2 + (\frac{\partial U}{\partial T_1})^2\sigma_{T_1}^2 + (\frac{\partial U}{\partial T_{1calc}})^2\sigma_{T_{1calc}}^2 + (\frac{\partial U}{\partial T_2})^2\sigma_{T_2}^2}$$

Variables defined

$$U = \frac{\rho_{2calc}T_{2calc}}{T_{1calc}}\frac{T_1}{T_2}$$

$$\frac{\partial U}{\partial k_2} = -\frac{T_{2calc} * T_1}{T_{1calc} * T_2}$$

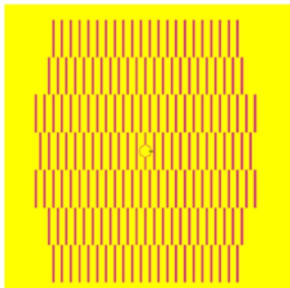
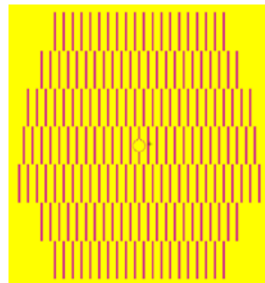
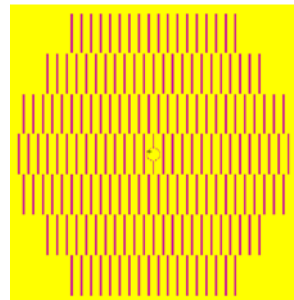
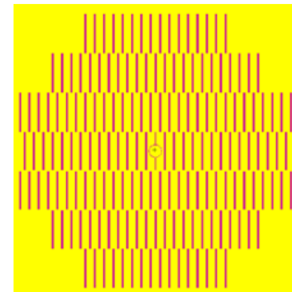
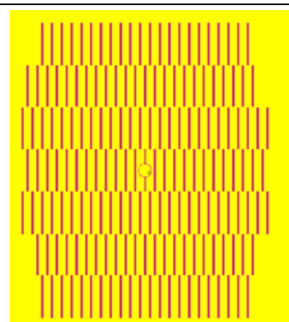
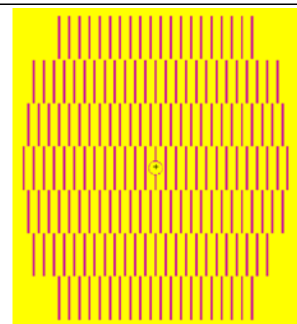
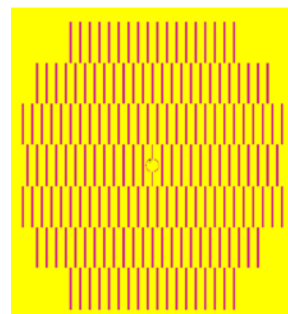
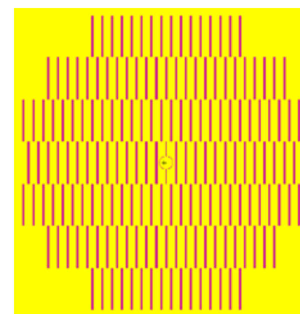
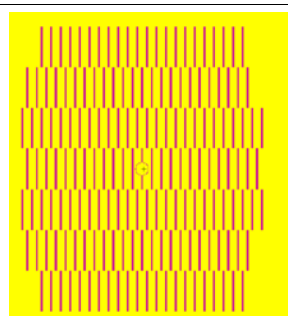
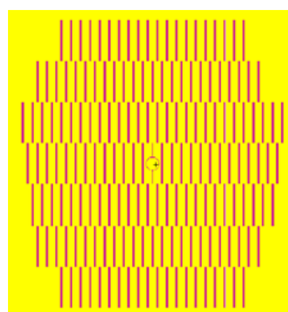
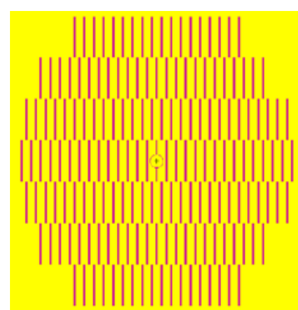
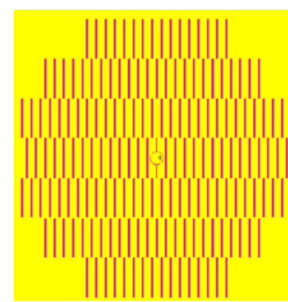
$$\frac{\partial U}{\partial T_{2calc}} = \frac{(1 - k_2) * T_1}{T_{1calc} * T_2}$$

