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THE MULTIDIMENSIONAL INSTRUCTIONAL EFFICIENCY

OF WORKED EXAMPLES

by

Danae Romrell

A dissertation

submitted in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy in the Department of Instructional Design

Idaho State University

Spring 2015

Committee Approval

To the Graduate Faculty:

The members of the committee appointed to examine the dissertation of DANAE ROMRELL find it satisfactory and recommend that it be accepted.

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December 5, 2014

Danae Romrell, MA/MS
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RE: Your application dated 12/4/2014 regarding study number 4205: The Multidimensional Instructional Efficiency of Worked Examples

Dear Ms. Romrell:

I agree that this study qualifies as exempt from review under the following guideline: 1. Research on educational practices in educational settings. This letter is your approval, please, keep this document in a safe place.

Notify the HSC of any adverse events. Serious, unexpected adverse events must be reported in writing within 10 business days.

You are granted permission to conduct your study effective immediately. The study is not subject to renewal.

Please note that any changes to the study as approved must be promptly reported and approved. Some changes may be approved by expedited review; others require full board review. Contact Tom Bailey (208-282-2179; fax 208-282-4723; email: humsubj@isu.edu) if you have any questions or require further information.

Sincerely,

Ralph Baergen, PhD, MPH, CIP
Human Subjects Chair

BYU IDAHO
Academic Administration
Brigham Young University–Idaho • 210 Kimball Building • Rexburg, ID • 83460-1690

August 8, 2014

Dear Danae,

Your request to use human subjects for the study entitled *The Multidimensional Instructional Efficiency of Worked Examples* is approved for 12 months from the date of this letter.

Please notify the IRB if you intend to make any significant modifications to the study's design or implementation.

Good luck with your study.

Regards,

Scott J. Bergstrom, Ph.D.
Chair, BYU-Idaho Institutional Review Board

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Abstract

This dissertation describes a study on the efficiency and effectiveness of using worked examples to provide learners with opportunities to practice new concepts and skills in a face-to-face calculus course. The independent variables considered in the study included the type of practice and the calculus readiness of the learner. The effect of readiness and type of practice on the efficiency (measured using a measure called the Multidimensional Instructional Efficiency (MIE)) and effectiveness (measured using student performance on a post-test) of calculus instruction were examined.

The Kemp Model for instructional design was used to design and develop the multimedia, online, worked examples to be used in the proposed study. The examples were created using Adobe Captivate, a LiveScribe Smartpen, and Mathematica. Two different types of examples were created: worked examples with self-explanation prompts and worked examples with instructional explanations.

In the proposed study, the two types of worked examples were compared to a control group where the learners completed traditional homework practice problems. The research questions were designed to test the hypothesis that the worked example groups were both more efficient and more effective than the control group. However, the worked examples with self-explanation prompts were expected to be more effective for learners with high calculus readiness and the worked examples with instructional explanations were expected to be more effective for learners with low calculus readiness.

The results of the experiment did not support the hypotheses and indicated that the type of practice the learners completed had no effect on the efficiency or effectiveness

of the calculus instruction. There was also no evidence of an interaction effect between type of practice and level of readiness.

A significant main effect of level of readiness was found, after adjusting for prior calculus knowledge and homework completion rates. In particular, learners with high readiness were found to perform better on the post-test than learners with low or medium readiness. Additionally, learners with high readiness had higher MIE scores, and thus higher efficiency, than learners with low or medium readiness. This emphasizes the critical importance of readiness in calculus instruction.

CHAPTER I

Introduction

As instructional designers consider new ways to design learning experiences for students in higher education mathematics courses such as calculus, it is important to know how to best provide opportunities for students to practice, especially in an online environment. Because calculus problems often require students to remember and apply skills from their prior algebra courses, calculus problems tend to have high element interactivity; that is, they consist of several interacting concepts or principles that cannot be learned in isolation (Sweller, 2010). This leads to math problems that carry a high cognitive load, especially with learners who have low readiness for calculus. In order to provide support for these students, this study looked at the effectiveness of using two different types of worked examples in an online context in order to reduce cognitive load and to facilitate efficient and effective practice in calculus courses.

Theoretical Framework

Cognitive Load Theory (CLT) is a theory of learning that is “based on a model of human cognitive architecture” (Ayres & Van Gog, 2009, p. 253) and addresses the limited capacity of working memory. According to Sweller (2004), the limited capacity of working memory is a mechanism which ensures that changes to long-term memory are made in a meaningful, ordered manner in order to facilitate the development of schemas by the learner. Cognitive load is the “demand for working memory resources…that [are] required for achieving the goals of a particular cognitive activity" (Kalyuga, 2007, p. 513).

Researchers have identified three different types of cognitive load: intrinsic, extraneous, and germane (Sweller, Van Merriënboer, & Paas, 1998). *Intrinsic load* is a measure of the complexity of the learning task, where tasks with high element interactivity carry a high intrinsic load. However, intrinsic load is modulated by the expertise, experience, and readiness of the learner. As such, not all learners will experience the same intrinsic load for a particular learning task (Hollender, Hofmann, Deneke, & Schmitz, 2010). *Extraneous load* is load imposed by poor design of learning tasks or by the presentation of information that is irrelevant to learning (Sweller, 2010). As found by several researchers (e.g. Reed, Corbett, Hoffman, Wagner, & MacLaren, 2012; Vogel-Walcutt, Gebrim, Bowers, Carper, & Nicholson, 2011), tasks with a high extraneous load require a significant amount of mental effort by the learner, but the mental effort expended does not lead to greater learning. *Germane load* is load that is required for learners to develop appropriate schemas. Unlike extraneous load, germane load is beneficial to learning (Hollender et al., 2010). Effective instructional designs reduce extraneous and/or intrinsic load while maximizing germane load (Ayres, 2006; Lee & Anderson, 2013). [See chapter 2 for a more detailed discussion of the three types of cognitive load.]

Researchers have identified several effects that occur when instructional changes are made that affect cognitive load (Sweller, 2004). One such effect is the worked example effect. According to Sweller (2010), the worked example effect "occurs when students learn less from problem solving than from studying the equivalent worked examples" (p. 129). Worked examples are thought to carry a lower extraneous load than conventional problem-solving problems because learners are able to focus their working memory resources on a specific step of the problem and what the next step should be for a problem in that particular problem state. Conventional problem-solving, on the other hand, requires the leaner to use working memory resources to consider many possible next steps, including many that are irrelevant for the particular problem (Sweller, 2010). By helping the learners focus on the *relevant* steps for solving the problem, the extraneous load is decreased and schema development is supported (Sweller & Chandler, 1991).

However using worked examples to reduce extraneous load does not automatically increase germane load (Van Gog & Paas, 2008). Two techniques that researchers have identified for ensuring that the cognitive resources freed by the use of worked examples are used to increase germane load include self-explanation prompts and providing instructional explanations (e.g. Hilbert & Renkl, 2009; Richey & Nokes-Malach, 2013; Van Gog & Paas, 2008). Instructional explanations are defined as "explanations provided to facilitate example processing [and]…help learners to detect inconsistencies in their own understanding" (Wittwer & Renkl, 2010, p. 395). Sweller et al. (1998) suggested that worked examples are even more effective at helping learners develop necessary schema when they use instructional explanations to "identify the critical features in the worked examples by annotating them with what they are supposed to illustrate" (p. 273-274). Self-explanation prompts are suggestions added to the worked example that encourage learners to engage in a “form of self-talk where a learner engages in an iterative personal dialog while engaged in problem solving” (Biesinger & Crippen, 2010, p. 1474).

In certain situations, self-explanation prompts and instructional explanations have been found to be effective at increasing germane load and supporting learning (Wittwer & Renkl, 2010). However, there is concern that students with low prior knowledge may not have the necessary background to provide clear and detailed answers to self-explanation prompts (Richey & Nokes-Malach, 2013). This may inhibit learning for students who provide responses to the self-explanation prompts that are unclear, or even incorrect (Berthold & Renkl, 2009). Based on this concern, instructional explanations may be a good alternative for learners with low prior knowledge because providing students with the explanation lowers cognitive load as compared with requiring them to answer self-explanation prompts. However, the concern is then whether or not instructional explanations are effective for learners with high prior knowledge, who may have been capable of providing good responses to self-explanation prompts.

Purpose of the Study

The purpose of this study was to explore the relationship among self-explanation prompts, instructional explanations, and the calculus readiness of the learners. Cognitive Load Theory indicates that replacing some of the practice problems with well-designed worked examples can reduce the extraneous load of the learners (Sweller et al., 1998) by helping them develop appropriate schema. As discussed above, adding self-explanation prompts and instructional explanations to worked examples may make the examples even more effective by increasing germane cognitive load. However, very little research has been done on using such techniques in a calculus context (see Chapter 2).

Because calculus problems rely so heavily on algebra skills, they tend to have very high element interactivity, and therefore carry a high intrinsic load. Because working memory is limited, and working memory resources are additive (Ayres, 2006), this requires instructional designers to pay particular attention to reducing extraneous load in order to allow for the higher intrinsic load of calculus practice problems. However, the intrinsic load of a calculus topic depends somewhat on the calculus readiness of the learner. If the learner is adequately prepared for calculus, they should have developed appropriate schemas related to algebra, and will not experience as high an intrinsic load as learners who have weak algebra skills and low calculus readiness.

In particular, this study looked at how self-explanation prompts, instructional explanations, and calculus readiness impact the effectiveness and efficiency of calculus instruction. In this context, *effectiveness* was measured based on student performance. *Efficiency* was measured using a construct called multidimensional instructional efficiency (MIE). As developed by Tuovinen and Paas (2004), MIE combines mental effort during learning, mental effort during testing, and student performance into one measure that gauges the efficiency of instruction. A learning technique with a large MIE would be very efficient because student performance would be very high with relatively low mental effort. A small value for the MIE indicates low instructional efficiency in that student performance is poor even though mental effort required for the learning task was relatively high.

Research Questions

The study addressed the following six research questions:

- Research Question #1: Is there a significant main effect of type of practice (worked examples with self-explanation prompts, worked examples with instructional explanations, or traditional homework) on multidimensional instructional efficiency (MIE) as measured using the MIE formula developed by Tuovinen and Paas (2004)?
- Research Question #2: Is there a significant main effect of calculus readiness (low readiness, medium readiness, or high readiness) on MIE as measured using the MIE formula developed by Tuovinen and Paas (2004)?
- Research Question #3: Is there an interaction effect between calculus readiness and type of practice on MIE?
- Research Question #4: Is there a significant main effect of type of practice (worked examples with self-explanation prompts, worked examples with instructional explanations, or traditional homework) on student performance as measured by posttest scores?
- Research Question #5: Is there a significant main effect of calculus readiness (low readiness, medium readiness, or high readiness) on student performance, as measured by posttest scores?
- Research Question #6: Is there an interaction between calculus readiness and type of practice on student performance?

Research Design

These research questions were answered using an experimental pre-test/post-test control group design (see Table 1). The experiment was conducted with students registered for two sections of Calculus I, both taught by the researcher. At the beginning of the semester students took a calculus readiness assessment in order to determine their level of readiness for the course. The learners were then randomly assigned to one of three experimental groups. A pre-test was administered and then the treatment was delivered online during the second unit of the course. The first group completed practice problems that consisted of faded worked examples with self-explanation prompts. The

Table 1

Research Design for the Proposed Study

Group 1: Worked Examples with Self-explanation Prompts	O_1	R	O_2	X_1	O_3	O_4	O_5
Group 2: Worked Examples with Instructional Explanations	O_1	R	O_2	X_2	O_3	O_4	O_5
Group 3: Control Group: Traditional homework	O_1	R	O_2	X_3	O_3	O_4	O_5

Note. O_1 represents the MDTP Calculus Readiness Test, R indicates random assignment to an experimental group, O_2 represents the Differentiation Pre-test, each X_i represents student practice using their assigned type of homework, O_3 represents the mental effort measurements taken during the learning phase, O_4 represents the mental effort measurements taken during the testing phase, and O_5 represents the Differentiation Post-test.

second group viewed the same examples as the first group, but the examples included instructional explanations, rather than self-explanation prompts. The third (control) group completed practice problems without viewing any worked examples.

As learners completed each learning task (either a worked example or a practice problem), they were asked to rate their level of mental effort. At the conclusion of the unit the learners took a post-test to determine student performance. The research questions were then answered using two separate two-way ANCOVA tests.

Definition of Terms

Calculus readiness. Readiness refers to the learners' mastery of concepts and skills that are required for success in a particular course. In calculus, the readiness of the student requires their understanding of the function concept, algebraic manipulation skills, and an understanding of trigonometry (Kay & Kletskin, 2012). In this study, calculus readiness was measured using the MDTP Calculus Readiness Test.

Cognitive load. Cognitive load is the "demand for working memory resources…that [are] required for achieving the goals of a particular cognitive activity" (Kalyuga, 2007, p. 513).

Conceptual knowledge. In mathematics, conceptual knowledge is defined as "explicit or implicit understanding of the principles that govern a domain and the interrelations between pieces of knowledge in a domain" (Rittle-Johnson & Alibali, 1999, p. 175).

Extraneous load. Extraneous load is one of three types of cognitive load. Extraneous load is defined to be load imposed by the poor design or presentation of instruction (Sweller et al., 1998). Extraneous load was important in the context of this proposed study because the worked examples under investigation were expected to reduce extraneous cognitive load.

Faded worked examples. “Faded worked examples gradually fade worked-out steps with increased levels of learner expertise by replacing these steps with problem solving sub-tasks” (Kalyuga, 2007, p. 529). In this study, the examples were faded as each assignment began with fully worked examples, followed by partially worked examples, and then practice problems.

Fully worked example. For the purposes of this study, a fully worked example was an example where the complete solution, including all solution steps, was presented to the learner.

Germane load. Germane load is one of three types of cognitive load. Germane load is beneficial to learning and refers to load that is required for the development of schemas (Sweller et al., 1998). Germane load was particularly important in the context of this proposed study because self-explanation prompts and instructional explanations were thought to increase germane load (Hilbert & Renkl, 2009).

Instructional explanations. Instructional explanations are defined as “explanations provided to facilitate example processing [and]…help learners to detect inconsistencies in their own understanding” (Wittwer & Renkl, 2010, p. 395). In this study, instructional explanations referred to explanations that appeared on the screen while a student was viewing a worked example that added additional explanations to those provided as part of the example.

Intrinsic load. Intrinsic load is one of three types of cognitive load. Intrinsic load is based on the level of complexity of the content of a learning task (Sweller et al., 1998). Intrinsic load is modulated by the prior knowledge, experience, and expertise of the learner (Hollender et al., 2010). Intrinsic load was important in the context of this study because the learning tasks being tested in this study had a higher intrinsic load for learners with low calculus readiness.

Mental effort. Mental effort refers to the “amount of resources actually allocated to accommodate the task demands” (Paas, Van Merriënboer, & Adam, 1994, p. 420). As explained by Paas et al. (1994), cognitive load can be conceptualized by considering both mental load and mental effort. Mental load is task-centered and is load imposed by the task or environment. Mental effort is human-centered and refers to the amount of effort actually used to complete the task. Mental effort is one element in the calculation of multidimensional instructional efficiency.

Multidimensional instructional efficiency. Instructional efficiency (IE) is a measurement that combines both mental effort and student performance into one measurement (Vogel-Walcutt et al., 2011). A task with high IE would indicate high performance with low effort, while a task with low IE would correspond to low performance with high effort. IE was first developed by Paas (1992) and then expanded to multidimensional instructional efficiency (MIE) by Tuovinen and Paas (2004). MIE differs from IE in that it considers two different measurements of mental effort: mental effort during learning and mental effort during testing (Tuovinen & Paas, 2004). MIE is computed using the formula $MIE = (z_P - z_{EL} - z_{ET})/\sqrt{3}$. In this formula MIE

represents the multidimensional instructional efficiency, z_P represents student performance, z_{EL} represents mental effort during learning, and z_{ET} represents mental effort during testing.

Partially worked example. For the purposes of this study, a partially worked example is a worked example where only some of the solution steps are provided for the learners. In a partially worked example most of the problem steps are provided, however the learner is asked to complete one or two of the problem steps.

Practice problem. See definition for traditional homework.

Procedural knowledge. In mathematics, procedural knowledge is defined as "action sequences for solving problems" (Rittle-Johnson & Alibali, 1999, p. 175)

Self-explanation prompts. Self-explanation prompts are suggestions added to a worked example that encourage learners to engage in a "form of self-talk where a learner engages in an iterative personal dialog while engaged in problem solving" (Biesinger & Crippen, 2010, p. 1474). In this study, self-explanation prompts referred to questions that appeared on the screen while a learner was viewing a worked example. These questions asked the learner to answer conceptual questions about the example they were viewing.

Traditional homework. In this study, traditional homework, or practice problems, referred to math problems similar to those given in calculus textbooks, which provide opportunities for learners to practice new skills. These problems are typically sorted into lists of problems of a similar type. Although these problems are traditionally done with paper and pencil, the traditional homework problems presented in this study were delivered online.

Worked examples. Worked examples are one instructional tool that has been used to reduce the cognitive load experienced by a learner. Worked examples provide "a step-by-step demonstration of how to perform a task or solve a problem" (Clark & Mayer, 2011, p. 22). In this study, worked examples referred to multimedia, online examples of calculus problems that learners viewed in place of completing traditional homework problems. These examples included audio, video, animations, and/or graphics.

Limitations and Delimitations

As with any study we made several key assumptions that were taken into consideration during the analysis of the results of the study. These assumptions led to limitations and delimitations of the study. A *limitation* is an aspect of the research design that poses a potential threat to internal validity, which is defined as the extent to which the differences seen in the treatment groups can be attributed to the experimental treatment (Campbell & Stanley, 1963). A *delimitation* is a threat to external validity, or the extent to which the study "can be generalized to different subjects, settings, [and] experimenters" (Bracht & Glass, 1968, p. 438). Bracht and Glass (1968) suggest there are two general types of external validity to consider: population validity and ecological validity.

Limitations. Campbell and Stanley (1963) identify eight common threats to internal validity. In the case of this controlled experiment, almost all of these threats (including history, maturation, testing, statistical regression, selection, and selection-maturation interaction) were mitigated by random assignment to treatment groups and the use of a control group. However, two of the threats suggested by Campbell and Stanley (instrumentation and experimental mortality) needed further consideration.

Campbell and Stanley (1963) noted that *instrumentation* can be a limitation when changes in how assessment instruments are graded leads to changes in the scores, especially if those changes favor one experimental group over the other. This is particularly a threat in the case of the pre- and post-tests for this experiment because they were subjectively graded (the other assessments were objectively graded and thus posed less of a threat). Because the post-test score was included in the computation of the MIE, this also could be a limitation for the MIE. Several measures were undertaken to ensure consistency in the grading procedures in order to mitigate this limitation of the study. Both the pre- and post-tests were graded by a single grader, not the researcher/instructor, who did not know to which experimental group each student belonged and who was trained on a rubric-based grading technique.

Experimental mortality refers to the limitation caused by having more students drop out of one group than another (Campbell & Stanley, 1963). The timing of this experiment made this limitation particularly concerning because the treatment occurred in Unit 2, immediately after learners took the Unit 1 Exam. The weakest students who do not do well on this first exam often drop the course at this point. Because there were relatively few students who fell in this category of being particularly weak in their mathematical abilities, special care was taken in the random assignment of students using the alternate ranks random assignment technique to ensure that those students were evenly distributed among the three experimental groups (Myers, Well, & Lorch, 2010).

Having these students drop the course had the potential to introduce a form of bias. In order to mitigate this potential limitation as much as possible, assignment to experimental groups was not done until after the learners had completed the Unit 1 Exam.

In addition to the possible threats to internal validity identified by Campbell and Stanley (1963), another limitation of this study is that it was impossible for the researcher to ensure that the study participants only saw the materials that were prepared for members of their experimental group. For example, it is possible that learners in one of the worked examples groups could have shared the worked examples they received as part of the treatment with members of the control group. In order to prevent this type of treatment contamination, the experimental groups each viewed different versions of the learning management system (LMS) and did not have direct access to the examples provided to the other groups. The importance of having learners view only the examples prepared for their group was explained to the learners and they were asked to not view materials prepared for other groups.

Delimitations. There were also several delimitations that were considered in this study. Specifically, there were threats to both population validity (threats dealing with generalizing the results of the study to other populations) and ecological validity (threats dealing with generalizing the results to other environments) (Bracht & Glass, 1968).

The population of learners that was available for this experiment might not have been representative of all calculus students, and this posed a threat to population validity (Bracht & Glass, 1968; Campbell & Stanley, 1963). Because this study was conducted with one instructor, in one class, at one university, the results need to be cautiously considered before being applied to other instructors, classes, or universities. However, this threat is somewhat lessened because the students who registered for this course were not aware that a study was being conducted as part of the course until the fifth day of class. Therefore, it is probably reasonable to assume that they were a fairly representative sample of calculus students at the particular university where the study took place. As such the results of this study can be generalized to other calculus students at this university, but the research should be repeated in other environments in order to apply the results to students in other courses or other schools.

This study also contains several threats to ecological validity. One such threat is the fact that the learners knew that their work was being examined as part of the experiment (Bracht & Glass, 1968; Campbell & Stanley, 1963). As such, they may have been particularly conscientious when viewing the worked examples used in the experiment. This may mean that the results would not be typical of students who were not participating in an experiment.