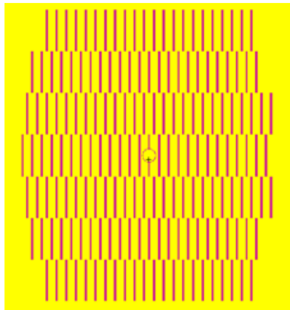
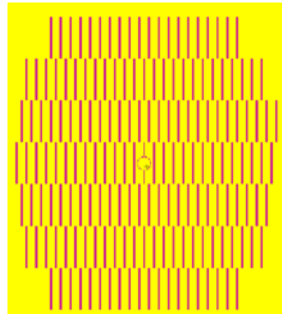
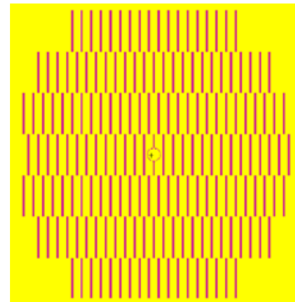
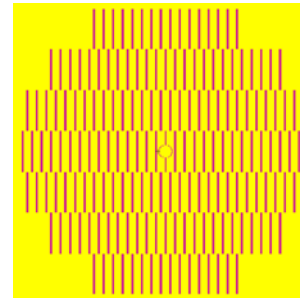
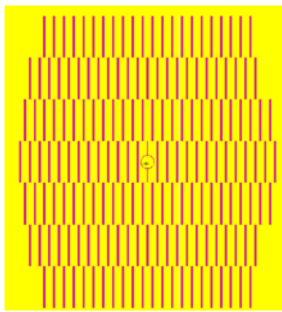
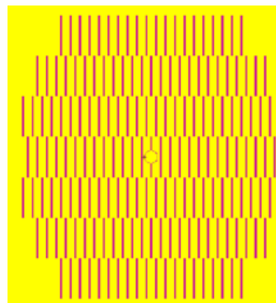
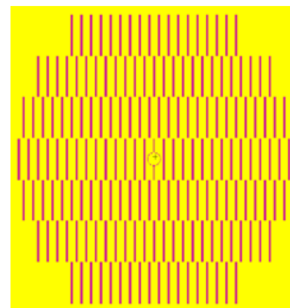
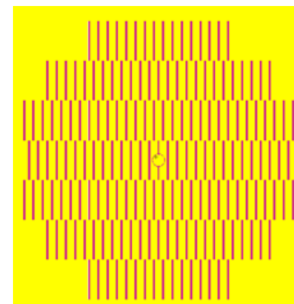
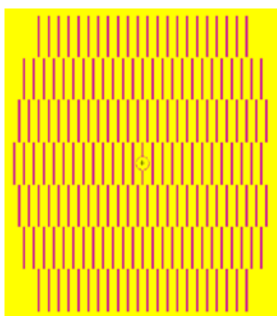
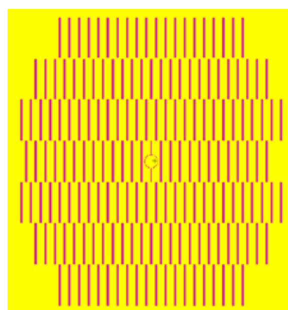
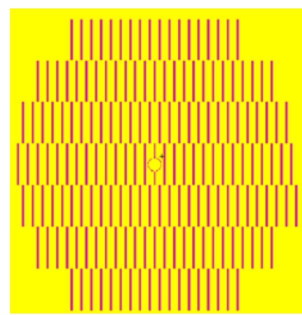
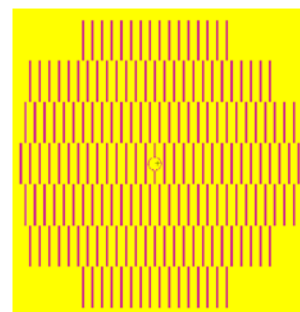
$$\frac{\partial U}{\partial T_1} = \frac{(1 - k) * T_{2calc}}{T_{1calc} * T_2}$$