The novelty of the worked examples also posed a threat to ecological validity (Bracht & Glass, 1968). The other units in the course used traditional paper-and-pencil homework problems selected from a calculus textbook. Therefore, the worked examples used in this experiment were new and novel to the learners. The results seen in this study might not be generalizable to a class where worked examples are used for an entire semester. However, it is worth noting that the three-week long period over which the examples were used is longer than most of the other studies in the worked examples literature.

There also may have been an experimenter effect. This threat to ecological validity refers to the fact that the attitudes and expectations of the experimenter might bias the learners in favor of certain style of practice problem (Bracht & Glass, 1968). This is especially worth consideration in this case because the researcher is also the instructor of the class. Special care was taken by the researcher to not reveal her biases to the class.

Two additional threats to ecological validity were pre- and post-test sensitization. The pre-test contained several conceptual questions where students were asked to explain their understanding of calculus concepts. Even if the learners did not know the answers to the pre-test questions, merely taking the pre-test may have alerted them to the fact that conceptual ideas are important and could be covered on the post-test. Thus, they may have attended more to conceptual ideas they encountered during the unit than they would have if they didn't take a pre-test. As such, it might not be possible to generalize the results to learners who do not take a pre-test (Bracht & Glass, 1968; Campbell & Stanley, 1963).

The choice of content and assessment instruments also posed a threat to generalizability of the environment. The decision was made to use the second unit of the course for this study. The assumption was made that this unit is representative of the other content included in calculus and that the results seen in this study would apply to other topics within the course. As part of the design of the experiment, the validity of the assessment instruments was established. This demonstrated that the assessment instruments selected accurately measured the intended constructs and that similar results would be seen with other similar assessment instruments (Bracht & Glass, 1968)

Finally, threats to ecological validity may have occurred because of the timing of the study. It may have been that the results seen in the study were due to other variables or events that occurred at the same time as the study and were not considered by the researcher. And the results seen in the post-test which was taken immediately at the end of the study might not be evident later in the course or later in the learners' academic careers (Bracht & Glass, 1968).

Significance of the Study

Researchers have studied worked examples extensively (i.e. Kalyuga, 2007; Salden, Koedinger, Renkl, Aleven, & McLaren, 2010; Sweller & Cooper, 1985; Van Gog, Paas, & Sweller, 2010). However, as will be described in more detail in Chapter II, a recent review of fifteen studies on worked examples in a mathematics context found that thirteen of the fifteen studies were conducted in an algebra or geometry class. Very little research has been done on using worked examples to teach mathematical principles in calculus. Because calculus problems tend to have higher element interactivity than algebra problems, they tend to carry higher cognitive load (Sweller, 2010). This is especially problematic for learners who have a low level of readiness because they may not be at the appropriate point in the development of their mathematical skills to understand the more conceptual ideas of calculus (Bruner, 1960; Pyzdrowski et al., 2013).

Additionally, much of the research on worked examples has been conducted using a single lesson, often outside of a classroom setting (see Chapter 2 for a more detailed discussion). This study adds to the existing literature on worked examples by considering their use over a longer period of time (three weeks) when delivered online as part of a full semester calculus course.

Because of positive results in the research on worked examples, it appears worked examples may be an appropriate tool for instructional designers to use to prepare online practice in a mathematics context, but more research is needed to see if these results apply in a calculus context.

CHAPTER II

Literature Review

The research questions for this study were intended to explore the effectiveness of using worked examples to provide opportunities for students to practice in a calculus class. In order to fully understand how these research questions fit within the theoretical and historical background of Cognitive Load Theory, a review of literature was conducted. A relatively broad search of relevant topics was conducted in order to develop a strong foundation for research on this topic. Table 2 details the database searches that were conducted to select articles for inclusion in this review. So as to focus on more recent technological trends, the search focused primarily on recent research published after 2009. Search terms such as multimedia, mathematics, undergraduate mathematics, cognitive load, worked examples, and mathematical readiness were used. The search identified 111 articles that were related to the topics of cognitive load, example-based learning, mathematics education, multimedia applications, or e-Learning.

The 111 relevant articles identified through this broad search were read, summarized, and classified. Three broad topics emerged as being important for providing a foundation for research on worked examples. Cognitive Load Theory provides an important theoretical perspective from which to consider examples, research on knowledge and learning in mathematics provided critical insight into the learning context, and research on e-Learning provided guidance on effectively incorporating technology. A Venn diagram was used to organize the articles based on how they addressed these three

Table 2

Summary of database searches for literature review

Date	Database or Journal	Search Terms	Filters	# of Results	# of Relevant Results
08/27/13	Clark & Mayer's Chapter on Worked Examples	Looked at their reference list and suggested resources		7	7
08/29/13	ERIC (EBSCOHost)	"Multimedia" AND "Mathematics"	Full-text, peer-reviewed, 2003-2013	56	13
09/14/13	Computers & Education Journal archive	"Undergraduate Mathematics"	After 2000	16	4
09/19/13	ETR&D; Computers & Education	Looked at all articles in five most recent issues	Five most recent issues	13	13
09/23/13	ETR&D	"Cognitive Load" AND "Mathematics"		38	6
10/03/13	PRIMO	"Gender Difference" AND "Worked example"	After 2009	6	1
10/03/13	PRIMO	"Gender Difference" AND "Worked examples"	After 2009	10	1
10/03/13	PRIMO	"Gender" AND "Worked Examples"	After 2010	22	7
11/13/13	PRIMO	"Worked Examples"	Peer-reviewed, After 2012	141	13
11/13/13	PRIMO	"Worked Examples"	Peer-reviewed, 2009-2011	323	22
02/10/14	Google Scholar	“Types of Cognitive Load”		1.9 million	Selected 2
03/18/14	PRIMO	“Mental Effort Measurement”		2	1
03/18/14	PRIMO	“Instructional Efficiency”	Peer-reviewed	44	6
04/02/14	PRIMO	“Conceptual knowledge” AND “Mathematics”	Peer-reviewed	97	12
10/14/14	Google Scholar	“Mathematical Readiness”	2000-2014	34,500	Selected 3

major foundational topics (see Figure 1). Based on this initial research, three main areas are discussed in the review: knowledge and learning in mathematics, cognitive load theory, and the theory of worked examples. Also, the gaps in the literature that are addressed by this proposed study will be identified.

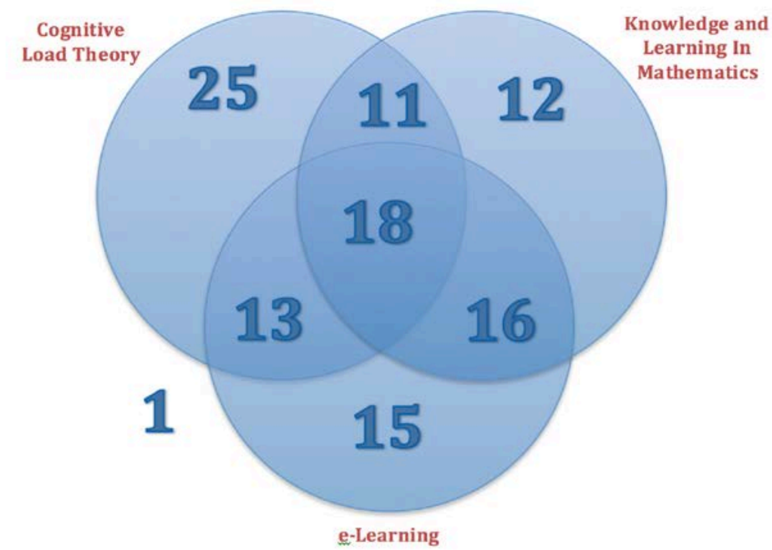

Figure 1. Venn Diagram classifying the 111 research articles considered for inclusion in this review of literature.

Knowledge and Learning in Mathematics

Mathematics instruction differs from instruction in other learning domains in a few significant ways. For example, Ayres (2006) suggests that mathematical tasks often have a high level of element interactivity. Element interactivity is defined as the extent to which the elements of the task "have to be assimilated simultaneously" (p. 288) rather than being learned in isolation. Learning abstract mathematical principles often requires learners to coordinate their understanding of several underlying concepts at once and, as such, abstract mathematical principles have high element interactivity and carry a high cognitive load. Another distinctive feature of learning in the mathematical domain is that topics that appear to be very algorithmic often require learners to understand complex conceptual ideas. This is important because learners who know algorithms without understanding concepts often apply those algorithms inappropriately (Hiebert & Lefevre, 1986). Additionally, learners often have low confidence and high anxiety related to learning mathematics (Loong & Herbert, 2012). High anxiety levels compete with cognitive resources and leave fewer cognitive resources available to focus on solving mathematical problems (Vytal, Cornwell, Arkin, & Grillon, 2012). The following discussion reviews research related to knowledge and learning in mathematics. In particular, in order to provide a research-based perspective on constructs that are essential to the proposed experiment, the discussion will address types of mathematical knowledge, teaching calculus, and mathematical readiness.

Conceptual vs. procedural knowledge. Hiebert and Lefevre (1986) note that there is a long history of division of mathematical content into conceptual and procedural knowledge. Many distinguished educational theorists including Dewey, Thorndike, Brownell, Gagné, and Bruner addressed the distinction and relationship between concepts and skills. Although earlier discussions of procedural and conceptual knowledge considered the two knowledge types as different and competing forms of knowledge, "current discussions treat [them] as distinct, but linked in critical, mutually beneficial ways" (Hiebert & Lefevre, 1986, p. 2). For example, Rittle-Johnson and Alibali (1999) describe procedural and conceptual knowledge as forming two ends of a continuum that cannot always be separated and do not develop independently. In fact, several researchers note the importance of emphasizing both types of knowledge (e.g. Newton, Star, &

Lynch, 2010; Star & Seifert, 2006). Newton et al. (2010) suggest that learners who have a solid understanding of both procedural and conceptual knowledge become more flexible problem solvers who are able to select the most effective and efficient procedure for a given mathematical task.

Teaching calculus. The concepts traditionally taught in calculus courses contain a higher level of abstraction than the concepts taught in a typical algebra course. Although, to a certain extent, abstraction is also required for the learning of algebra, the introduction of the limit concept in calculus significantly increases the level of abstraction required of learners of calculus (Sofronas et al., 2011). According to White and Mitchelmore (1996), “a feature of all advanced mathematics is the need for abstract concepts” (p. 80). The ability to understand abstraction in a mathematical setting comes as a result of procedural knowledge that is supported by conceptual knowledge (White & Mitchelmore, 1996). Therefore, the learning of procedures in calculus must be carefully connected to conceptual knowledge in order “to foster the development of understanding” (Star & Seifert, 2006, p. 281) and help learners gain better understanding of the connection between calculus concepts and skills (Sofronas et al., 2011).

Mathematical readiness. As theorized by Bruner (1960), cognitive readiness is an essential aspect of learning. Bruner explained that mathematical principles can be taught to children at any age, as long as they are presented in a form that aligns with the readiness of the learner. For example, “it can be demonstrated that fifth-grade children can play mathematical games with rules modeled on highly advanced mathematics…They will flounder, however, if one attempts to force upon them a formal mathematical description of what they have been doing" (Bruner, 1960, p. 38). In order to understand formal mathematical language, learners must attain formal operational thought.

In general, calculus requires a higher level of cognitive readiness from the learner than algebra because of its higher level of abstraction (White & Mitchelmore, 1996). However, even learners with adequate cognitive readiness may not be ready for calculus if they have not learned the required prerequisite content. Learners with low content readiness are often not successful at learning. Additionally these learners have other disadvantages as compared to their more prepared classmates. Schwonke et al. (2013) found that learners with low readiness often do not use learning aids in a "learning-oriented way" (p. 138). Additionally, they are easily overwhelmed when presented with new material and have poor metacognitive skills. Schwonke et al. found that providing metacognitive support for learners helps them use external learning resources, such as tutors and help resources, more strategically and efficiently.

Content readiness research conducted in a calculus context shows that readiness is correlated with student performance (Pyzdrowski et al., 2013). In particular, research shows that in order to succeed in calculus, learners need to understand the function concept, have adequate algebraic manipulation skills, and have an understanding of trigonometry (Kay & Kletskin, 2012). Universities with successful calculus programs typically have some way to assess and ensure the content readiness of learners who register for calculus (Rasmussen, Ellis, & Zazkis, 2012).

Careful consideration must be given on how to teach abstract calculus concepts to learners with varying degrees of cognitive and content readiness. Cognitive Load Theory provides a helpful theoretical perspective from which learning in calculus can be considered.

Cognitive Load Theory

Cognitive Load Theory (CLT) is a theory of learning that is based on knowledge of human cognitive architecture. Models of human cognitive architecture describe two types of memory: working memory and long-term memory. The working memory can be equated with consciousness; individuals "are only aware of information in working memory" (Sweller, 2004, p. 12). However, working memory is limited. Learners can only retain a limited number of informational items in their working memory at any given time (Sweller, 2004). Long-term memory, however, provides for a more permanent system for storing information. Unlike the working memory, long-term memory is practically limitless in the amount of knowledge and information it can store (Ayres & Van Gog, 2009).

Building knowledge in long-term memory occurs as individuals organize new information and skills in their working memory and transfer that knowledge to long-term memory through the development of schemas. Schemas, or collections of related information that can be considered as a single construct (Hollender, Hofmann, Deneke, & Schmitz, 2010), allow learners to categorize pieces of information in their long term memory "in the manner in which they will be used" (Sweller, Van Merriënboer, & Paas, 1998, p. 255). This presents a particular advantage because each schema consists of numerous pieces of information that can then be processed in working memory as a single item. Sweller (2004) illustrated this principle by describing the schema for *restaurant*. This single word represents a large collection of informational pieces including “much of what we know of food, the preparation of food, eating, the serving of food, aspects of a financial system and its relation to goods and services, the architecture of buildings and furniture, social relations between humans, etc.” (p. 13). However, because all of these informational pieces are linked in a single schema in long-term memory, an individual can consider them as a single entity in working memory.

As described by Sweller (2004), CLT assumes that “the purpose of instruction is to build knowledge in long-term memory” (p. 21). This occurs as learners develop schemas that connect new information to their prior knowledge. However, the limited nature of working memory requires instructional techniques to be as efficient as possible, so as to not overload the working memory. This is particularly important in light of research that shows that learning is most effective when working memory resources are not overloaded (Ayres & Van Gog, 2009). As such, instructional designers should carefully consider the cognitive load required for a given instructional activity. Cognitive load refers to the “demand for working memory resources…that [are] required for achieving the goals of a particular cognitive activity” (Kalyuga, 2007, p. 513). Researchers have discussed several different learning effects that help optimize the cognitive load of instructional activities including, among others, the worked-example effect, the split-attention effect, the expertise reversal effect, the element interactivity effect, and the goal free effect. These effects are all “based in part on the assumption that instructional design should be structured to facilitate alterations in long-term memory" (Sweller, 2004, p. 12).

Types of cognitive load. As researchers have considered techniques for optimizing the cognitive load of instructional activities they have identified three different types of cognitive load: germane, intrinsic, and extraneous (Hollender et al., 2010; Sweller et al., 1998). Germane load refers to the working memory resources that are necessary for the development of schema (Hollender et al., 2010). Because the development of appropriate schema is the goal of instruction, germane load is beneficial to learning. It is essential that instructional designs allow for adequate cognitive resources to be allocated for germane load.

Intrinsic load depends on the complexity and element interactivity of the information that is to be learned. To illustrate the difference between a task with low intrinsic load and a task with high intrinsic load, Sweller et al. (1998) explains that learning vocabulary in a foreign language has low intrinsic load because each word can be considered and learned independently of other words. Alternately, learning correct grammar structure in a foreign language has high intrinsic load because in order to correctly understand grammatical structure the learner cannot consider each of the words in a sentence one word at a time, but rather must consider the sentence and its meaning collectively, as well as the order and relationship among the words in the sentence. Intrinsic load is relative to the expertise, experience, and prior knowledge of the learner. Not all learners will have the same intrinsic load for a particular learning task (Hollender et al., 2010).

Extraneous load is load imposed by learning tasks or presentation of information that is irrelevant to learning (Sweller, 2010). Tasks with a high extraneous load require significant mental effort by the learner, but the mental effort expended does not lead to greater learning. For example, consider a learning task where a learner is provided with a graphic and text that mutually describe a new idea, but the graphic and the text are not placed next to each other. This situation carries high element interactivity because the learner must maintain information from the graphic in their working memory while reading the text, and vice versa. Sweller et al. (1998) note that the cognitive load required to integrate the text and the graphic is extraneous "because it is caused entirely by the format of the instruction rather than by the intrinsic characteristics of the material" (p. 263). Schnotz (2010) notes that in addition to being due to high element interactivity, extraneous load can be due to "other forms of unnecessary usage of resources, namely resources such as time and effort" (p. 317). For example, consider a situation where a learner views a graphic that clearly describes a new idea without requiring additional explanation, but then the learner is asked to read written text that provides the same information as the graphic. This task carries high extraneous load, not because of element interactivity, but because reading the text is a waste of the learner's time and effort.

Germane, intrinsic, and extraneous load are additive, which means that reducing one type of load leaves more available cognitive function available for the other types of cognitive load (Ayres, 2006), as long as the total load is not greater than available working memory resources (Sweller et al., 1998). Because germane load is beneficial to learning, appropriate instructional designs will decrease extraneous and/or intrinsic load while simultaneously increasing germane load (Lee & Anderson, 2013). Extraneous load is strongly influenced by the instructional design. Ayres (2013) explains that unlike intrinsic load, which occurs naturally, extraneous load is created by instructional designers when they create poorly designed instructional materials. In particular, “poorly designed learning materials, where learners spend considerable effort in trying to follow or understand the procedures, are high in extraneous load” (p. 116).

Because intrinsic load depends on the complexity of the learning task, it is the most difficult type of load to affect through instructional design. Either increasing the prior knowledge of the learner or reducing the element interactivity of the learning task can decrease intrinsic load. One common strategy for decreasing intrinsic load through instructional design is the isolated elements strategy, where tasks with high element interactivity are broken down and part of the task is presented before presenting the entire task (Ayres, 2013).

Merely reducing the extraneous and/or intrinsic load does not automatically increase the germane load. As explained by Sweller et al. (1998), in order to increase the germane load, learners’ attention must be directed away from tasks that are not relevant to learning to cognitive processes that help with the development of schemas. Some specific techniques for increasing germane load include increasing the motivation of the learner (Sweller, 2010), including design elements such as self-explanation prompts that help the learner identify essential elements of a learning task (Sweller et al., 1998), and providing worked examples in place of problem solving so that learners can focus on problem steps to aid in the development of schemas (Van Gog, Kester, & Paas, 2011).

Measuring cognitive load. Because of its complexity, cognitive load is difficult to measure (Sweller et al., 1998). In particular, measuring specific types of cognitive load has proven to be quite difficult and there is currently much interest in the development of assessments that measure different types of cognitive load (Ayres & Van Gog, 2009; Kalyuga, 2009; Kirschner, Ayres, & Chandler, 2011; Van Gog, Kester, Nievelstein, Giesbers, & Paas, 2009). Early research on cognitive load relied heavily on performance-based measures (Paas et al., 1994), but in the 1990s, Paas and his colleagues (see Paas et al., 1994; Paas & Van Merriënboer, 1993; Paas, 1992) developed a mental effort rating scale that began to be widely used as a technique to measure cognitive load. Although mental effort is not the same as cognitive load, mental effort is generally accepted as reflecting the actual cognitive load (Ayres & Van Gog, 2009; Paas, Tuovinen, Tabbers, & Van Gerven, 2003; Sweller et al., 1998; Van Gog & Paas, 2008). Mental effort is defined as "the aspect of cognitive load that refers to the cognitive capacity that is actually allocated to demands imposed by the task" (Paas et al., 2003, p. 64).

Mental effort has been measured using a variety of different subjective and psychomotor techniques (Paas et al., 2003, 1994). Paas et al. (1994) found that a subjective rating scale was less intrusive and more sensitive to changes in mental effort than a psychomotor technique. The subjective scale developed by Paas (1992) is widely used as a measurement of mental effort (e.g. Boekhout, Van Gog, Van de Wiel, Gerards-Last, & Geraets, 2010; Nievelstein, Van Gog, Van Dijck, & Boshuizen, 2013; Van Gog et al., 2011). The subjective measurement scale (see Table 3), which has since become known as the Paas' Mental Effort Measurement Scale (PMEMS), is a nine-point Likert scale self-rating that varies from a score of 1 (very, very low mental effort) to a score of 9 (very, very high mental effort).

Table 3

Paas' Mental Effort Measurement Scale (PMEMS) (Tuovinen & Paas, 2004, p. 142)

Numerical Rating	Interpretation
1	Very, very low mental effort
2	Very low mental effort
3	Low mental effort
4	Rather low mental effort
5	Neither low nor high mental effort
6	Rather high mental effort
7	High mental effort
8	Very high mental effort
9	Very, very high mental effort

Instructional efficiency. Research studies that just look at mental effort do not really provide enough information to make conclusions about the effectiveness of the instruction because, based on CLT, the goal of instruction is not just to have low mental effort. Rather, the goal is to have an appropriate level of mental effort relative to the performance of the learners. Paas et al. (1994) developed the concept of instructional efficiency in order to provide one measure that combines mental effort and performance of the learners. Note that in this context a learning task with high instructional efficiency is considered to be efficient relative to the amount of mental effort expended by the learners. Instructional efficiency as defined by Paas et al. (1994) does not take time into account.

The instructional efficiency calculation first proposed by Paas et al. (1994) was based on mental effort ratings collected during the testing phase of an experiment, rather than during the instructional phase. Later researchers who used the concept of instructional efficiency changed the computation (most likely unintentionally) by using mental effort ratings that were collected during the instructional phase of an experiment (Van Gog, Kirschner, Kester, & Paas, 2012). Van Gog et al. (2012) note that by measuring the mental effort during different phases of the experiment, the resulting calculations actually measure different constructs. Using the original measure, researchers can find the instructional efficiency of the *learning objectives*, while researchers who use the modified measure can determine the instructional efficiency of the *learning process* (Van Gog et al., 2012). Both of these constructs are ultimately useful, and as such, Tuovinen and Paas (2004) describe a multidimensional approach to computing learning efficiency that incorporates mental effort during the learning phase and mental effort during the testing phase, along with performance. This multidimensional approach is the one that will be used in this proposed study.

Cognitive Load Theory versus Constructivism. Cognitive Load Theory (CLT) and constructivism are two different theories of learning that are often seen as being at odds one with another. Both of these theories of learning are based on the development of schemas, but the theories differ in *how* they propose schemas are developed. CLT suggests that instructional designers should reduce extraneous load by making it as simple as possible for learners to develop schemas (Sweller & Chandler, 1991). Constructivism, on the other hand, suggests that learners develop schemas by constructing their own personal learning experiences, which often carries a very high cognitive load (Vogel-Walcutt et al., 2011).

In order to compare the two theoretical perspectives, Vogel-Walcutt, Gebrim, Bowers, Carper, & Nicholson (2011) compare two online lessons, one that uses a CLT approach and one that uses a constructivist approach (problem-based learning). Their findings show that the CLT approach is efficient, in that the learners spend significantly less time on the learning tasks, but that there is not a significant difference in student knowledge between the two approaches. Overall, researchers conclude that a CLT approach leads to more *efficient* learning (Sweller & Chandler, 1991; Sweller & Cooper, 1985; Vogel-Walcutt et al., 2011).

Proponents of CLT note that learning designed based on a CLT approach is often better at helping less capable or more novice learners than learning designed based on constructivism (Kalyuga, 2007). When considered from a CLT perspective, learning activities that require learners to construct their own meaning carry a very high cognitive load, which leads to difficulties for learners with low prior knowledge. According to CLT, constructivist activities would be most successful among learners with high prior knowledge for whom the activities carry a lower intrinsic load (Vogel-Walcutt et al., 2011).

The Theory of Worked Examples

Clark and Mayer (2011) define a worked example as "a step-by-step demonstration of how to perform a task or solve a problem" (p. 224). Cognitive load theorists recommend the use of worked examples based on their observations of the worked example effect. The worked example effect states that "in initial skill acquisition, it is more favorable to learn from examples with worked solutions than to solve problems” (Schwonke et al., 2009, p. 258). The worked example effect is based on the theory that learning from examples helps learners “focus attention on problem states and their associated moves” (Sweller & Chandler, 1991, p. 353). Additionally, "worked examples prevent students from unproductive cognitive activities" (Schwonke et al., 2009, p. 265). This reduces extraneous cognitive load and aids in the development of schemas (Cooper & Sweller, 1987; Schwonke, Renkl, Salden, & Aleven, 2011). Although early research on worked examples focused on skill acquisition for procedural knowledge in well-structured situations, more recent research has looked at using worked examples in more conceptual settings and in ill-structured domains (i.e. Jarodzka, Van Gog, Dorr, Scheiter, & Gerjets, 2013; Nievelstein et al., 2013; Richey & Nokes-Malach, 2013; Rourke & Sweller, 2009). The chart given in Appendix A summarizes some of the recent research on worked examples. The following discussion will highlight some of the key findings of that research.

Recent research on worked examples. Much of the research on worked examples has been done using paper-based examples (e.g. Ayres, 2013; Boekhout et al., 2010; Gentner, Loewenstein, & Thompson, 2003; Hilbert & Renkl, 2009; Moreno & Valdez, 2007; Newton et al., 2010; Ngu & Yeung, 2012, 2013; Nievelstein et al., 2013; Quilici & Mayer, 1996; Richey & Nokes-Malach, 2013; Rourke & Sweller, 2009; Sweller & Cooper, 1985; Van Gog et al., 2011; Van Gog & Kester, 2012). The increased use of technology in educational settings has created a variety of new ways to create and present worked examples. Other formats of worked examples that have been studied include video-based examples (e.g. Kay & Edwards, 2012; Kay & Kletskin, 2012; Van

Gog, 2011; Wong, Leahy, Marcus, & Sweller, 2012), examples provided via an eLearning application (e.g. Corbalan, Paas, & Cuypers, 2010; Darabi, Nelson, Meeker, Liang, & Boulware, 2010; Jarodzka et al., 2013; Nievelstein et al., 2013; Scheiter, Gerjets, & Schuh, 2009; Vogel-Walcutt et al., 2011), examples provided through a cognitive tutor (e.g. Booth, Lange, Koedinger, & Newton, 2013; Reed et al., 2012; Salden et al., 2010; Schwonke et al., 2009, 2011), and in-class examples (e.g. Miller, 2010).

Static vs. animated examples. Scheiter et al. (2009) describe a comparative study on the effects of adding animations to algebra worked examples, contrasting traditional print worked examples with multimedia examples that utilized an animation, “where a realistic animation of the problem [was] morphed into a more abstract representation of the problem statement” (p. 492). Based on their study of 32 ninth-grade algebra students, the authors find that learners who viewed the worked examples with animations exhibited better problem-solving skills on far transfer problems, or problems that use similar skills in a method not previously encountered by the learners, than those who viewed the printed worked example. Wong et al. (2012) also replace static graphics with animations in worked examples designed to teach origami to children ages 10-11. Based on their study of 66 children, the authors conclude that animations are effective if they are provided in short segments. Providing long segments significantly increases the cognitive load and thus negates the positive effects of using the animation.

Reed et al. (2012) compare three different types of worked examples: static table, static graphic, and interactive graphic. The examples were used in a high-school Algebra

I course and were delivered on a computer. All three types of examples present the same information, but differ in how the information is organized. Static table examples display the example in a table where an explanation accompanies each step of the example, static graphics use an image to help demonstrate the concept, and interactive graphic examples use an interactive image as part of the example. Based on the results of their study involving 128 students, the authors conclude there is no difference among the three types of examples. All were found to be equally efficient, led to approximately the same number of errors on subsequent practice problems, and resulted in equivalent performance on paper-and-pencil posttests. The authors note that although these types of examples were competitors in the context of the study, they work well together and they recommend using a mixture of all three example-types to provide instruction.

Structure and design of worked examples. The structure, design, and organization of worked examples have been considered by several recent studies. For example, in a high school Algebra I context, Booth et al. (2013) find that providing learners with a mixture of both incorrect and correct examples is more effective than providing correct examples alone; students who are asked to explain incorrect examples gained more knowledge about the conceptual features of the problems. A series of articles by Van Gog and her colleagues (Van Gog et al., 2011; Van Gog & Kester, 2012; Van Gog, 2011) examined the sequencing of worked examples with practice problems. When considered collectively, the results show that viewing an example prior to completing a practice problem is more effective than completing the tasks in the opposite order. This is due to the reduced cognitive load provided by viewing the example first.

However, the analysis showed that the order the problems are presented in does not matter as much as how many opportunities learners are given to practice a particular problem solving technique. This emphasizes the fact that cognitive load is not the only consideration in determining how to help learners obtain new knowledge and skills. It is also essential to ensure that learners have opportunities to practice and to apply their new skills. This supports this proposed study in that the worked examples in the proposed study were created specifically in order to provide learners with more opportunities to practice.

Schwonke et al. (2011) conducted a study in a ninth-grade geometry class to determine whether the ratio of worked steps to to-be-solved steps in faded worked examples affects student learning. Based on their study of 125 students, they find that problem solving carries a higher cognitive load than worked examples, regardless of the number of worked steps. They also find that, for difficult problems, the highest post-test scores are found for examples with the highest number of worked steps. The authors observe an expertise reversal effect; the worked examples are most effective for content with which the learners were not familiar and are not as effective for familiar material. However, the expertise reversal effect was not as pronounced for conceptual content. The authors conclude, “worked examples might lose their effectiveness later – or potentially not at all – for the development of conceptual understanding” (p. 61).

Boekhout et al. (2010) determined whether or not an expert or a novice model should be used in a video worked example. The video worked examples in the study were used by 134 physiotherapy students to learn to diagnose physical complaints of patients.

The authors find that learners who learn from an expert model do significantly better on tests of far transfer than learners who learn from a novice model.

In a series of three experiments conducted with undergraduate statistics students Quilici and Mayer (1996) find that worked examples that emphasize the statistical structure of the problem are more helpful for schema development than examples that emphasize surface details of the problem. This was especially true for lower ability students. Corbalan et al. (2010) explain that when linear algebra examples are faded and students are asked to complete to-be-solved steps, the examples are most effective when feedback is provided on each step rather than just at the end of the problem. This conclusion is based on their finding that providing feedback at each step of the problem reduced the overall mental effort required to complete the problem. In a study of 54 eighth grade algebra students, Ayres (2013) determine that isolating elements of problems where learners are known to struggle and having learners practice those skills in isolation is beneficial to learners with low readiness. When considering case-based business examples, Gentner et al. (2003) find that asking learners to compare two examples and look for similarities helps increase transfer.

Benefits of worked examples. Several research studies have compared groups of learners who are provided with worked examples to groups of learners who are not provided with worked examples. These studies highlight some of the benefits of using worked examples for instruction. Some of these benefits include increased ability to develop mental mathematical models (Darabi et al., 2010), better course performance (Miller, 2010; Ngu & Yeung, 2013), increased flexibility in problem solving (Newton et al., 2010), more efficient learning (Nievelstein et al., 2013; Schwonke et al., 2009), and deeper conceptual understanding (Schwonke et al., 2009).

Self-explanation Prompts vs. Instructional Explanations

Theoretically, worked examples are beneficial to learning because they help emphasize the steps for solving a problem; focusing on the problem steps decreases extraneous cognitive load and helps facilitate the development of schemas. However, worked examples can be even more beneficial to learning when they are combined with additional instructional tools that increase germane load. Two such instructional tools are self-explanation prompts and instructional explanations.

Self-explanation prompts are questions added to worked examples to encourage learners to consider more carefully the details of the example (Biesinger & Crippen, 2010). The use of worked examples is thought to free up cognitive resources through a reduction in extraneous load. However, this does not guarantee that the learner will use the newly available cognitive resources in a productive way. Self-explanation prompts encourage learners to focus on features of the problem that are essential for the development of schemas, and therefore should increase germane load (Hilbert & Renkl, 2009). Because self-explanation prompts encourage learners to think about more than just the steps for solving a problem, self-explanation prompts can lead to better conceptual knowledge than the use of worked examples alone (Booth et al., 2013).

As an example, consider the self-explanation prompt given in the worked example shown in Figure 2. This related rates example is one that was designed for use in the proposed study. Note that the question displayed in the gray box on the screen asks the learner to explain when it is possible to substitute the value of a variable into an equation before taking its derivative. Understanding this concept is essential for completing the next step of this problem, as well as for completing other similar related rates problems.

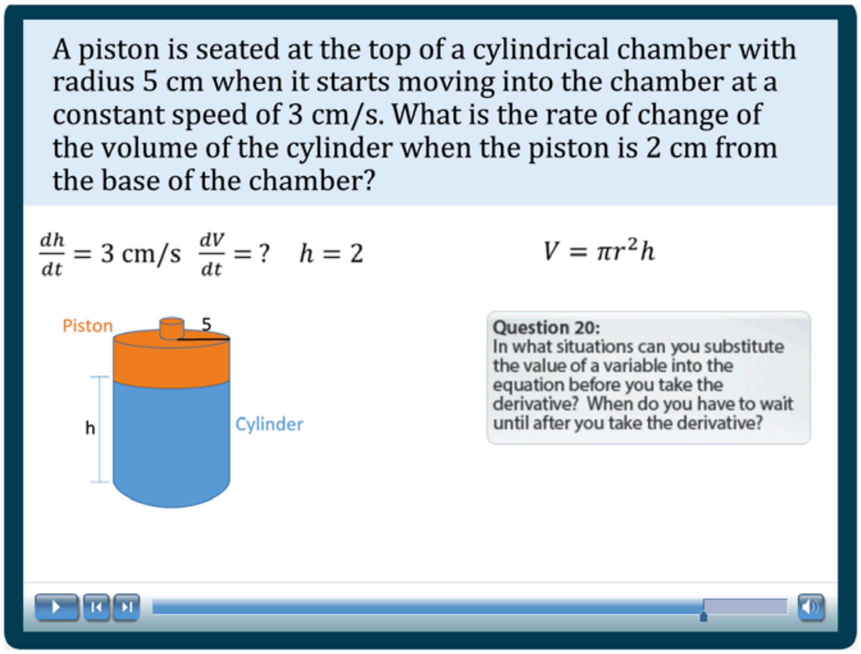

Figure 2. Screen shot from a multimedia worked example with self-explanation prompts.

Instructional explanations are defined as "explanations provided to facilitate example processing" (Wittwer & Renkl, 2010, p. 395). Wittwer and Renkl (2010) describe a meta-analytical review of research on instructional explanations. They find that adding instructional explanations to worked examples did lead to an increase in conceptual knowledge, although the effect size was only significant in studies where the learning domain was mathematics. However, they find that providing instructional explanations is not more effective than providing self-explanation prompts.

The worked example shown in Figure 3 is an example of a worked example with an instructional explanation. The instructional explanation shown in the gray box is intended to provide additional conceptual explanations for the next step of the problem. Although the learner could follow the steps of the problem without seeing this explanation, the instructional explanation provides additional information that help the learner understand *why* the next step of the problem is possible.

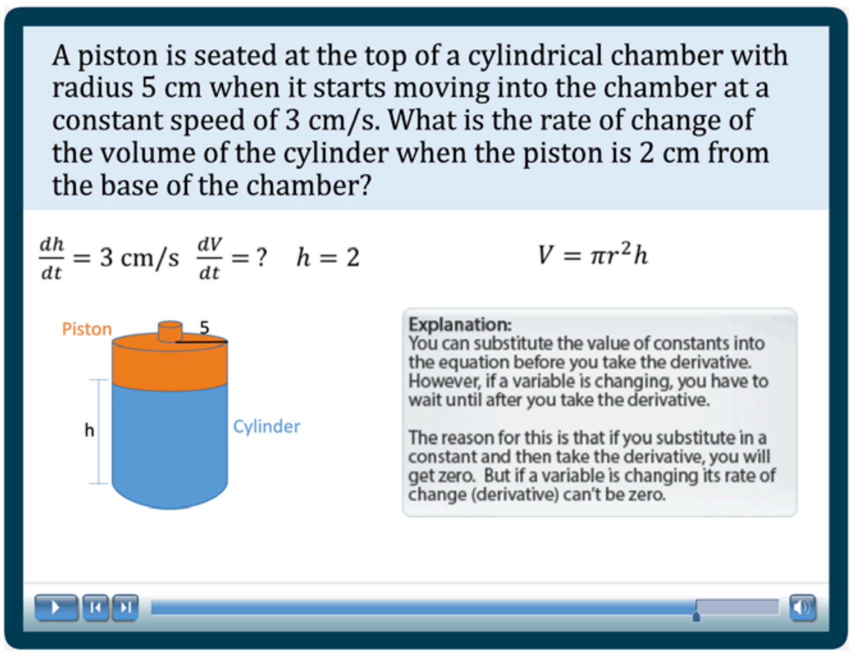

Figure 3. Example of a screen shot from a multimedia worked example with instructional explanations.

Comparing Figure 2 to Figure 3 illustrates the difference between self-explanation prompts and instructional explanations; although the prompt and the explanation have the same purpose, one is presented in question form that the learner must answer and the other is provided in explanation form. One concern related to the use of self-explanation prompts is that students with low prior knowledge might not have the necessary cognitive skills to construct good responses to self-explanation prompts (Richey & Nokes-Malach, 2013). Another concern is that the construction of incorrect responses to self-explanation prompts has been found to impair learning (Berthold & Renkl, 2009). Providing instructional explanations, rather than self-explanation prompts, is an alternative that is expected to address these concerns (Wittwer & Renkl, 2010).

Context of Prior Worked Example Research

Most research on worked examples has been conducted with high-school age learners in an algebraic context. As can be seen in the Summary Table of Worked Example Research included in Appendix A, of the 29 worked example studies included in this review of literature, fifteen of them address topics in the mathematical domain. Of those, only two (Corbalan et al., 2010; Miller, 2010) are at the level of calculus or beyond. The study conducted by Corbalan et al. (2010) was conducted in a linear algebra course (for which calculus is typically a prerequisite) and focuses on the most effective ways to provide feedback in a worked example and the study by Miller (2010) focuses on a method for using worked examples in an in-class setting in a calculus class with the teacher providing the example. These studies provide evidence that worked examples are beneficial for adult learners at the calculus level, but additional research is needed to address using worked examples in calculus in order to provide opportunities for learners to practice new knowledge and skills outside of class.

Most of the studies included in this review were conducted outside of a regular classroom setting. As can be seen in the chart in Appendix A, 20 of the 29 studies included in this review were conducted as a one-time experiment with student volunteers. Of the remaining nine studies that were situated in a classroom setting, all but one (Newton et al., 2010) were conducted for three or fewer lessons. The one exception was the study by Newton et al. (2010) that was conducted in a three-week long remedial/review summer school algebra course. Based on the literature included in this review, research has not been done using worked examples over an extended period of time within a classroom setting.

Most of the studies included in this review of literature used worked examples as a form of instruction. Only six (Booth et al., 2013; Corbalan et al., 2010; Darabi et al., 2010; Newton et al., 2010; Ngu & Yeung, 2012, 2013) of the 29 studies combine the use of worked examples with in-class instruction. By combining the worked examples with in-class instruction, the purpose of using worked examples shifts from providing instruction to providing opportunities to practice and to support (rather than replace) face-to-face classroom instruction.

The review of literature represented in Appendix A also indicates variability in techniques used in these studies for measuring and assessing cognitive load. None of the 29 worked example research studies included in this review used MIE. The use of MIE as a dependent variable was particularly illustrative in this study in light of the extended length of the learning phase of this study because MIE considers mental effort both during learning and during testing.

Gaps in the Literature

The results of this study add to the existing body of literature on worked examples by providing new information on the connection between the readiness of the learner and the use of self-explanation prompts and instructional explanations. The study helps address concerns that self-explanation prompts may carry too high of a cognitive load for learners with low readiness, and thereby helps determine whether self-explanation prompts or instructional explanations are best for students with low, medium, and high readiness.

The study provides more information on using worked examples for practice (rather than instruction) in the more abstract setting of calculus and with adult learners. Additionally, the study provides insight on the use of worked examples over an extended period of time while situated in a classroom setting.

CHAPTER III

Methodology

This quantitative experiment was designed in order to answer the six research questions posed earlier in this paper (see page 6). The goal of the experiment was to determine the efficiency (measured using MIE) and effectiveness (measured using student performance) of three different types of worked examples intended to provide opportunities for students to practice in a face-to-face calculus course. All three groups saw the same problems; however, for Groups #1 and #2, approximately half of the practice problems were replaced with worked examples. Prior to completing practice problems, learners in Group #1 viewed worked examples with self-explanation prompts and learners in Group #2 viewed worked examples with instructional explanations. As a form of control, learners in Group #3 did not view any worked examples. Data analyses were conducted to determine which of the three types of practice problems had the highest multidimensional instructional efficiency, which produced the highest student performance, and whether there was an interaction effect between type of practice and student readiness.

Research Design

As illustrated in Table 1 (see page 7), the study used an experimental, pre-test/post-test control group design to answer the six research questions. The study included two independent variables: type of homework completed by the student (traditional homework, worked examples with instructional explanations, or worked examples with self-explanation prompts) and readiness of the student (low readiness, medium readiness, or high readiness). There were two dependent variables: student performance and multidimensional instructional efficiency (MIE). As described by Tuovinen and Paas (2004), MIE is a function of mental effort during the learning phase, mental effort during the testing phase, and student performance. MIE incorporates all three of these individual measures into one computational measure to assess the efficiency of the instructional technique. There were also two covariates: homework completion rate and prior knowledge of calculus (as measured by a pre-test).

Population and Sampling

The population of interest in this study was university-level students taking their first semester of calculus at a private northwestern university. The sample consisted of students from two sections of Calculus I during the winter 2015 semester. As will be discussed in more detail later in this chapter, the students who register for Calculus I each semester have a fairly consistent demographic. As such, this convenience was representative of the population of interest. Each section of Calculus I has a maximum class size of 49 students. During the semester in which the study took place, a total of 93 students from two sections of Calculus I signed an informed consent form and agreed to participate in the experiment. According to the results of a *G*Power* a priori power analysis (Faul, Erdfelder, Lang, & Buchner, 2007), for a moderate effect size of 0.35 (measured using Cohen's f), the total sample size would need to be 83 students to have 80% power. Therefore the sample of 93 students should have been large enough to statistically detect a meaningful difference among experimental groups, if one existed.