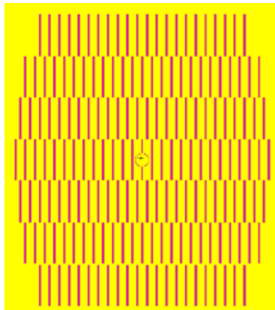
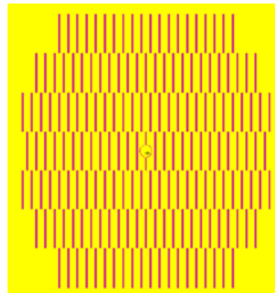
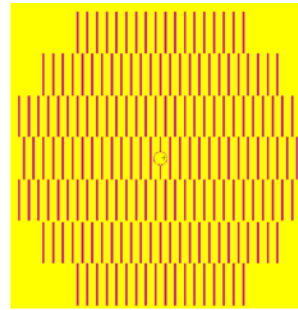
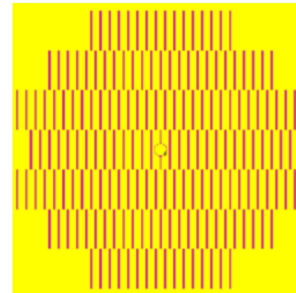
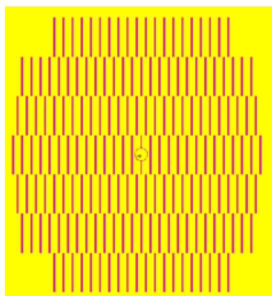
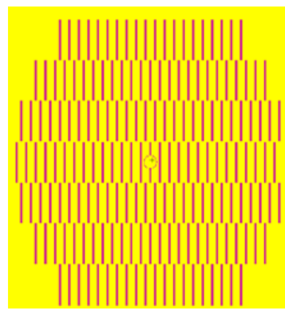
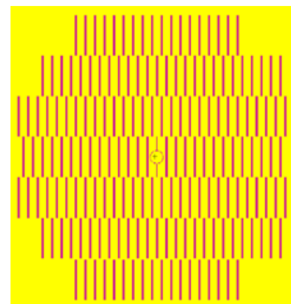
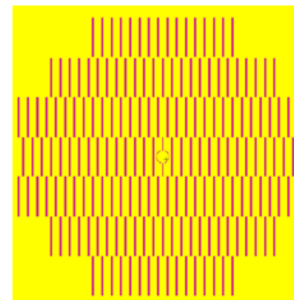
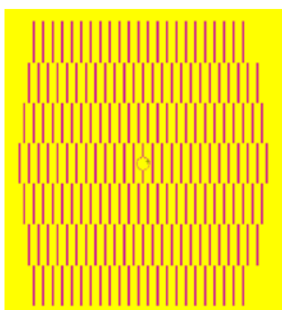
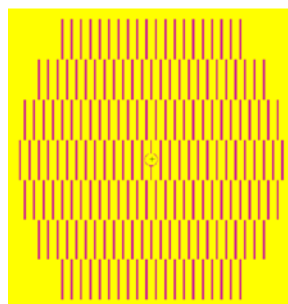
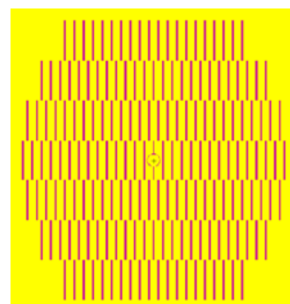
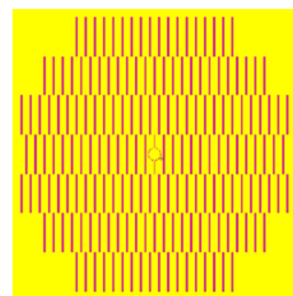
$$\frac{\partial U}{\partial T_{1calc}} = -\frac{(1 - k) * T_{2calc} * T_1}{T_{1calc}^2 * T_2}$$