Materials

Examples have traditionally played a big role in the teaching and learning of mathematics (Sweller & Cooper, 1985). Worked examples are commonly included in textbooks or presented by an instructor during face-to-face classes in order to help students learn to solve mathematical problems. The worked examples used in this study were similar to such examples in that they provided students with worked out solutions to mathematical problems. However, the worked examples used in this study had characteristics and advantages that went beyond those provided by traditional worked examples included in textbooks or presented by an instructor. The multimedia nature of the experimental examples provided a more interactive experience for the learners. The learner was able to control the speed of the example, to skip forward or go back, and to watch the example multiple times. Additionally, unlike traditional worked examples that don't necessarily require a learner response, the experimental examples were embedded into the course LMS and learners were expected to respond to questions related to each example. This additional scaffolding helped ensure that learners interacted with the examples in meaningful ways.

Two different versions of each example were created for use in this study. One version of the example utilized self-explanation prompts, while the other version used instructional explanations. Figures 4 and 5 show a comparison of the same problem as presented to each of the three experimental groups. As can be seen in the figures, all three groups were exposed to the same problem. However, as can be seen by the screenshot on the left in Figure 4, the worked example for Group 1 included a question that students answered by typing their response in the learning management system. In this particular example, students were asked to explain the meaning of the answer obtained through the calculations shown in the example. The screenshot on the right in Figure 4 shows the same example as prepared for Group 2. Notice that the only difference between the two examples is that the example for Group 2 provided students with an explanation of the meaning of the answer obtained in the example, rather than asking them to provide an explanation. Figure 5 shows the same problem as provided to the control group. In this case, students were asked to solve the problem without an example being provided.

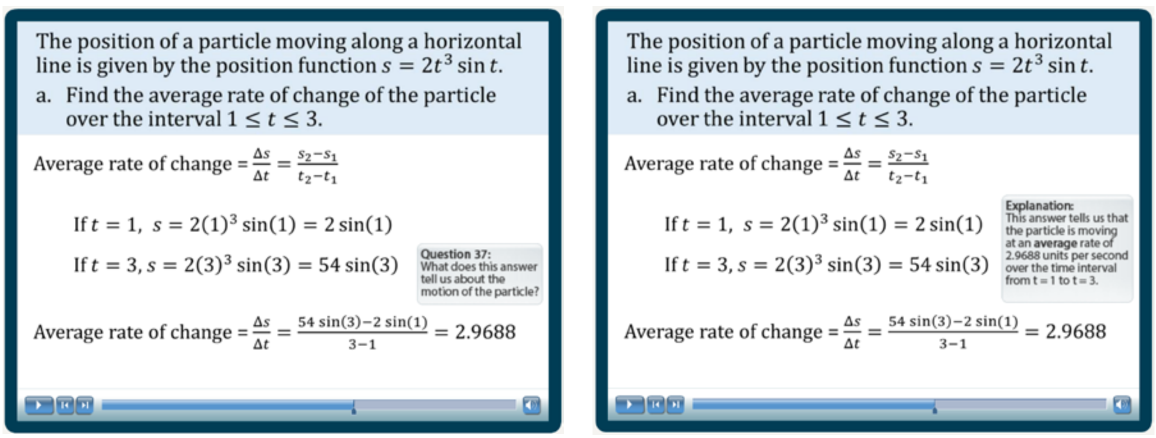

Figure 4. Side-by-side comparison of the two types of worked examples. Although both examples demonstrate how to solve the same mathematical problem, the screenshot on the left shows a self-explanation prompt while the screen shot on the right shows an instructional explanation.

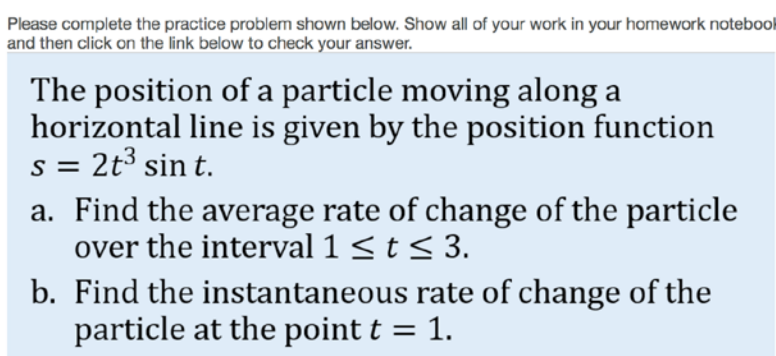

Figure 5. Screenshot from a practice problem, as used for Group 3.

Table 4

Distribution of fully worked examples, partially worked examples, and practice problems used in the study.

	Group 1: Self-explanation Prompts	**Group 2:** Instructional Explanations	**Group 3:** Practice Problems (control)
Number of fully worked examples	47	47	0
Number of partially worked examples	30	30	0
Number of practice problems	69	69	146

As can be seen in Table 4, some of the worked examples were fully worked examples while others were partially worked examples. As suggested by the name, fully worked examples showed a complete solution to the problem, whereas partially worked examples provided solutions to some steps, but asked learners to fill in the details on other steps. Some partially worked examples asked learners to complete intermediate steps before the example proceeded with the rest of the solution (see Figure 6 for an example); while other partially worked examples began the problem and then asked the learner to complete the remainder of the problem (see Figure 7 for an example). For the treatment groups the assignments were faded, meaning that for each type of problem the problems on the assignment gradually progressed from fully worked examples, to partially worked examples, to practice problems. Appendix B includes a list of all the mathematical problems used in the study; their classification as a fully worked example, a partially worked example, or a practice problem; and their alignment to the objectives of the unit.

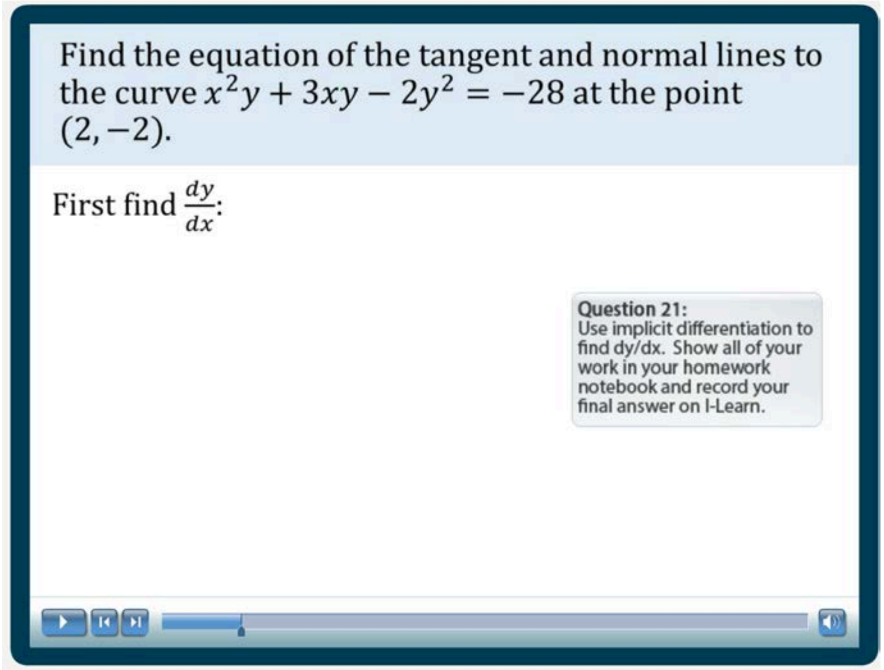

Figure 6. Screenshot from a partially worked example. Notice this example asks the learner to find dy/dx. This is the first step of the problem and the example will continue with the rest of the solution after learners complete this step.

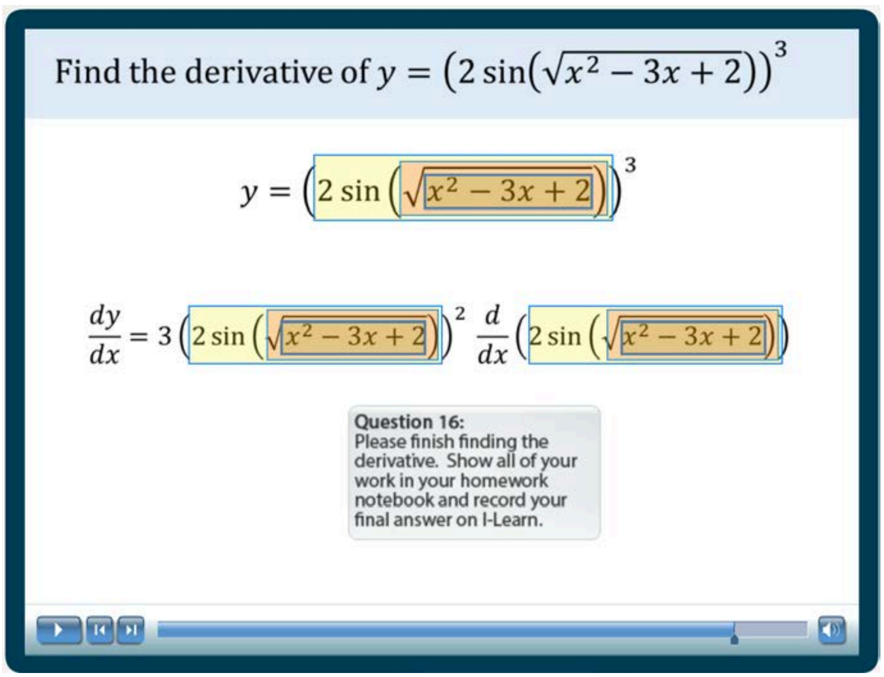

Figure 7. Screenshot from a partially worked example. Notice this worked example sets up the problem and then asks the learner to complete the remaining steps to finish the problem.

Instructional Design of Materials

The worked examples used in this experiment were designed using the Kemp Model (Morrison, Ross, Kalman, & Kemp, 2011). The Kemp Model was originally developed by Kemp (1985) and then later improved and redesigned by Kemp, Morrison, and Ross (1994). This study used the most current version of the Kemp Model as described by Morrison et al. (2011). As illustrated in Figure 8, the Kemp Model consists of nine design elements (indicated by the circles in the center of the figure) and eight processes that occur throughout the design (indicated in the two outer rings).

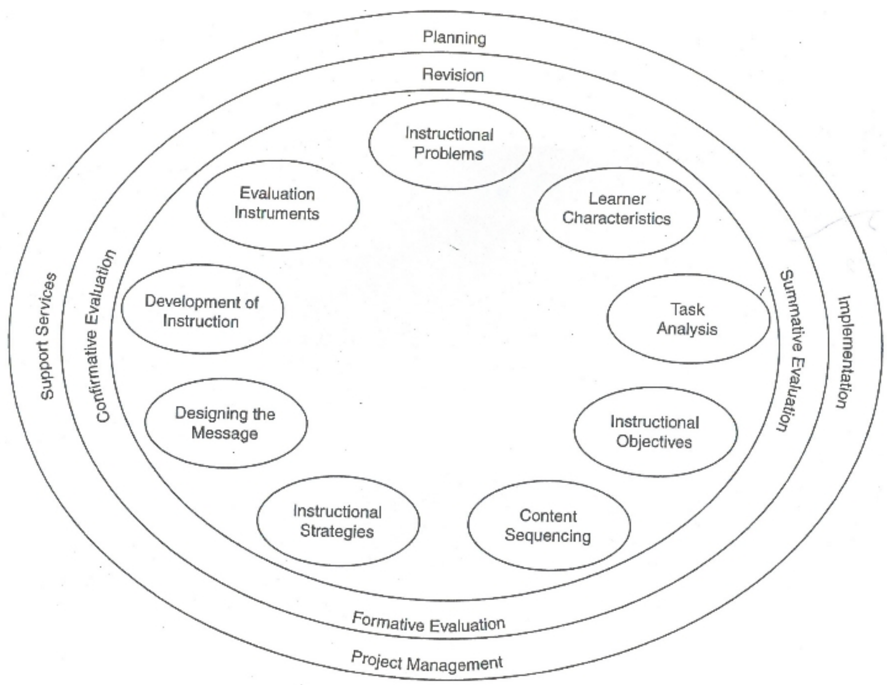

*Figure 8***.** The Kemp Model (Morrison et al., 2011, p. 12). The nine design elements are represented with circles in the center of the diagram and the two outer rings represent the eight ongoing processes.

The instructional design process was carried out beginning in June 2014 with analysis of the instructional problem and of the learners. The design and development of the worked examples was conducted between July and September 2014. In September and October 2014 the developed examples were tested during a piloting semester. Based on the results of the piloting semester, the examples were revised and finalized during November and December 2014 in order to be ready for implementation in the experiment in January 2015. The experiment was conducted in January and February 2015.

Instructional problem. The problems and worked examples that were developed were intended to provide opportunities for calculus learners to practice applying new knowledge and skills. Traditionally, calculus students in face-to-face classes are given a series of practice problems after hearing an in-class lecture on a new topic (Sweller & Cooper, 1985). However, the practice problems then carry a very high cognitive load for many learners as they are asked to solve problems with little or no scaffolding. Additionally, students registering for university-level calculus courses are often under-prepared (Pyzdrowski et al., 2013; Rasmussen et al., 2012). For these students, practice problems carry an even greater cognitive load (Kay & Kletskin, 2012). Learners often use their entire available cognitive load to figure out *how* to solve the problem and have few cognitive resources, or the motivation, to focus on *why* the technique works.

In order to provide increased scaffolding for learners before they are given independent practice problems, this study compared two different types of worked examples to traditional problem solving. The first type of examples, worked examples with self-explanation prompts, provided learners with questions to consider that helped them focus on the concepts behind problems and provided motivation for the learners to think about why processes work. However, there was concern that learners with low calculus readiness may not have the cognitive ability to adequately answer the questions posed in the self-explanation prompts. As such, worked examples with self-explanation prompts were compared to worked examples with instructional explanations. These examples also provided helpful scaffolding for the learners without the increased cognitive load required by self-explanation prompts.

Learner characteristics. In order to determine demographic information for students who typically register for Calculus I, a historical sample of two previous sections of the course was analyzed (see Appendix C). The combined sample of 96 students who registered for Calculus I in the previous fall and spring semesters showed that 12.5% of the students were female and 87.5% of the students were male. The majority of the students were freshmen (40.6%) and sophomores (33.3%), although a few juniors (19.8%) and seniors (6.3%) also registered for the course. The majority of the students were majoring in STEM fields (science, technology, engineering, and math) (82.3%) with the most common major being engineering (49.0%). International students, for whom English was not their first language, comprised 10.42% of the class.

In order to ensure that these statistics are consistent across semesters, the two courses comprising the sample were compared. Chi-square tests for homogeneity of populations were used to compare the two classes in four different categories: Gender, Class Standing, STEM/Non-STEM majors, and International students. Table 5 shows the results of the chi-squared tests. As can be seen in the table, none of the tests were significant at the $\alpha = .05$ level of significance. This indicates that the two sections of the course had similar demographics. This provided evidence that the demographic information collected as part of the learner analysis gave a reasonable description of the sample that was later used in the experiment.

Table 5

Comparison of Spring and Fall sections of Calculus I. None of the P-values were significant at the $\alpha = 0.05$ level of significance. This indicates the two sections of the course have similar demographics.

Category	**Spring**	**Fall**	**Chi-Square**	***p*-value**
Male	87.2%	87.8%	$\chi^2(1) = 0.006$	$p = .938$
Female	12.8%	12.2%		
Freshman	29.8%	51%	$\chi^2(3) = 5.116$	$p = .164$
Sophomore	42.6%	24.5%		
Junior	21.3%	18.4%		
Senior	6.4%	6.1%		
STEM Major	87.2%	77.6%	$\chi^2(1) = 1.544$	$p = .214$
Non-STEM	12.8%	22.4%		
International	12.8%	8.2%	$\chi^2(1) = 0.545$	$p = .461$
Non-international	87.2%	91.8%		

Calculus I has a prerequisite of precalculus (or as an alternative students may take both college algebra and trigonometry). Some of the students have taken precalculus (or college algebra/trigonometry) in college while other students may have taken precalculus in high school. For example, during the piloting semester, 44% of the students had taken precalculus in high school; 51% had taken precalculus at the university where the study took place; and 5% had taken precalculus at another college or university. For some students, their experience with precalculus occurred in the prior one or two semesters, but for other students, their experience with precalculus could have been several years prior to registering for Calculus I. During the piloting semester, only 41% of the students had completed precalculus in the six months prior to the beginning of Calculus I; the mean number of months since the student had taken precalculus was 39 months. Based on information provided by the mathematics department about the learners in this course, many of the learners had taken another calculus course prior to taking Calculus I. Of those learners, many took calculus in high school, but did not earn college credit for it. Others failed calculus on their first attempt and were retaking the course

Instructional objectives. The examples developed for this project covered one unit of the Calculus I course. The Calculus I course consists of five units: 1) Limits and Continuity, 2) Differentiation, 3) Applications of Differentiation, 4) Integration, and 5) Applications of Integration. The Mathematics Department sets the curriculum for the course and selected the topics to be included in the course. As such, changing the topics selected for inclusion in the course was not an option for this project. This project covered the second unit on differentiation. The unit is divided into four lessons each consisting of three sub-lessons. Several instructional objectives were identified for each of the sub-lessons. A complete list of the forty instructional objectives for the unit on differentiation can be found in Appendix D.

The instructional objectives were revised and validity was verified through a survey of three subject matter experts. Although minor changes were made to the objectives based on the recommendation of the subject matter experts, all three subject matter experts indicated that the objectives were appropriate for this unit. Complete survey results are given in Appendix E.

Task analysis. A task analysis was conducted to determine what tasks learners would need to complete in order to meet each of the objectives. For each objective, a series of tasks was identified and then each task was categorized as to knowledge type, difficulty, duration, and importance. The knowledge type was classified using three classifications of knowledge types: declarative, procedural, and structural (Jonassen, Tessmer, & Hannum, 1999). Declarative knowledge is defined as knowledge of facts; procedural knowledge is knowledge of processes required to perform a task; and structural knowledge is knowledge of concepts and how different concepts are related. The difficulty, duration, and importance were each rated as low, medium, or high. These somewhat subjective classifications were initially assigned by the instructional designer and then validated by subject matter experts.

The task analysis was revised and face validity was verified through a survey of subject matter experts. With a few minor exceptions, the subject matter experts agreed that each task was required, that no tasks were missing, and that the tasks were appropriately categorized. Minor changes were made to the tasks in order to address the suggestions made by the subject matter experts. The complete, final version of the task analysis is given in Appendix F

Content sequencing. Although the topics to be covered in the unit on differentiation were mandated by the mathematics department, the researcher determined the sequencing of the content. The content in the unit is very hierarchical and concepts build one upon the other (see the learning hierarchy in Appendix G). Based on this learning hierarchy, a concept-related sequencing scheme was selected for this project. The content was divided into four lessons that each covered one of four overarching themes of the unit: 1) the concept of the derivative, 2) differentiation rules, 3) rates of change, and 4) differentiation of inverses. Each lesson was further divided into three sub-lessons. The practice problems were intended to provide opportunities for the learners to practice after the topic was covered in class. Originally, each sub-lesson was expected

Table 6

Content Sequencing of the Differentiation Unit

Lesson #: Lesson Content		**# of Class Periods**	**# of Examples /Practice Problems**
Lesson 1: The Concept of the Derivative			
Lesson 1A:	Tangent Lines	1	13
Lesson 1B:	Derivatives at a Point	1	10
Lesson 1C:	The Derivative as a Function	1	12
Lesson 2: Differentiation Rules			
Lesson 2A:	Basic Rules	1.5	12
Lesson 2B:	Trigonometric Rules	1.5	15
Lesson 2C:	The Chain Rule	1	12
Lesson 3: Rates of Change			
Lesson 3A:	Applications	1	11
Lesson 3B:	Implicit Differentiation	1.5	15
Lesson 3C:	Related Rates	1.5	12
Lesson 4: Derivatives of Inverses			
Lesson 4A:	Exponential and Logarithmic Functions	1	11
Lesson 4B:	Logarithmic Differentiation	1.5	12
Lesson 4C:	Inverse Trig Functions	1.5	12

to be covered in one class period. However, based on the results of a pilot of the sequencing scheme and practice problems during the Fall 2014 semester, it was determined that more time was needed to adequately cover some topics. As such, the schedule was adjusted. As shown in Table 6, fifteen class periods were needed to cover the unit. Because the Calculus I course met five days per week, this comprised three weeks of instruction.

Designing the message. The design of the worked examples was based on recent research on creating effective worked examples. Learner controls were provided within the examples that allowed the learner to pause, go forward, and go backward. A progress bar was also included. The examples were divided into short segments and the video automatically paused after each step of the worked example (Wong et al., 2012). The examples utilized equation animations (where long or complicated mathematical equations appeared in stages in sync with the audio rather than all at once) and animated graphs (Scheiter et al., 2009; Wong et al., 2012). The examples were faded in that they progressed from fully worked examples, to partially worked examples, to practice problems (Schwonke et al., 2011).

The calculus problems to be used in the worked examples were chosen carefully to align with the objectives and the tasks given in the task analysis. In order to ensure that the user interface was easy to use, four prototype examples were developed and reviewed by subject matter experts and students. Based on their feedback, a Worked Example Wireframe was developed that detailed all formatting and interface features to be used in the examples (see Appendix H).

Development of the instruction. The examples were developed using a combination of PowerPoint slides, Adobe Captivate, a Livescribe Smartpen, and screen captures of animations and demos created with Mathematica. The library of examples can be found at this link: www.dromrell.com/WorkedExampleLibrary/index.htm

Evaluation instruments. The differentiation post-test was used to determine whether or not the learners met the objectives of the course. More specific details on the post-test are given in the instrumentation section of this chapter.

Eight ongoing processes. The Kemp et al. Model describes eight ongoing processes that should be done throughout a design project. These eight processes include formative, summative, and confirmative evaluation; revision; planning; project management; support services; and implementation (Morrison et al., 2011).

Evaluation of the effectiveness of the worked examples was conducted in several different ways. Initially, prior to development, four prototype examples were developed and reviewed by four subject matter experts and six students. Based on their feedback, the examples were revised and prepared for development (see Appendix I for the complete survey results). Some of the major revisions made based on this survey were: 1) improved learner controls, including adding a pause button and a progress bar; 2) a change to the way videos were imported to the Captivate file in order to improve navigation; 3) improved audio calibration to ensure the volume was consistent throughout the example; 4) increased font size throughout the examples; and 5) revision of LMS assignment settings.

Evaluation of the examples continued after development as the examples were used during the piloting semester in Fall 2014. Based on the results of the pilot and feedback from learners involved in the pilot, further revisions were made to the examples prior to their implementation in the experiment. In particular, changes were made to the method users used to report their answers to the practice problems and the segmenting of the examples was refined to reduce the number of times the example paused during viewing.

Support services during the development of the examples for the project were provided by the Faculty Technology Center and by employees on the University Instructional Development Team.

Instrumentation

Readiness assessment. The initial skills test selected for use in the study was the Calculus Readiness Test developed by the Mathematics Diagnostic Testing Project (MDTP). The MDTP tests were created by schools in the University of California system and were used by permission for this study (Mathematics Diagnostic Testing Project [MDTP], n.d.). The purpose of the MDTP Calculus Readiness Test is to assess how well students are prepared for a first-semester calculus course by assessing learners' understanding of prerequisite topics (see Table 7 for a list of topics covered). The test contains 40 multiple-choice questions and was given in a proctored setting during the first week of the course.

The first version of the MDTP test was created in 1986 and numerous studies have been done to establish the reliability and validity of the test. The MDTP Manual

Table 7

Topics included on MDTP Calculus Readiness Test (MDTP, n.d.)

Topic	**# of Questions**
Rational Expressions and their Graphs	5
Exponents and Radicals	4
Linear Equations and Inequalities; Absolute values and their graphs	8
Polynomials and Polynomial Functions	5
Functions	5
Trigonometry and Geometry	7
Logarithmic and Exponential Functions	6

(MDTP, n.d.) gives numerous references to independent studies that have been conducted concerning the validity and reliability of the MDPT Calculus Readiness Test. In particular, the test has been found to have face validity, content validity (in that it accurately assesses the topics necessary for calculus), and predictive validity (in that it is a good predictor of future success in calculus). Validity coefficients were reported for eighteen different studies of predictive validity conducted at various universities in California (MDTP, n.d.). The highest correlation reported was .61 and was reported based on a sample of 459 students at UCLA. The lowest correlation reported was .30 and was based on a sample of 143 students at Sacramento State. Most of the reported correlations (11 out of 18) were between .40 and .61. These values indicate that the assessment has fair predictive validity. The reliability of the assessment was reported based on eleven different studies (MDTP, n.d.). Reliability was measured using the Kuder-Richardson 20 (KR20) reliability coefficients. The KR20 values ranged from .76

to .87, with all but one of the studies having a value above .84. Because these values are all close to 1.00, they indicate the assessment is reliable.

Mental effort ratings. Learners reported the mental effort required to complete problems both while practicing on their homework and while taking the test. Mental effort was measured using Paas' (1992) Mental Effort Measurement Scale (PMEMS). As noted in Chapter II, PMEMS is widely used, has been found to be reliable and valid, and is sensitive to small changes in mental effort. Paas et al. (1994) claimed the measurement scale was reliable based on two different studies where the values of Chronbach α were found to be .90 and .82, respectively. The same two studies were also used to establish the sensitivity of the rating scale. Using an ANOVA test to compare the mental effort rating scale of two groups with known differences in mental effort, Paas et al. (1994) found there was a statistically significant difference in both the first, $F(2, 39) = 9.14$, $p < .05$, and second, $F(1, 56) = 6.5$, $p < .025$, studies.

In this experiment, the mental effort ratings were collected after each learning task (e.g. practice problem or worked example). Additionally, the mental effort rating was collected after each exam question on the post-test. Once students (in all three experimental groups) completed a learning task they were asked to rate the mental effort involved on a scale of 1 – 9 with 1 being very, very low effort, and 9 being very, very high effort. The decision to ask learners to rate their mental effort after every learning task was based on the results of a study by Van Gog et al. (2012). They found that when, and how often, the measurements of mental effort were taken had an effect on the results of a study and that it was preferable to collect measurements after each learning task.

Performance assessments. As detailed by Van Gog and Paas (2008), performance measurements taken during the learning phase of instruction cannot be considered predictive of the final learning outcomes of the instruction. Rather, in order to reliably measure performance, a final assessment must be given at the completion of the instruction. Based on this recommendation, performance during the learning phase was not considered during the analysis of the data in this study. Rather, pre- and post-tests were administered at the beginning and end of the unit. A list of questions for each of these performance assessments, as well as their alignment to the instructional objectives, can be found in Appendix J. The pre- and post-test were very similar and consisted mostly of similar problems with different numbers. The purpose of the pre-test was to confirm that the three treatments groups had similar levels of knowledge about differentiation at the beginning of the treatment. Including the pre-test score as a covariate in the data analysis controlled for variation in prior knowledge. The post-test was given in order to determine student performance and to assess whether or not the learners have met the objectives for the unit.

Because these assessment instruments were designed specifically for this study, the reliability and validity of the assessment needed to be established. As explained by Worthen, White, Fan, and Sudweeks (1999), establishing content validity is a two-part process. First, the assessment was systematically designed by the researcher to cover each learning objective of the unit. See Appendix J for a list of the assessment questions and their alignment to the learning objectives. Second, three subject matter experts reviewed the assessments and based on their suggestions minor changes were made to the assessment. All three subject matter experts agreed that the assessment was appropriate for determining whether or not the learners had met the objectives.

The piloting semester of the experiment was used to help establish the reliability of the assessment. The assessment was administered to 39 students during the piloting semester. A rubric was developed to use with the grading (see Appendix K). A teaching assistant who was trained on the use of the rubric graded the exams and the exams were also graded by the researcher. The correlation between scores assigned by the teaching assistant and the scores assigned by the researcher was computed to be $r = .97$ and indicates a high level of interrater reliability. As a measure of the internal consistency reliability of the tests, Cronbach's alpha was computed using SPSS and found to be $\alpha = .87$. This value of α is indicative of an instrument with relatively high internal consistency reliability.

Procedures

Table 8 provides an overview of the four phases of the study. On the fifth day of class, the instructor described the study to the students and invited them to complete an informed consent form. Students who declined to participate in the study were assigned to the control group, but none of their data was included in the data analysis.

During the first week of the semester, the MDTP readiness test was administered in order to determine the calculus readiness of learners. The top third of the students were classified as having high readiness, the middle third with medium readiness, and the bottom third as having low readiness. Students were randomly assigned to experimental groups using the alternate ranks method as described by Myers, Well, and Lorch (2010).

All random assignments were done using the random number generator in Excel. Using the alternate ranks method, students were ranked based on their score on the readiness test. Ties on the readiness test were broken using a random ranking of students with the same score. Then, the top three students were randomly assigned to the three experimental groups. Then the next three students in the readiness ranking were randomly assigned to the three experimental groups, and so on. This continued until all students were assigned to an experimental group. This procedure for assigning students to experimental groups helped ensure that students with very high readiness or very low readiness were equally distributed among the three groups (Myers et al., 2010)

Table 8

Summary of Procedures

Phase	Description	Timeframe
Preliminary Procedures	• Distribute and Collect Consent Form	• Fifth Day of Class
Initial Assessment Phase	• Administer MDTP - Initial skills readiness assessment • Randomly assign participants to experimental groups, by readiness level. • Administer Differentiation Pre-test in the testing center	• First week of class • After the learners have taken the exam on the first unit in the course. • During week 2 of the calculus course
Learning Phase	• Students complete homework assignments that correspond to their assigned treatment group. Mental effort is assessed on each problem.	• Weeks 4-6 of calculus course (see schedule in Table 6)
Testing Phase	• Students take Differentiation Post-test in the testing center. Mental effort is assessed on each problem.	• At the end of week 6 of calculus course

During the second week of the course, the students took a Differentiation Pre-test in order to determine their prior knowledge of differentiation. The results of the pre-test were used to verify that the treatments groups each had approximately the same level of prior knowledge regarding differentiation.

The learning phase of the study lasted approximately three weeks (see sequence in Table 6). During that time, the students in each group completed practice assignments for each lesson. The practice assignments were delivered online through the LMS. Although all groups had the same problems on each of the practice assignments, the format of the assignment problem was different for each group. Approximately half of the problems for learners in Groups #1 and #2 were worked examples with the remaining problems being practice problems. The learners in Group #1 viewed worked examples with self-explanation prompts while the learners in Group #2 viewed worked examples with instructional explanations. Group #3 (the control group) was given only practice problems and did not view any worked examples. Students in all experimental groups were also asked to rate the mental effort required for each problem on each homework assignment; this rating was inserted into the homework via a multiple choice question at the end of each practice problem in the LMS.

The final phase of the study was the testing phase. All learners took the same Differentiation Post-test to assess their knowledge. The paper/pencil post-test was administered in a non-timed, proctored setting in the university’s testing center. Using a rubric, each test item was scored on a scale of 0 (no answer) to 4 (correct concept selected with correct answer) by the same teaching assistant who graded the pre-test. In order to determine the mental effort during the testing phase, each problem on the exam asked the learner to rate the mental effort required for that question.

Data Analysis

After the data were collected they were analyzed in order to answer the six research questions posed by this study. Mental Effort was measured during both the learning and testing phases of the experiment. The mean mental effort during learning (E_L) was found by averaging the 146 mental effort ratings reported by each learner during the learning phase and the mean mental effort during testing (E_T) was found by averaging the 20 mental effort ratings reported by each learner on the post-test. Additionally, based on the result of the post-test, a performance score (P) was computed for each learner.

In order to compute the multidimensional instructional efficiency (MIE) measure for each treatment, the grand mean and standard deviation were computed for student performance, as measured by post-tests scores, and each type of mental effort using data from the combined sample of all experimental subjects. Using this grand mean and standard deviation, the z-score for each subject for student performance (z_P), mental effort during the learning phase (z_{EL}), and mental effort during the testing phase (z_{ET}) were each computed. The z-score values were used to compute the MIE measure for each student using the formula: $MIE = (z_P - z_{EL} - z_{ET})/\sqrt{3}$ (Tuovinen & Paas, 2004). The individual MIE scores for students in each group were then averaged to find an average MIE score for each of the three experimental treatments. An MIE score of zero corresponds to neutral efficiency where mental effort is balanced with performance. A positive MIE value would indicate that the mental effort is lower relative to performance, whereas a negative MIE value would indicate the mental effort was high relative to performance. Positive MIE values were ideal in this context.

As described in Chapter IV, Research Questions #1, 2, and 3 were answered using a two-way ANCOVA. The two-way ANCOVA looked at the multidimensional instructional efficiency and determined whether there was a main effect due to readiness, a main effect due to the type of practice, and whether or not there was an interaction effect for readiness and type of practice. Pre-test scores and practice completion rates were used as covariates in the analysis. Similarly, research questions #4, 5, and 6 were also answered using a two-way ANCOVA. However, this time the dependent variable was student performance. Again, the ANCOVA test determined if there was a main effect due to readiness, a main effect due to the type of practice, and whether or not there was an interaction effect between readiness and type of practice, using pre-test scores and practice completion rates as covariates. For all research questions, a planned Dunn-Bonferroni analysis was conducted to compare the treatment means for the main effects that were found to be significant. If an interaction effect was found to be statistically significant, then simple main effects analysis were employed prior to mean comparisons. Simple main effects analyses, where needed, were followed by post hoc mean comparisons using the Tukey method. All follow-up statistical tests were conducted at a family-wise error rate of $FWE = .05$.

CHAPTER IV

Results

This study was designed to determine the efficiency and effectiveness of worked examples as a tool for providing calculus students with opportunities to practice. In particular, the study compared worked examples with self-explanation prompts to worked examples with instructional explanations, and examined whether using a combination of worked examples and practice problems was more efficient and effective than problem solving alone. Additionally, the study examined whether the readiness of the learner impacted the efficiency and/or effectiveness of worked examples.

There were six research questions included in the study. The first three questions investigated the *efficiency* of the worked examples as measured using MIE.

- Research Question #1: Is there a significant main effect of type of practice (worked examples with self-explanation prompts, worked examples with instructional explanations, or traditional homework) on multidimensional instructional efficiency (MIE) as measured using the MIE formula developed by Tuovinen and Paas (2004)?
- Research Question #2: Is there a significant main effect of calculus readiness (low readiness, medium readiness, or high readiness) on MIE as measured using the MIE formula developed by Tuovinen and Paas (2004)?
- Research Question #3: Is there an interaction effect between calculus readiness and type of practice on MIE?

Questions four through six explore similar questions about the *effectiveness* of the worked examples, as measured by post-test scores.

- Research Question #4: Is there a significant main effect of type of practice (worked examples with self-explanation prompts, worked examples with instructional explanations, or traditional homework) on student performance as measured by posttest scores?
- Research Question #5: Is there a significant main effect of calculus readiness (low readiness, medium readiness, or high readiness) on student performance, as measured by posttest scores?
- Research Question #6: Is there an interaction between calculus readiness and type of practice on student performance?

This chapter will describe the results of the study that was conducted to answer these research questions. In particular, the discussion will include a description of the sample, a summary of the results of each assessment included in the experiment, and the findings of the data analysis used to answer the research questions.

Description of the Sample

The sample was made up of students registered for two sections of Calculus I at a large private university in the Northwestern United States during January and February 2015. One of the sections had 51 registered students and the other section had 52, giving a total possible sample of 103 students. Of those students, 11 declined to participate in the study and one student dropped the course during the second week of the experiment.

One additional student was removed from the sample because of her extensive prior knowledge of calculus concepts and theory. As a senior majoring in Applied Mathematics, she had taken most of the math courses required for a bachelor's degree in math, including real analysis (advanced calculus). She was retaking calculus to improve her grade, but as demonstrated by a pre-test score that was 30 percent higher than any other student in the class, her prior knowledge of mathematics was very atypical for a first semester calculus student. Because the student really did not belong in the population of Calculus I students, the decision was made to remove her information from the data set (Mitchell & Jolley, 2009).

After the learners had completed the unit, two more students were removed from the sample. The first student was removed because he did not take the pre-test and only completed 39% of the assigned homework problems in the unit. For the entire sample, the homework completion rate was 95% ($SD = 8\%$) and his homework completion rate was 18% lower than any other student in the sample. The second student was removed from the sample because the testing center reported that the student was found to be cheating on the post-test. After removing these students from the study, the final sample size was 88 learners. This is larger than the sample size of 83 required by the power calculations presented in Chapter III (see page 46).

Figure 9 provides a visual representation of the alternate ranks randomization technique that was used in this experiment. The students were ranked based on their score on the readiness exam. Any ties were randomly broken. The students were then assigned a ranking number from #1 to #92. The numbers in the boxes on the figure represent these ratings. The students were then grouped in blocks of three and the students within each block were randomly assigned to one of the three treatment groups, as shown in the figure. The blocks were then divided into thirds to classify them as high, medium, and low readiness.

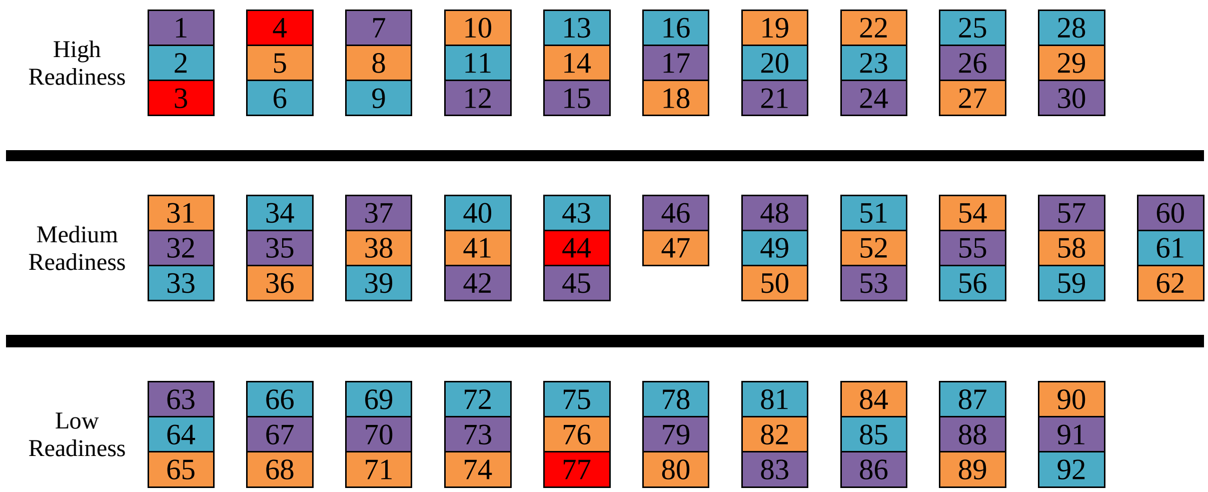

Figure 9. Diagram showing the alternate ranks randomization scheme. The number in each box represents the rank of the learner on the MDTP Calculus Readiness Assessment. The color of the box represents the assignment to treatment groups: Group #1 is purple, Group #2 is blue, and Group #3 is orange. The red boxes represent learners who were removed from the sample after random assignment to treatment groups.

The students who were removed from the sample after assignment to treatment groups are shown in red in Figure 9. Two of the removed students had high readiness, one had medium readiness, and one had low readiness. Two of the students were assigned to Group 1 and the other two students were from Group 3. Because the students who were removed from the sample were spread across different levels of readiness and different types of practice, removing these students from the sample should not introduce much bias. There is a little cause for concern because the 3rd and 4th ranked students were removed from the sample. Since both of these students had high readiness this may introduce some bias into the results. Fortunately, the two students were from different experimental groups, which somewhat mitigates this concern.

Table 9 provides demographic information for the sample of 88 students included in the final data analysis. As can be seen in the table, most of the students were male, freshmen or sophomores, and majoring in a STEM major. Engineering was the most common major with about half of the students (48%) majoring in civil, mechanical, electrical, or computer engineering.

During the instructional design process, a learner analysis was completed and similar demographic information was collected for students in prior semesters of Calculus I (see Table 5). The data shown here in Table 9 is consistent with the information collected during the learner analysis.

Table 9

Demographic information for the sample of 88 students included in the data analysis

Category	**Number**	**Percentage**
Male	69	78%
Female	19	22%
Freshman	31	35%
Sophomore	28	32%
Junior	21	24%
Senior	8	9%
STEM Major	70	80%
Non-STEM	18	20%
International	2	2%
Non-international	86	98%

Calculus I has a prerequisite of precalculus (or college algebra and trigonometry). According to information provided by the learners on a demographic survey, 65% of the learners satisfied the prerequisite by taking precalculus at the university where the study was conducted. The remaining students satisfied the prerequisite by taking precalculus at another college or university (9%), by taking precalculus in high school (22%), or by learning the prerequisite material and passing the calculus placement exam that allowed them to register for calculus without formally completing the prerequisite (4%).

Although many of the learners were taking calculus for the first time (61%), there were a significant number of students who had already taken calculus (39%). Based on student responses to a survey, the most commonly reported reasons learners were retaking calculus were to improve a low or failing grade (33%), to review the concepts before proceeding to Calculus II (33%), because their first calculus course did not transfer or satisfy their major requirements (18%), or because they did not pass the AP Calculus Exam (12%).

Assessment Results

Several assessments were included in this experiment. A readiness test was given in order to determine the calculus readiness of the learners, a pre-test was given in order to determine the learners' prior knowledge of the calculus topic of differentiation, mental effort ratings were collected, and a post-test was given in order to assess the performance of the learners. The mental effort ratings and post-test scores were then used to compute the multidimensional instructional efficiency (MIE) measure. The assessment results for each of these instruments are discussed below.

Readiness test results. The MDTP Calculus Readiness Exam was given during the first week of the semester. The test was administered online, in a proctored setting, in the University Testing Center. In order to accommodate all students on the computers available in the Testing Center, the students were able to take the exam at their convenience over a three-day period during the first week of classes.

Table 10 shows the results of the readiness test for the total sample of 88 students. Based on the results of the readiness test, learners were assigned to an experimental group using an alternate ranks randomization technique (Myers, Well, & Lorch, 2010). This technique ensured that learners in the three experimental groups (worked examples with self-explanation prompts, worked examples with instructional explanations, and the control group) had similar levels of readiness. This was confirmed by the use of a one-way ANOVA test that showed there was not a significant difference in readiness test results among the three experimental groups, $F(2, 85) = 0.04,\ p = .96,$ partial $\eta^2 = .001$.