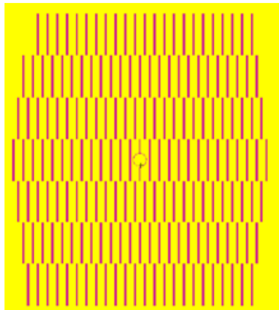
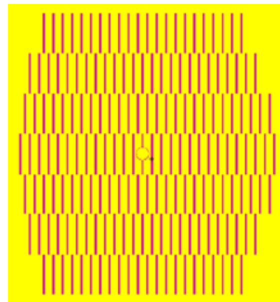
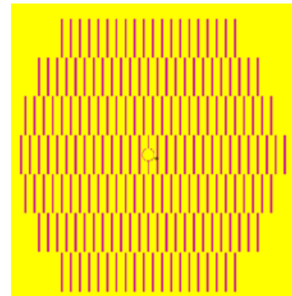
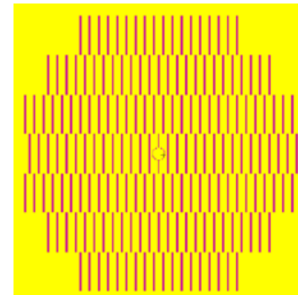
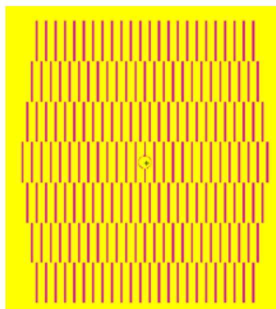
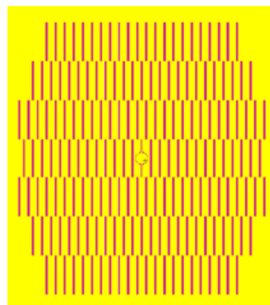
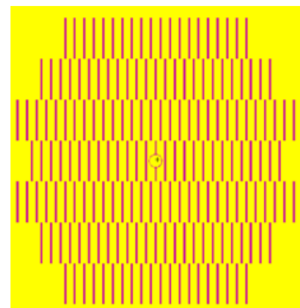
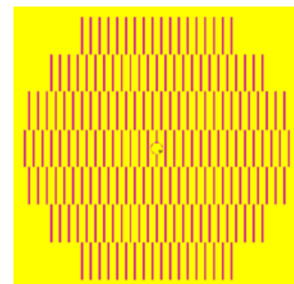
$$\frac{\partial U}{\partial T_2} = -\frac{(1 - k) * T_{2calc} * T_1}{T_{1clac} * T_2^2}$$

Appendix H: **Kcode Fuel Plate Configurations**

Fuel Plates	Configuration 1	Configuration 2	Configuration 3	Configuration 4
332				
334				
336				

Fuel Plates	Configuration 1	Configuration 2	Configuration 3	Configuration 4
338				
340				
342				

Fuel Plates	Configuration 1	Configuration 2	Configuration 3	Configuration 4
344				
346				
348				

Fuel Plates	Configuration 1	Configuration 2	Configuration 3	Configuration 4
350				
352				
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