Table 10

Summary of results on the MDTP Calculus Readiness Assessment

Group	***N***	**Mean**	***SD***
Combined Sample	88	54.60	14.48
Type of Practice:			
Self-Explanation Prompts	29	54.91	14.21
Instructional Explanations	30	54.92	15.75
Control	29	53.97	13.86
Level of Readiness:			
Low Readiness	29	38.45	7.27
Medium Readiness	31	55.24	3.89
High Readiness	28	70.63	7.57

The readiness test results were also used to divide the sample into three groups based on their readiness. As shown in Figure 9, blocks of learners were placed in a low readiness, medium readiness, or high readiness group based on their readiness test score. The top 10 blocks of learners were classified as high readiness, the middle 11 blocks were classified as low readiness, and the bottom 10 blocks were classifies as low readiness. A one-way ANOVA test verified that the three readiness groups were significantly different, $F(2, 85) = 180.37,\ p < .001,\ \text{partial}\ \eta^2 = .81.$ The Tukey HSD test found that the low readiness group was significantly different from the medium ($p < .001$) and high ($p < .001$) groups. The medium readiness group was also found to be significantly different from the high readiness group ($p < .001$). This confirmed that the three readiness groups represented three distinct levels of readiness

Pre- and post-test results. The pre- and post-tests consisted of similar questions and were intended to assess the learners' knowledge of the calculus topic of differentiation. The pre-test is different from the readiness test in that the readiness test was intended to assess prerequisite knowledge from algebra and trigonometry, while the pre-test was intended to assess calculus knowledge. The questions included on the pre- and post-tests can be found in Appendix J. Both tests were given in a proctored setting in the University's Testing Center. The pre-test was given during the second week of the class, prior to beginning the unit on differentiation. The post-test was given at the end of the sixth week of the course. The average lengths of time students spent taking the pre- and post-tests were 1 hour 35 minutes and 1 hour 51 minutes, respectively.

Table 11 shows the results of the pre- and post-test for the combined sample of 88 students. The scores are also broken down into subgroups for the three types of practice and the three levels of readiness.

Table 11

Summary of pre- and post-test scores

		Pre-test		Post-test	
Group	***N***	**Mean**	***SD***	**Mean**	***SD***
Combined Sample	88	9.84	12.39	67.51	13.53
Type of Practice:					
Self Explanation Prompts	29	13.08	15.60	67.11	15.23
Instructional Explanations	30	6.76	9.10	68.29	11.80
Control	29	9.79	11.26	67.51	13.86
Level of Readiness:					
Low Readiness	29	5.93	9.36	61.55	14.41
Medium Readiness	31	9.54	9.43	68.02	10.95
High Readiness	28	14.23	16.38	73.13	13.06

Pre-test Results. A two-way ANOVA was used to investigate the prior knowledge of the calculus topic of differentiation for each experimental group. Table 12 gives the results of the ANOVA test. As can be seen in the table, there was not a significant difference in pre-test scores among the three practice groups. This verified that the alternate ranks randomization technique was effective in creating groups with similar levels of prior knowledge before receiving the treatment. There was also not a significant interaction effect.

As can also be seen in Table 12, there was a significant difference in pre-test scores among the three readiness groups. A follow-up analysis using Tukey's HSD Post-hoc test showed that there was a significant difference in readiness test scores between the high and low readiness groups ($p = .030$), but that there was not a significant difference between low and medium ($p = .485$) or medium and high ($p = .301$) groups. This result indicates that learners with particularly low readiness were more likely to not do as well on the pre-test and learners with particularly high readiness were more likely to do well on the pre-test. This result supports the conclusion that learners with low algebra readiness are likely to not have had prior calculus experience and learners who have had exposure to calculus topics are likely to have stronger algebra skills.

Table 12

Two-way ANOVA for pre-test scores showing that there is not a significant difference in pre-test scores based on type of practice, but there is a significant different in pre-test scores based on level of readiness.

Dependent Variable: Pre-test Score

Source	$\boldsymbol{SS}$	$\boldsymbol{df}$	$\boldsymbol{MS}$	$\boldsymbol{F}$	$\boldsymbol{p}$	**Partial** $\boldsymbol{\eta^2}$
Intercept	8612.43	1	8612.43	58.90	.000	.43
Type of Practice	563.15	2	281.58	1.93	.153	.05
Level of Readiness	1008.727	2	504.36	3.45	.037	.08
Practice * Readiness	222.61	4	55.65	0.38	.822	.02
Error	11550.70	79	146.21			
Total	21879.33	88				

Post-test results. The post-test scores are significantly higher than the pre-test scores, $\mathrm{t}(87) = -36.61, \mathrm{p} < .001$. This demonstrates that learners who participated in the unit had a significant improvement in knowledge about the calculus topic of differentiation. The boxplots in Figure 10 give a visual representation of the post-test results for each experimental group. Notice that observation 85 is an extreme outlier. This student had high calculus readiness but did not do well on the post-test. In order to determine the effect of this outlier on the data analysis, all calculations were computed both with and without the outlier present in the data. Removing this outlier did not have any effect on the significance of any variables. So, the outlier was left in the sample for the final data analysis.

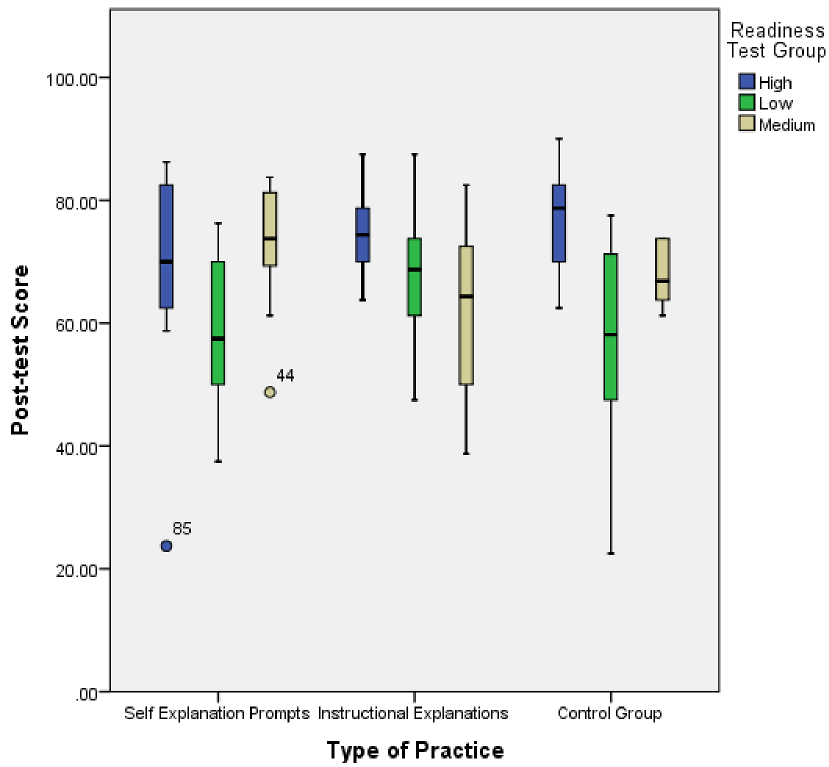

Figure 10. Side-by-side boxplots comparing post-test scores for each type of practice and each level of readiness.

Mental effort rating results. The learners rated their mental effort during two different phases of the experiment: during the learning phase and during the testing phase. These mental effort ratings were collected using the Paas Mental Effort

Measurement Scale (PMEMS). The PMEMS is a 9-point Likert scale with effort ratings ranging from 1 (very, very low mental effort) to 9 (very, very high mental effort). During the three-week learning phase of the experiment, the learners were asked to rate the mental effort required for each homework problem they completed (both worked examples and practice problems). The 146 mental effort ratings reported by each learner were then averaged to find the mean mental effort during learning. The PMEMS was also used to collect mental effort ratings on the post-test. The post-test contained 20 questions and the mean mental effort rating during testing was computed by averaging the 20 ratings provided by each learner. Table 13 summarizes the mean mental effort ratings for the learning phase and for the testing phase for each group and for the combined sample.

Table 13

Summary of mean mental effort ratings during the learning and testing phases of the experiment.

		Learning Phase		**Testing Phase**	
Group	***N***	**Mean**	***SD***	**Mean**	***SD***
Combined Sample	88	5.05	1.10	5.40	1.22
Type of Practice:					
Self Explanation Prompts	29	5.11	0.91	5.46	1.10
Instructional Explanations	30	4.84	1.38	5.37	1.46
Control	29	5.20	0.94	5.38	1.11
Level of Readiness:					
Low Readiness	29	5.20	1.14	5.77	1.17
Medium Readiness	31	5.50	0.64	5.63	0.71
High Readiness	28	4.39	1.17	4.78	1.49

As can be seen in the scatterplots shown in Figure 11, there was a high correlation ($r = .85$) between mental effort ratings reported by learners during the learning phase and those reported during the testing phase. A regression analysis showed that there was a significant linear relationship between the score reported during the two phases of the experiment, $t(86) = 14.88,\ p < .001$. The positive correlation indicates that learners who reported high (or low) mental efforts on the homework tended to also report high (or low) mental effort on the exam.

Notice that the grouped scatterplot on the left in Figure 11 shows that the lowest mental effort ratings were reported by learners with high readiness for calculus, while the highest mental effort ratings were reported by learners with low readiness for calculus. Learners with medium readiness for calculus tended to report more average mental effort scores. Notice that these types of trends are not apparent when the scatterplot is grouped by type of practice, as shown in the scatterplot on the right.

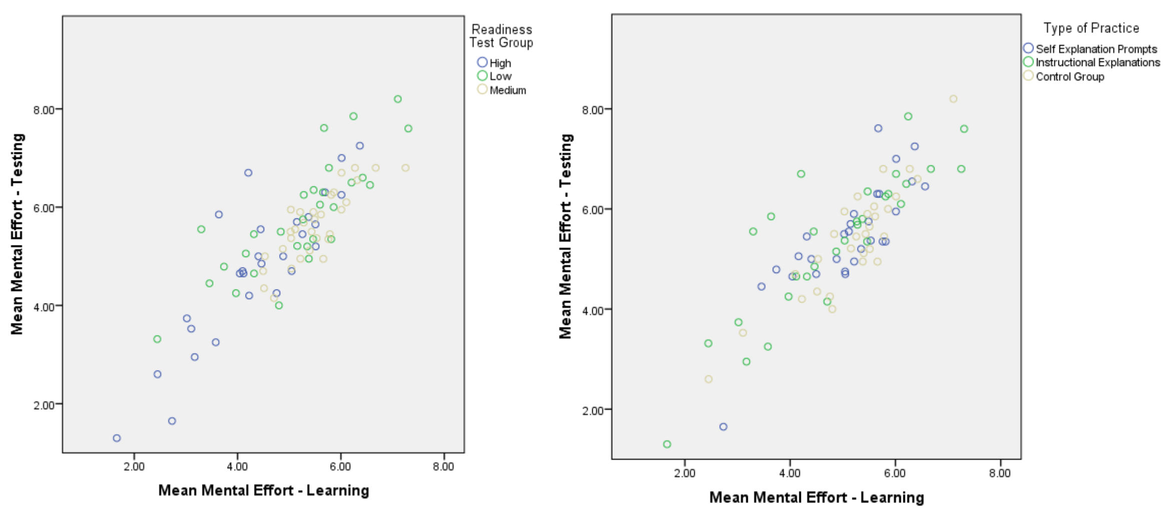

Figure 11. Grouped scatterplots comparing the mean mental effort during learning to the mean mental effort during testing. The plot on the right classifies each point by type of practice and the plot on the left classifies each point by level of readiness.

Homework completion rates. When considering mental effort ratings, it is important to recognize that not all students completed all of the problems. Although students had opportunities to complete 146 homework problems (including worked examples and/or practice problems) and were asked to report the mental effort rating 146 times, there were some students who did not complete all of the problems. In order to consider this in the data analysis, the homework completion rate was included as a covariate when the research questions were answered. Table 14 provides a summary of the homework completion rates for each group and for the combined sample.

Table 14

Summary of homework completion rates

Group	***N***	**Mean**	***SD***
Combined Sample	88	95.40%	8.01%
Type of Practice:			
Self Explanation Prompts	29	93.81%	9.96%
Instructional Explanations	30	97.24%	3.70%
Control	29	95.09%	8.92%
Level of Readiness:			
Low Readiness	29	95.68%	5.49%
Medium Readiness	31	94.21%	10.90%
High Readiness	28	96.43%	6.37%

Notice that the mean homework completion rate of 95.40% was relatively high. A two-way ANOVA was used to determine whether there was difference in the completion rates among different treatment groups. The analysis found no difference in homework completion rates based on type of practice, $F(2, 79) = 1.32,\ p = .27,$ partial $\eta^2 = .032$), or level of readiness, $F(2, 79) = 0.53,\ p = .59,$ partial $\eta^2 = .013$. There was also no interaction effect between type of practice and readiness, $F(4, 79) = 0.66,\ p = .62,\ \text{partial}\ \eta^2 = .032$.

MIE calculations and results. The multidimensional instructional efficiency (MIE) measure, as described by Tuovinen and Paas (2004), was computed using standardized scores for mental effort during learning (z_{EL}), mental effort during testing (z_{ET}), and post-test scores (z_P) with the formula $MIE = (z_P - z_{EL} - z_{ET})/\sqrt{3}$. A positive value for the MIE indicates that the performance was high relative to mental effort; a negative value indicates the performance was low relative to effort. Table 15 provides a summary of the MIE measure for the combined sample as well as each group.

Figure 12 provides a visual summary of the MIE measurements by type of practice and by level of readiness. Notice that there were a few outliers in the MIE data. The most concerning outlier is observation #23. The data for this student show that she had the lowest score on the post-test and had the third highest mean mental effort during learning and the highest mean mental effort during testing. Because her performance was extremely low and her mental effort was extremely high, this led to her having a particularly low MIE measurement. In order to examine the effect of this outlier on the data analysis, all computations involving the MIE were completed both with and without the outlier present. Removing the outlier did not affect the significance of any of the variables except for the homework completion rate covariate. When evaluating the impact of homework completion rates it will be important to consider this outlier.

Table 15

Summary of MIE measurements

Group	***N***	**Mean**	***SD***
Combined Sample	88	0	1.42
Type of Practice:			
Self Explanation Prompts	29	-0.08	1.31
Instructional Explanations	30	0.16	1.57
Control	29	-0.09	1.40
Level of Readiness:			
Low Readiness	29	-0.51	1.40
Medium Readiness	31	-0.32	0.95
High Readiness	28	0.88	1.50

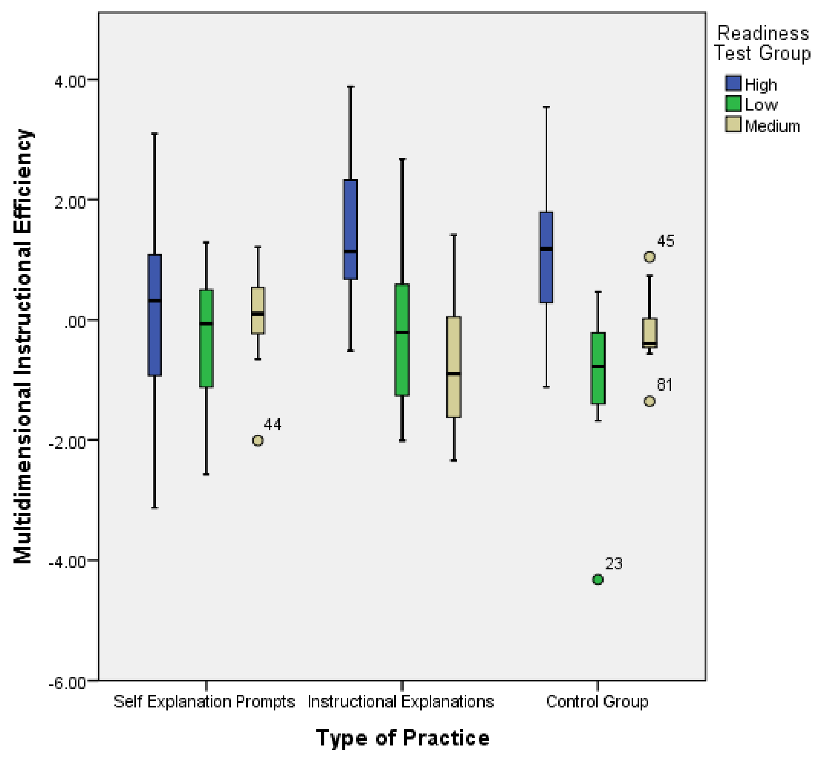

Figure 12. Side-by-side boxplots comparing MIE for each type of practice and each level of readiness.

Results for Research Questions 1 - 3

Research Questions 1-3 investigate the efficiency of the treatments using the MIE. A two-way ANCOVA with two covariates was used to determine whether or not there was a main effect due to type of practice, a main effect due to level of readiness, and/or an interaction effect on MIE. In addition to two independent variables (type of practice and level of readiness), the ANCOVA test took into account two covariates (pre-test scores and homework completion rates). Including the covariates in the analysis allowed us to determine the effect of type of practice and readiness on the efficiency of instruction (MIE) after adjusting for prior knowledge (pre-test scores) and differences in the number of learning tasks completed by each learner (homework completion rates).

An ANCOVA test was shown to be an appropriate procedure for this situation because the following five assumptions of an ANCOVA test were satisfied. First, the data used in the experiment was generated by randomly assigning subjects to experimental groups, so the groups were independent random samples. Second, Levene's test verified homogeneity of variances, $F(8, 79) = 0.70,\ p = .69$. Third, as shown in Figure 13, the residuals for the MIE were normally distributed, making it reasonable to assume normality of treatment groups. Fourth, regression analyses verified that there was a linear relationship between the covariates and the dependent variable. In particular, there was a linear relationship between pretest scores and MIE, $t(86) = 3.30,\ p = .01,\ r = .34$, and between homework completion rates and MIE, $t(86) = 2.73,\ p = .01,\ r = .28$. Fifth, homogeneity of regression slopes was verified through tests for an interaction between type of practice and level of readiness on each covariate. Non-significant interactions were found both for pre-test scores, $F(4, 79) = 0.38,\ p = .82$, and for homework completion rates, $F(4, 79) = 0.66,\ p = .62$.

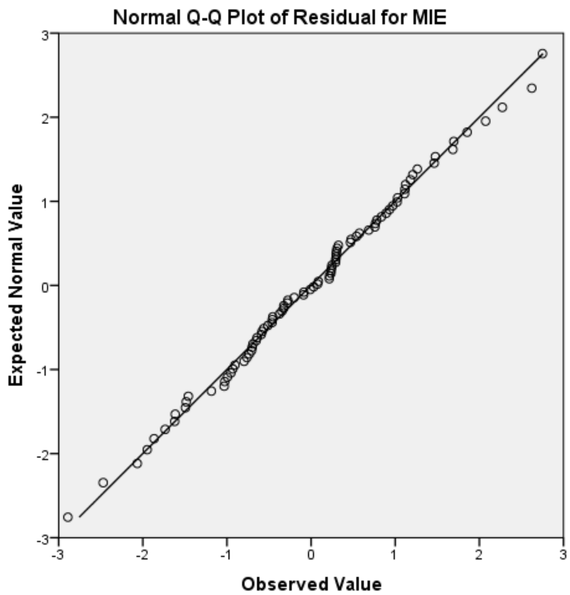

Figure 13. Normal probability plot verifying normality of the residuals for the MIE.

Table 16 shows the results of the ANCOVA test. Note that both covariates (pre-test score and homework completion rate) are significant. Additionally, there was a significant main effect for level of readiness. Type of practice and the interaction term were not significant. However, recall that there was one student with an unusually low value for the MIE. Including this extreme outlier in the data analysis did not affect any results except for the significance of the homework completion rate covariate. As shown in Table 16, with the outlier present homework completion rate was a significant predictor of the MIE, $F(1, 77) = 5.30,\ p = .024,$ partial $\eta^2 = .064$. If the outlier is removed from the data the homework completion rate is not a significant predictor of the

MIE, $F(1, 76) = 3.86,\ p = .053,$ partial $\eta^2 = .048$. Notice that the effect size for homework completion rate is relatively small both with and without the outlier. Regardless of whether or not it is considered to be a significant predictor of MIE, homework completion rate does not account for much of the variance in MIE scores.

Table 16

Two-way ANCOVA for MIE, including the interaction term.

Dependent Variable: Multidimensional Instructional Efficiency (MIE)						
Source	SS	df	MS	F	p	**Partial** η^2
Intercept	8.71	1	8.71	6.11	.016	.073
Pre-test Score	10.12	1	10.12	7.09	.009	.084
HW Completion	7.55	1	7.55	5.30	.024	.064
Type of Practice	1.30	2	0.65	.45	.637	.012
Level of Readiness	18.83	2	9.42	6.60	.002	.146
Practice*Readiness	11.66	4	2.92	2.04	.097	.096
Error	109.85	77	1.43			
Total	175.75	88				

Research Question 1. Research question #1 asked if there was a main effect due to type of practice (worked examples with self-explanation prompts, worked examples with instructional explanations, or practice problems). As can be seen in Table 16, the type of practice was not significant, $F(2, 77) = 0.45,\ p = .64,\ \eta^2 = 0.012$. This result indicates that the three types of practice had equivalent instructional efficiency, after adjusting for pre-test scores and homework completion rates. This does not support the hypothesis that the worked example groups would be more efficient than the control group.

Research Question 2. Research question #2 asked if there was a main effect due to level of readiness (high, medium, or low). Table 16 shows that level of readiness was significant when adjusted for pre-test scores and homework completion rates. A planned Dunn-Bonferroni analysis was conducted to compare the mean MIE among the three readiness groups. Table 17 provides the results of the post-hoc test. There is evidence to show that the high readiness group has a distinct mean MIE when compared to the low and medium readiness groups. However, there is no evidence of a difference in the mean MIE for the low and medium readiness groups. Because the mean MIE is positive for the high readiness group ($M = 0.88, SD = 1.50$) and negative for the other groups (medim readiness: $M = -0.32, SD = 0.95$ and low readiness: $M = -0.51, SD = 1.40$) the test provides evidence that the homework assignments were more efficient for learners with high readiness than they were for learners with low or medium readiness.

Table 17

Planned Dunn-Bonferroni analysis to compare mean MIE among the three readiness groups.

Comparison Groups	t	df	p
Low readiness vs. Medium readiness	-0.61	58	$.545$
Medium readiness vs. High readiness	3.72	57	$.000$*
Low readiness vs. High readiness	-3.62	55	$.001$*

Note: *Significant at $FWE = .05$

Research Question 3. The third research question asked whether there was an interaction effect between type of practice and level of readiness on MIE. As was shown in Table 16, there was not a significant interaction effect, $F(4, 77) = 2.04,\ p = .097$, partial $\eta^2 = .096$. This does not support the hypothesis that the most efficient type of practice depends on the level of readiness of the learner. However, even though the result was not statistically significant at the $\alpha = .05$ level of significance, the effect size is still relatively large. The value of partial η^2 shows that approximately 9.6% of the variability in MIE values can be explained by the interaction between the type of practice and level of readiness of the learner. The lack of statistical significance may be due to a lack of power in the test for the significance of the interaction effect. The a priori power calculations completed in Chapter III (see page 46) were for the power of the main effect. Similar a priori power calculations indicate that in order to have 80% power when testing the interaction effect, the sample size would need to be at least 103 students. As such, it is possible that the lack of a significant interaction in this experiment was due to low statistical power.

Results for Research Questions 4 – 6

The first three research questions investigated the *efficiency* of the experimental treatments. The last three research questions asked similar questions regarding the *effectiveness* of the treatments. The post-test was used as a measure of effectiveness. Similar to the first three questions, research questions #4-6 were also answered using a two-way ANCOVA with two covariates.

The five assumptions for an ANCOVA test were again verified, this time using the post-test (rather than the MIE) as the dependent variable. First, the data used in the experiment was generated by randomly assigning subjects to experimental groups, so the groups were independent random samples. Second, Levene's test verified homogeneity of variances, $F(8, 79) = 1.60,\ p = .14$. Third, as shown in Figure 14, the residuals for the post-test were normally distributed, making it reasonable to assume normality of treatment groups. Fourth, regression analyses verified that there was a linear relationship between the covariates and the dependent variable. In particular, there was a linear relationship between pre- and post-test scores, $t(86) = 3.50,\ p = .001,\ r = .35$, and between homework completion rates and post-test scores, $t(86) = 3.77,\ p < .001, r = .38$. Fifth, homogeneity of regression slopes was verified through tests for an interaction between type of practice and level of readiness on each covariate. Because this computation does not depend on the dependent variable, the calculations done while verifying the assumptions for the ANCOVA test used to answer research questions #1-3 still apply. Therefore, it was once again reasonable to assume homogeneity of regression slopes.

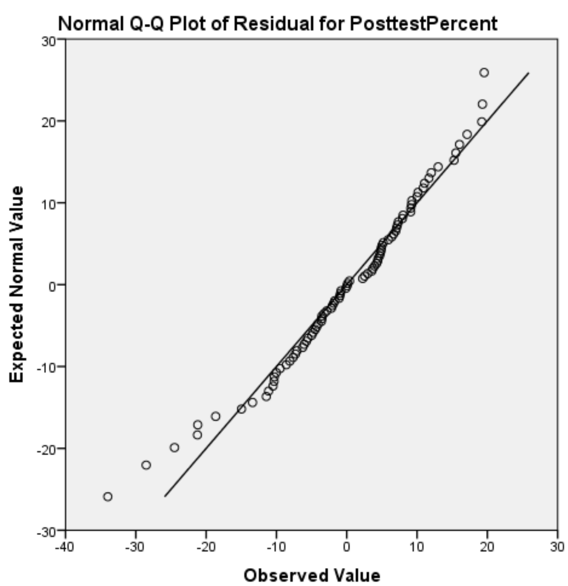

Figure 14. Normal probability plot verifying normality of the residuals for the post-test.

The results of the ANCOVA analysis for the post-test score are shown below in Table 18. The table shows that both covariates (pre-test score and homework completion rate) are significant. Additionally, there is a significant main effect for level of readiness. Type of practice and the interaction effect were not significant.

Table 18

Two-way ANCOVA for post-test scores, including the interaction term.

Dependent Variable: Post-test scores						
Source	SS	df	MS	F	p	**Partial** η^2
Intercept	37.50	1	37.50	0.30	.587	.004
Pre-test Score	1031.02	1	1031.02	8.19	.005	.096
HW Completion	1797.99	1	1797.99	14.27	.000	.156
Type of Practice	28.15	2	14.07	0.11	.894	.003
Level of Readiness	1072.48	2	536.24	4.26	.018	.100
Practice*Readiness	972.23	4	243.06	1.93	.114	.091
Error	9699.03	77	125.96			
Total	417054.69	88				

Research Question 4. Question #4 asked whether there was a significant main effect due to type of practice on student performance. The ANCOVA test showed that type of practice was not significant (see Table 18). This result does not support the hypothesis that the type of practice problems learners completed (worked examples with instructional explanations, worked examples with self-explanation prompts, or practice problems) would affect their performance on the post-test. Because the a priori power calculations described in Chapter 3 indicate that the sample size was large enough for the test to have sufficient power, it is unlikely that this result reflects a Type II error. Additionally, the low value of partial η^2 (.003) indicates that only 0.3% of the variation in post-test scores can be explained by the type of practice completed by the learners.

Therefore, this result indicates that the type of practice really had no effect on post-test scores.

Research Question 5. Research question #5 asked if there was a main effect on post-test scores due to level of readiness (high, medium, or low). Table 18 shows that level of readiness was significant when adjusted for pre-test scores and homework completion rates. A planned Dunn-Bonferroni analysis was conducted to compare the mean post-test scores among the three readiness groups. Table 19 provides the results of the post-hoc test. In this case there is evidence to show that the low readiness group is significantly different from the high readiness group. However, there is not enough evidence to distinguish the medium readiness group from either the low or the high readiness groups. Because the mean post-test score for learners in the high readiness group ($M = 73.13, SD = 13.06$) is greater than the mean post-test score for learners in the low readiness group ($M = 61.55, SD = 14.41$), there is evidence that the homework assignments were more effective for learners with high calculus readiness than for learners with low calculus readiness.

Research Question 6. The final research question asked if there was an interaction effect between type of practice and level of readiness on post-test scores. As

Table 19

Planned Dunn-Bonferroni analysis to compare the mean post-test score among the three readiness groups.

Comparison Groups	t	df	p
Low readiness vs. Medium readiness	-1.97	58	$.054$
Medium readiness vs. High readiness	-1.63	57	$.108$
Low readiness vs. High readiness	-3.17	55	$.002$*

Note: *Significant at $FWE = .05$

shown in Table 18, the interaction effect was not significant. As such, this experiment does not provide evidence to support the hypothesis that the most effective type of practice depends on the readiness of the learner. However, as was seen for research question 3, the lack of statistical significance may be due to low statistical power. The relatively large effect size indicates that 9.1% of the variability in post-test scores can be explained by the interaction.

Summary of Results

This experiment examined the effect of type of practice and level of readiness on the efficiency and effectiveness of opportunities to practice in a calculus course. The results of the experiment showed that type of practice did not have a significant effect on the efficiency or effectiveness of the instruction. Additionally, there was not an interaction effect between type of practice and readiness.

The level of readiness was found to have a significant main effect on both the efficiency (as measured by MIE) and effectiveness (as measured by post-test scores) of calculus practice, after adjusting for homework completion and pre-test scores. The assignments were found to have significantly higher efficiency and effectiveness for learners of high readiness than for those with low or medium readiness. As such, readiness was found to be critically important for success in calculus.

CHAPTER V

Conclusions

The purpose of this research study was to explore the efficiency and effectiveness of using worked examples to provide opportunities for learners with varying levels of readiness to practice in a Calculus I course. As part of the study, the researcher developed two sets of worked examples, one set with self-explanation prompts and the other set with instructional explanations. These examples were used on homework assignments over a three-week period during the unit on differentiation in a Calculus I course. The learners in the course were divided into three groups: two experimental groups and a control group.

This chapter will provide a discussion of the experimental results and will offer conclusions based on the research. In particular, this chapter includes a discussion of the results of the experiment, a discussion of the effectiveness of the instructional design process in developing the examples, a discussion of technical issues encountered during the experiment, recommendations for practitioners, and recommendations for future research.

Discussion of Experimental Results

The results of this experiment answered six research questions on the efficiency and effectiveness of calculus instruction for learners with different levels of readiness and who completed different types of practice. Based on the results described in Chapter IV, we see that this study highlights the critical importance of readiness for calculus.

Additionally, the study found that the type of practice (worked examples or traditional homework) did not impact the effectiveness or efficiency of the instruction. Also, this experiment did not have a statistically significant interaction and thus did not provide support for the hypothesized interaction effect that suggested that the best type of practice would depend on the level of readiness of the learner. However, the test for the significance of the interaction term had relatively low power and that may have contributed to the lack of significance.

The critical important of readiness. This experiment clearly demonstrated the critical important of calculus readiness for learners in a Calculus I course. The readiness of the learner was found to affect both the effectiveness and the efficiency of the assignments, after adjusting for prior calculus knowledge and homework completion rates.

The effectiveness of the instruction was measured using post-test scores. The mean post-test scores for low, medium, and high readiness learners (with standard deviation in parentheses) were 61.55 (14.41), 68.02 (10.95), and 73.13 (13.06), respectively. As shown in Chapter IV, research question #5 demonstrated that there was a significant difference in the mean post-test scores of the three readiness groups, even after adjusting for prior calculus knowledge and homework completion rates. Further analysis of the post-test scores showed that 20% of the learners in the combined sample failed the test (received a score below 60%). However, a greater percentage of students in the low readiness group failed the exam (41%) than in the medium (12%) or high (7%) readiness groups.

This result on the effect of readiness on the effectiveness of instruction supports and confirms existing literature on the readiness of the learner. The MDTP Calculus Readiness Test assessed knowledge on topics related to algebraic manipulation skills, the function concept, and trigonometry (see Table 7 on page 62 for a more detailed list of topics covered on the MDTP). The results of this experiment support Kay and Kletskin’s (2012) claim that these three topics are necessary for success in calculus. Additionally, the results of this experiment provide support for Pyzdrowski et al.’s (2013) finding that calculus readiness is correlated with student performance.

The efficiency of the instruction was measured using the MIE. Recall that MIE measures the efficiency of the instruction by comparing student performance to mental effort, both during learning and during testing. A positive value for the MIE is desirable because it shows that student performance is high relative to the level of mental effort. A negative value for the MIE indicates inefficiency in the instruction because student performance is low relative to the level of mental effort. Research question #2 found that there was a significant difference in MIE scores among the three readiness groups. Further analysis of the data found the 95% confidence intervals (CIs) for the MIE for low, medium, and high readiness learners were [-0.85, 0.05], [-0.70, 0.16], and [0.23, 1.16], respectively. Note that because the CIs for low and medium readiness learners contain zero, there is not enough evidence to determine whether those groups have a positive or negative MIE. Although these CIs do not definitively determine the efficiency or inefficiency of the instruction for the low and medium readiness groups, they do show that the high readiness group was more efficient than either the low or medium groups.

Because the 95% CI for MIE for the high readiness group includes only positive values, we can be 95% confident that for high readiness learners, the MIE score is positive and thus represents high instructional efficiency.

One possible explanation for this result could be the finding of Schwonke et al. (2013) that leaners with low readiness do not use learning aids in a learning-oriented way. The possible inefficiency in the instruction for low and medium readiness learners may be due to how they used the examples. For example, the learners in the worked example groups were provided with cognitive aids such as self-explanation prompts or instructional explanations. Failure to use these cognitive aids appropriately, or other non-learning oriented behaviors, may have reduced the overall instructional efficiency for those groups.

No effect due to type of practice. Research questions #1 and #4 found that the type of practice completed by the learners had no effect on either the effectiveness or efficiency of the instruction. Several previous researchers have found that although there was no difference in student performance between learners who viewed worked examples and learners who solved practice problems, there was a difference in the efficiency and worked examples were found to be more efficient (i.e. Boekhout et al., 2010; Hoffman & Nadelson, 2009; Nievelstein et al., 2013; Vogel-Walcutt et al., 2011). Therefore the finding that the type of practice had no effect on the *effectiveness* of the instruction is consistent with earlier research, but the finding that the type of practice had no effect on the *efficiency* of the instruction contradicts prior worked example research.

One possible explanation for the difference between the result of this study and the result of previous research could be the length of the experiment. Because the experiment was conducted over a three-week period and was situated in a classroom setting, it was much more difficult to control for other variables than it would have been in prior research that was conducted in a laboratory setting or for a shorter length of time. During the three-week period of the experiment, students received instruction from the instructor during class and also were able to get help from tutors. These other variables may have lessened the impact of the type of practice completed by the learners.

In order to explore the effect of tutoring on the survey results, the learners were asked to report the number of times they met with a tutor during the experiment. Sixty-three percent of the learners indicated that they received help from a tutor during the three weeks of the experiment. The mean number of tutoring visits per student during the three weeks was 4.16 ($SD = 4.59$). However, the mean number of visits did not vary by type of practice, $F(2, 79) = 0.47,\ p = .63,\ \text{partial}\ \eta^2 = .012$, or by level of readiness, $F(2, 79) = 1.30,\ p = .28,\ \text{partial}\ \eta^2 = .03$. There was also not an interaction effect on the number of tutoring visits between type of practice and readiness, $F(4, 79) = 0.60$, $p = .66,\ \text{partial}\ \eta^2 = .030$). This indicates tutoring probably was not a confounding variable.

Lack of interaction between readiness and type of practice. It was hypothesized that there would be an interaction effect between level of readiness and type of practice. As demonstrated through the results of research questions #3 and #6 in Chapter IV, this study did not provide statistical evidence of an interaction between readiness and type of practice for either student performance or MIE. All types of practice were found to be similarly effective (as measured by MIE) for learners with all levels of readiness. However, the lack of interaction may have been due to low statistical power. The interaction effect had a moderate effect size on both MIE (partial $\eta^2 = .096$) and student performance (partial $\eta^2 = .091$). These medium effect sizes are similar to values of partial η^2 that were reported for significant results obtained by other researchers (i.e. Boekhout et al., 2010; Booth et al., 2013; Hilbert & Renkl, 2009; Ngu & Yeung, 2012; Richey & Nokes-Malach, 2013; Wong et al., 2012) in worked example research (see effect sizes reported in summary of worked example research in Appendix A).

Due to the expertise reversal effect, it was expected that learners with high readiness would benefit most from completing practice problems only (with no worked examples) while learners with low or medium readiness would benefit most from viewing worked examples. Several researchers (i.e. Kalyuga, 2007; Salden, Aleven, Schwonke, & Renkl, 2009; Schwonke et al., 2009) have found evidence that worked examples are more effective for novices than they are for experts. However, as explained in detail in Chapter II, most studies on worked examples have been conducted in algebra or geometry using highly structured problems. The introduction of the limit concept in calculus leads to more abstract concepts than those in algebra. Consequently, calculus problems tend to be less-structured than the highly-structured problems typical of an algebraic context. The lack of an expertise reversal effect on problems that are less-structured than algebra problems would be consistent with Nievelstein et al. (2013) who found that the expertise reversal effect did not occur when worked examples were used with less-structured problems.

Although the interaction effect in this experiment failed to achieve statistical significance, the results of the experiment do not necessarily contradict prior research that did find an interaction effect. In particular, Berthold and Renkl (2009) found that self-explanations helped foster conceptual knowledge when learners answered the prompts correctly, but that they hindered learning for learners with low readiness who lacked the necessary cognitive skills to answer the prompts correctly. Based on Berthold and Renkl's (2009) finding, the hypothesis for this experiment was that worked examples with self-explanation prompts would be more beneficial for learners with medium or high readiness while worked examples with instructional explanations would be better for learners with low readiness. Although the interaction effect was not statistically significant, the means plot (see Figure 15) shows results that are consistent with this hypothesis.

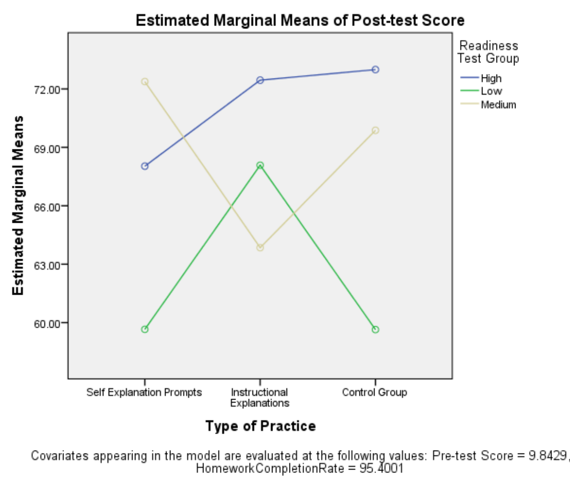

Figure 15. Means plot for post-test scores showing the direction of the comparison among treatment conditions supports the hypothesis that that worked examples with self-explanation prompts would be more beneficial for learners with medium or high readiness while worked examples with instructional explanations would be better for learners with low readiness.

Discussion of Effectiveness of Instructional Design Process

Chapter III detailed use of the Kemp Model (Morrison et al., 2011) in following a rigorous instructional design process to design and develop the worked examples. This experiment provides a source of confirmative evaluation for the examples as called for by the Kemp Model (see Figure 8 on page 51).

The results of this experiment show that providing a combination of worked examples and practice problems is as effective as providing learners with practice problems only. Prior to the development of the examples, the instructor was using only practice problems on the homework assignments. This study confirms that worked examples can be used to replace some of the practice problems with equal efficiency and effectiveness. However, the design and development of the worked examples was much more time consuming than developing practice problems. It was probably not worth the extra time required to develop the examples because they were not found to be more effective or efficient than using practice problems.

Although the faded examples were found to be as efficient and effective as using only practice problems, only high readiness learners were found to have positive instructional efficiency (as measured by MIE). This indicates that changes need to be made to the instructional design of the unit in order to accommodate learners with low or medium readiness. Remediation activities that help improve the algebra skills of low or medium readiness learners might help improve the instructional efficiency and effectiveness of the unit for these learners and should be incorporated into the course.

In a survey administered to the students at the end of the unit, the learners in the two worked example groups were asked what they found to be the most helpful aspect of using the worked examples. Table 20 summarizes the responses to this open-ended question. Note that the students appreciated seeing step-by-step solutions to problems, liked being able to watch the video more than once outside of class, and benefited from the additional explanations provided by the examples of both concepts and procedures.

Table 20

Summary of survey responses to the question: "What was the most helpful aspect of viewing the video examples?"

Response	**Percentage**
Provided a step-by-step solution	25%
Allowed for repetition of instruction outside of class	23%
Provided added explanations of concepts	15%
Provided added explanations of procedures	11%
Provided a visual example	7%
Helped solve the practice problems	5%
No response	5%
Hearing the audio	3%
They were a convenient source for answers to questions	3%
Allowed me to pause when I didn't understand	2%
Provided a variety of examples	2%

In addition to asking the learners what they felt was most helpful about the video worked examples, the survey also asked what they did not like about viewing the examples. Table 21 summarizes the results to this open-ended question. The top three responses were that viewing the videos took longer than it would have taken to just complete the practice problems, that there were occasional technical problems, and that there was nothing they didn't like about the videos. Indicating that the videos over-explained or took too long on easy problems was a response that was reported more often by learners with high (38%) or medium (38%) readiness than it was for learners with low readiness (24%).

Based on these responses, it appears that the instructional design might be improved by providing different examples for learners based on their readiness. For high readiness learners in particular, it would be beneficial to find ways to shorten the videos and ensure that the video examples are examples of more complicated problems where they are most likely to be useful and not examples of the simpler homework problems. Also, in this experiment the learners were required to watch all of the videos. When the videos are used with classes in the future, it might be beneficial to consider making some video examples optional so students can chose whether or not to view the videos.

Table 21

Summary of survey responses to the question: “What did you not like about viewing the video examples?”

Response	**Percentage**
Videos over-explained or took too long on easy problems	34%
Occasional technical problems	20%
Nothing – I liked the videos	10%
No response	8%
Videos didn’t help with practice problems	5%
Didn’t like self-explanation prompts	5%
Couldn’t ask questions while watching the videos	3%
Videos didn’t provide enough explanations	3%
Didn’t like the segmentation of the videos	3%
Videos were boring	2%
Provided a variety of examples	2%
Would have liked to see more videos and fewer practice problems	2%
Would have liked to see fewer videos and more practice problems	2%
Too many types of questions on the assignments	2%

Technical Issues

The examples developed for use in this experiment utilized video and audio and were delivered online. The experiment was conducted in a face-to-face class and the experimental worked examples were assigned to the learners as part of their homework assignments for the class. Using online examples requires particular care in order to ensure that technical issues are kept at a minimum. This experiment utilized a careful formative evaluation process in order to minimize the technical issues encountered during the experiment. The following discussion will address some of the technical problems encountered during the development of the examples and during their use in the experiment. In particular, technical issues that were resolved during the piloting semester and those that arose during the actual experiment will be discussed.

Technical issues during the piloting semester. During the Fall 2014 semester, the complete set of examples was used in a Calculus I course. This piloting semester identified several important technical issues that were able to be resolved.

The most severe technical issue that was encountered and resolved during the piloting semester dealt with the method used for providing feedback to the learners, especially on the practice problems. The original design called for the learners to complete the problems, using pencil and paper, in a homework notebook. They would then type their answer into the LMS, which would automatically check to see if the answer was correct and provide the learners with feedback. The purpose of having them type the answer into the LMS was to allow for immediate feedback on the correctness of their solution. The LMS supposedly had a fairly robust method for checking if two equations were functionally equivalent, so the LMS should have been able to determine the correctness of the solution, even when the student typed the solution into the computer in a format different from that entered by the instructor. Two problems arose based on this design for entering answers into the LMS. First, the LMS was not always as accurate at determining functional equivalence as the documentation provided by the LMS supplier claimed it would be. Second, the learners struggled to type equations into the LMS correctly. Even when using an HTML equation editor, the learners frequently missed parentheses or other important features of equations that led the LMS to mark the problem as incorrect, even when the learner had the correct answer written in their homework notebook.

The combination of these two problems made it very difficult for the learners to trust the feedback they got from the LMS, which was a very significant source of frustration to the learners. It quickly became apparent to the researcher that these technical problems were overshadowing any positive effects of using worked examples on the assignments. So, in the middle of the piloting semester a change was made to the way to the learners received feedback on the problems. Rather than typing the answer into the LMS, the students were provided with a button they could click that would show them the correct answer to the practice problem. They were still required to show their work in their homework notebook, but they could check their answers without having to type the equation into the LMS. Anecdotal evidence of the success of this modification was provided through an email sent to the researcher from one of the top students in the class. In an email sent on October 9, 2014, he said:

> I personally prefer this way so much better. The problem with the last type was that I spent time half for my algebraic skills but the other half for technical issues. This was difficult because I had to focus on technical things before I could pay attention [to] what I was learning. Since I do not have to deal with those technical issues anymore, I can do the homework faster, and I can actually focus only on my calculation[s] (what matters most regarding of LEARNING).

The piloting semester was also helpful in identifying design changes that would improve the quality of the worked example. In particular, the learners reported that the segmentation of the worked examples was frustrating and interrupted their learning. The decision to segment the examples had been based on prior research that showed segmentation helped improve the effectiveness of the examples (Wong et al., 2012). However, when the segmentation was reviewed based on the feedback provided during the piloting semester, it was determined that the videos paused more often than they needed to. As such, the segmentation of the videos was reviewed and revised at the end of the pilot study.

Technical issues during the experiment. During the piloting semester, most of the potential technical issues were identified and were able to be corrected prior to implementing the worked examples in the experiment. As such, there were relatively few technical issues that occurred during the actual experiment. However, a few unanticipated issues did arise.

Two students reported that their Internet filter was blocking the web page that was used to provide answers to the practice problems to the learners. For an unknown reason, the Internet filter had classified the website as adult/mature content. Because the students who encountered this problem were using rented computers, they did not have access to the password needed to override the block. This prevented them from completing their homework on their own computers. Both students were able to complete the homework by borrowing a computer from someone else, but this was a very inconvenient solution. The researcher contacted the Internet filtering company and asked them to reclassify the website as educational. The company complied and within a few days the affected students quit seeing the block and were able to access the answers on their personal computers.

The LMS that was used to distribute the homework assignments experienced intermittent campus-wide downtime during the experiment. These downtimes were infrequent and typically quite short, but caused frustration to students who were unable to access the homework assignments during the outage. Additionally, similar campus-wide problems with the LMS occasionally caused content embedded in the LMS to load more slowly than normal. At the end of the experiment the learners completed a survey and were asked if they encountered technology issues during the experiment. If they answered yes, they were asked to describe the problems they encountered. Of the 87 students who responded to the survey, 63% said they encountered no technology problems while working on the homework. Twenty-nine percent of the students reported there were times when the videos loaded slowly (either due to campus-wide LMS issues or due to limited Internet bandwidth in student apartments), 14% reported occasional LMS outages, and 6% of the learners reported the LMS occasionally did not save their submitted responses (a known problem related to the LMS outages). All of the technology problems reported by the students were due either to LMS outages or limited Internet bandwidth on campus or in student apartments.

Although these technical issues were a source of frustration to some learners, they were not significant enough to impact the final results of the study. Many of the survey responses indicated that the technical issues occurred "once" or "a few times", but the issues did not occur on most of the homework. Also, homework completion rates were very high (mean completion rate of 95%) which indicates that LMS outages or slowly loading videos did not prevent the learners from completing the assignments. Additionally, a two-sample t-test was used to compare the post-test scores of learners who reported they encountered technology problems with those who did not and found that there was not a significant difference, $t(63.06) = 0.11,\ p = .91$. Similarly, there was also no difference in MIE scores, $t(61) = -0.10,\ p = .92$.

Recommendations for Future Practice

This study illustrated the critical importance of calculus readiness. Learners who were not ready for calculus did not perform as well in the course as learners with adequate calculus readiness. Additionally, the assignments were found to be less efficient for learners with low readiness than they were for learners with high readiness. Therefore, it is highly recommended that instructional designers, math departments, and calculus instructors carefully consider techniques that ensure the calculus readiness of their students. This could include both restricting registration to students who demonstrate adequate readiness and providing remediation for learners with lower readiness who register for calculus. This study adds support to the recommendations of Rassmussen, Ellis, and Zazkis (2013) who found that successful calculus programs typically have a method in place to assess and ensure the content readiness of their learners prior to registration.

This experiment also found that worked examples were as effective and as efficient as practice problems for calculus students. Therefore, providing worked examples to give learners opportunities to practice would be one possible technique that could be used in calculus courses. These worked examples should use tools such as instructional explanations and self-explanation prompts in order to help the learners develop adequate schema.

The instructional design process used in this experiment depended on a careful formative evaluation process. Future practitioners are encouraged to ensure that the worked examples are well designed and thoroughly tested prior to their implementation.

Recommendations for Future Research

This research study added to the existing literature on worked examples by providing evidence of their effectiveness and efficiency when used in a university-level calculus classroom. Very little research had been done previously on using worked examples with learners at the calculus level. Most prior research had been conducted with middle-school and high-school students in algebra or geometry courses. Although this study extends those results to more mature learners with more abstract content, there is additional research that should be done.

This experiment failed to find evidence of an interaction effect, possibly because of low statistical power. It would be beneficial to repeat the experiment with more learners to see if the increased power provided by a larger sample provides evidence that different types of practice are more efficient and/or effective for learners with different levels of readiness.

This experiment focused on one of the five units in a Calculus I class. The unit on differentiation was selected as the unit to be used in this study because it is a unit that contains a good mix of conceptual and procedural ideas that depend greatly on the algebra skills of the learner. Future research should look at whether the same results would be seen if worked examples were used for a different unit or for the entire course.

The examples used in this study were embedded in the LMS. As explained earlier in this chapter, there were some problems with the speed and stability of the LMS. Future research might use other methods to distribute the worked examples in order to eliminate some of the technological issues related to an unstable LMS.

An unsolicited email received from a student during the experiment highlighted an unexpected effect of the treatments. Referring to the experimental homework assignments, the student stated “I love doing homework this way…it is easier to internalize the materials when they’re tailored by the teacher for the student, rather than generic problems out of the book.” To this student, the benefit of the homework assignment was that it was created by the instructor. Because the researcher was also the instructor of the course, this perception of personalization of the content may have been an unintended benefit of the experimental assignments. Future research should look at the difference when the worked examples and practice problems are developed by the instructor and when they are developed by someone other than the instructor of the course.

This study considered student performance by looking at scores on a post-test. The post-test contained both conceptual and procedural questions, but the data analysis considered the total post-test grade and did not distinguish between questions that assessed conceptual knowledge and questions that assessed procedural knowledge. The worked examples (both with self-explanation prompts and instructional explanations) were expected to improve conceptual understanding more than problem solving because the examples specifically identified connections between mathematical concepts and mathematical procedures. Although this study found no difference in total post-test scores among the three treatment groups, there may have been a difference in conceptual understanding among the learners. Future research should specifically assess conceptual and procedural knowledge in order to determine how the two types of worked examples compare to problem solving in the development of conceptual understanding.

Currently, cognitive load theorists are researching methods to individually measure the three types of cognitive load (germane, extraneous, and intrinsic) (Ayres & Van Gog, 2009; Kalyuga, 2009; Kirschner et al., 2011; Van Gog et al., 2009). The worked examples used in this study were expected to be more effective and efficient than practice problems because they were expected to decrease extraneous cognitive load. However, the mental effort measurement used in this study was a measurement of total cognitive load and did not distinguish among the three types of load. Cognitive load is additive (Ayres, 2006). This means that two learners could have the same total cognitive load, even though one learner had a high extraneous load and low germane load and the other learner had a low extraneous load and high germane load. Therefore, because this study relied on total cognitive load measurements, we are unable to determine whether the three groups differed on the level of extraneous cognitive load. Future research should look at the effect of worked examples with self-explanation prompts and worked examples with instructional explanations on extraneous cognitive load.

The examples prepared for use in this experiment were intended to provide learners with an opportunity to practice applying new calculus knowledge and skills. They were not intended to be the sole source of instruction on the topic of differentiation. The examples were used in a face-to-face class and the instructor provided in-class instruction and in-class activities that gave the learners their first exposure to the concepts and procedures. Learners used the worked examples as part of their homework that was done outside of class. Future research should determine whether different results would be found if the worked examples were used to replace instruction. Worked examples similar to those used in this study could be a valuable tool in an online course and research that investigates the effectiveness and efficiency of such examples in a calculus context would add valuable new insight to the existing worked examples literature.

The Joint Mathematics Meetings that were held in January 2015 included 33 talks that mentioned a flipped or inverted classroom in the title of the talk (“Joint Mathematics Meeting Full Program,” 2015). The frequency of flipped classrooms as a topic at this large conference of university-level mathematics faculty shows that flipped classrooms are currently a popular topic among university mathematics faculty. In a flipped classroom, learners are provided with activities that help introduce the content prior to attending class. Worked examples like the ones developed in this study could potentially be used to provide learners with examples they could view prior to attending class. This study found that the worked examples were as effective as practice problems. Future research could consider whether such examples are an effective tool for providing instruction prior to class in a flipped classroom.

Summary

This dissertation details the results of an experiment to determine the effectiveness and efficiency of worked examples as a technique for providing learners in a Calculus I course with opportunities to practice. In addition to considering the type of practice completed by the learner, the readiness of the learner was also incorporated into the analysis.

The theoretical framework for this study was based in Cognitive Load Theory. The worked examples were expected to reduce the extraneous load experienced by learners as they completed their homework. By reducing the extraneous load, more cognitive resources were available to devote to the development of schema. In order to facilitate the development of schema and increase the germane load of the learners, self-explanation prompts and instructional explanations were added to the examples.

The data analysis answered six research questions. The first three research questions addressed the instructional efficiency of the examples, as measured using MIE. These three questions were answered using a two-way ANCOVA with two covariates.

The results of the first question showed that the type of practice (worked examples with self-explanation prompts, worked examples with instructional explanations, or practice problems) completed by the learner did not affect the MIE, while the results of the second question showed that the readiness of the learner did have an effect on the MIE after adjusting for prior calculus knowledge and homework completion rates. In particular, learners with high readiness had positive instructional efficiency that was significantly higher than the instructional efficiency of learners with low or medium readiness. The third research question showed that there was no interaction effect on MIE between type of practice and readiness of the learner.

The final three research questions answered similar questions about the effectiveness of the instruction, as measured by post-test scores. Again, these questions were answered using a two-way ANCOVA with two covariates. The results for the fourth question found that type of practice did not have a significant main effect on student performance while the results for the fifth question indicated that level of readiness did have a significant main effect, even after controlling for prior knowledge and homework completion. The findings for the final research question showed that there was not an interaction effect on student performance between type of practice and level of readiness.

These results highlight the critical importance of the readiness of the learner and future practitioners are advised to consider ways to ensure the readiness of the learner, either through placement testing or through remediation. The results of this experiment also show that worked examples can be used to replace some practice problems on homework without adversely affecting the efficiency or effectiveness of the homework assignments.

One of the key features that ensured the success of this experiment was the use of a careful instructional design process, including a detailed evaluation procedure. The Kemp Model was very useful in outlining the steps of the instructional design process. Following the outlined steps led to a final version of the worked examples that was greatly improved when compared to the first version. Although there were some technological issues during the experiment, most of the potential technology problems were identified and solved prior to the final implementation because of the feedback obtained through the instructional design process.

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APPENDIX A

Summary Table of Worked Example Research

Table A1

Summary Table of Worked Example Research

Author	**Date**	**Type of Examples**	**Content Area**	**Age of Subjects**	**In a classroom setting?**	**Research Questions**	**Findings**	**Effect Size**
Ayres	2013	Paper	Algebra	Year 8 students ($n = 54$)	No	They hypothesized that isolating the elements of problems that typically cause students the most difficulty because of the high cognitive load and having the students spend time practicing that part of the solution would help students learn. In order to test this hypothesis they compared the performance of students in three groups: the targeted-isolated elements group, the equal-isolated elements group, and the full worked-example group. The targeted-isolated elements group spent more time studying just those parts of the problem that had high cognitive load. The equal-isolated elements group spent equal time studying each part of the problem. The worked examples group studied the entire worked example.	They found that the targeted-isolated group did better than the full worked-example group on knowledge, cognitive load, and transfer. There was an interaction between prior knowledge and group where students with low prior knowledge did best with the equal-isolated elements and students with high prior knowledge did equally well.	Not reported
Boekhout, Van Gog, Van de Wiel, Gerards-Last, & Geraets	2010	Paper	Medical (diagnosing physical complaints of patients)	Undergrads ($n = 134$)	No	They explored the difference between 1st and 2nd year students (to see the effect of expertise). Also looked at the difference in using expert models and student models to create the examples. The hypothesis was that there were be an interaction between model expertise and student expertise.	All students performed better when viewing the expert example. But the 1st year students required less mental effort - so their learning was more efficient.	Effect of student expertise (1st year students vs. 2nd year students) on mental effort $\eta^2 = .122$ and on retention $\eta^2 = .031$

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Booth, Lange, Koedinger, & Newton	2013	Cognitive tutor	Algebra I	Experiment 1: high school students ($n = 134$); Experiment 2: 8th grade students ($n = 64$).	Yes. Using a total of eight worked examples.	Compared three groups: Correct examples only, Incorrect examples only, and a combination of correct and incorrect examples. "In Experiment 1, students working with the Algebra I Cognitive Tutor were randomly assigned to complete their unit on solving two-step linear equations with the traditional Tutor program (control) or one of three versions which incorporated examples; results indicate that explaining worked examples during guided practice leads to improved conceptual understanding compared with guided practice alone. In Experiment 2, a more comprehensive battery of conceptual and procedural tests was used to determine which type of examples is most beneficial for improving different facets of student learning. Results suggest that incorrect examples, either alone or in combination with correct examples, may be especially beneficial for fostering conceptual understanding" (p.24).	"Experiment 1 indicated that combining guided practice with worked example problems benefited students' conceptual knowledge (Hypotheses 1a and 1b); Experiment 2 measured the impacts of particular types of examples. Results indicated that students performed best after explaining incorrect examples; in particular, students in the Combined condition gained more knowledge than those in the Correct only condition about the conceptual features in the equation, while students who studied only incorrect examples displayed improved encoding of conceptual features in the equations compared with those who only received correct examples (Hypothesis 2)" (p. 31)	Experiment 2: Main effect of condition $\eta^2 = .009$; multivariate effect of condition $\eta^2 = .13$
Corbalan, Paas, & Cuypers	2010	eLearning	Linear Algebra	Undergrads ($n = 9$ and $n = 34$)	Yes. For 3 sets of problems	In this article the authors describe the results of two studies investigating whether feedback should be provided on each step of a problem or provided all at once at the end of the problem. In particular, they wanted to know how the timing of the feedback effected the motivation and transfer of students. The second study extended the first by including more subjects and by looking at student performance, mental effort (a measurement of cognitive load), motivation and transfer in addition to looking at student preferences.	The first study found that students preferred having feedback provided at each step of the solving process rather than just getting feedback on whether their answer was correct. In the second study they found that feedback on each step led to the best results in each of these areas. Mental effort was higher for students in the feedback on final step group	Effect on Mental effort $d = 0.36$

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Darabi, Nelson, Meeker, Liang, & Boulware	2010	eLearning	Engineering	Undergrads ($n = 32$)	Yes. For three class sessions: one introduct-ory, one learning, and one testing	Looked at the effect of worked examples vs. conventional problem solving in helping learners develop mental models	The worked example participants progressed more in their development of the mental model than the problem solving participants did.	None reported
Gentner, Loewenstein, & Thompson	2003	Paper Case-based examples	Business	Undergrads ($n = 48$, $n = 128$, $n = 158$)	Yes. For one lesson	In these three experiments they hypothesized that having students compare two examples to look for similarities would help increase transfer.	Their findings supported their hypotheses	Not reported
Hilbert & Renkl	2009	Paper	Concept mapping	Experiment 1: Police academy ($n = 30$) Experiment 2: 11th graders ($n = 76$)	No	They conducted two experiments - in the first they compared a group that viewed worked examples to one that didn't - they found no significant difference. So they conducted a second experiment where they included self-explanation prompts	Self-explanation prompts led to better performance.	Experiment 2: self-explanation prompts vs concept mapping on their own - effect on performance $\eta^2 = .076$
Jarodzka, Van Gog, Dorr, Scheiter, & Gerjets	2013	Computer-based with eye-tracking	Fish Identifica-tion	Undergrads ($n = 75$)	No	Used eye-tracking data from expert models to create examples that helped guide the learner's eye to the look at the correct place on the screen. Would help the learners know what important information to attend to, more than the learners who viewed examples without eye-tracking data.	Students who viewed the example with the eye-tracking data had more coherent eye-tracking data on the performance assessment	$\eta^2 = .39$

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Kay & Edwards	2012	Video podcasts	Middle School Math	Middle School Students ($n = 136$)	No. Learners viewed one example	This study looked at student attitudes and performance as a result of using video podcast worked examples to teach middle-school mathematics principles.	"Students were positive about the quality of worked example video podcasts and appreciated the step-by-step, easy-to-follow explanations, diagrams, and being able to control the pace of learning. Learning performance increased significantly after using worked example video podcasts. There were no gender or grade level differences in attitudes toward worked example video podcasts or learning performance" (p. 1).	Pre- vs. Post-test: $d = 2.72$
Kay & Kletskin	2012	Video podcasts	Pre-calculus	Undergrads enrolled in calculus ($n = 288$)	No. Participa-tion was voluntary.	"Four key research questions were addressed regarding the use of problem-based video podcasts in the realm of pre-calculus mathematics: 1) Why do students choose to use or not to use video podcasts? (open-ended response); 2) How often are video podcasts used? (tracking data and student feedback); 3) How did students rate the usefulness and quality of video podcasts? (survey data); and 4) Did student understanding of pre-calculus knowledge improve as a result of using video podcasts? (survey questions)" (p 622)	Two-thirds of the students chose to use the podcasts. The most commonly cited reasons for not using them were that they felt they already knew the material. Almost 90% of those who used the podcasts said they were useful. Student self-reported data says precalculus understanding increased.	None reported
Miller	2010	In-class examples	Calculus	Undergrads ($n = 22$)	No. Students volunteered to participate	Used the three step method 1) review a computer worked example as a class 2) have the teacher go over an example in class, and 3) have the students work an example in small groups.	Participants in these voluntary review sessions did better than non-participants - but the fact that they participated is confounded with the type of review session.	None reported (The significant p-values were $p = .06$ and $p = .08$)
Moreno & Valdez	2007	Paper cased-based examples	Pre-service teacher education	Experiments 1 and 2: pre-service teachers ($n = 53$) and ($n = 55$)	Yes . Not fully integrated with classroom instruction	Research questions: 1) Does the format of the case affect student recall? 2) Does the presentation of the example affect transfer? and 3) Do presentation and/or format affect learning perceptions?	Students did better with video examples than with other static formats. Viewing several examples for one concept helped.	For tests of transfer, effect size for group was $\eta^2 = .17$.

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Newton, Star, & Lynch	2010	Paper	Algebra	High school students ($n = 6$)	Yes . In a 3-week long remedial/ review summer school course	In this study the authors used worked examples to help teach students flexibility in solving algebra problems. In this context of this study they defined flexibility as the awareness of different methods of solving problems and the ability to choose a method appropriate for a particular problem. The authors were particularly interested in students who struggled in math.. The course specifically focused on promoting flexibility. Data was collected through a pre-test, an intermediate assessment, and a post-test as well as a series of interviews with the students.	The authors found that the students were often aware of multiple approaches to solve problems and could solve them when prompted, but did not typically use non-standard approaches unless prompted. If students did choose to use a non-standard approach it was usually on more difficult problems where using a non-standard approach could make the problem easier to complete (e.g. by avoiding fractions). Additionally, they found that if a student was very familiar with one approach to solving a problem, they were less likely to use an alternate approach. Overall, students were more concerned with the accuracy of a solution that with the efficiency of their method. They did not find that students were confused by seeing different approaches (which was a concern for struggling students).	None reported
Ngu & Yeung	2012	Paper	Algebra problems in a chemistry context	Experiments 1, 2, 3, and 4: Grade 11 Students $n = 23$, $n = 33$, $n = 40$, and $n = 43$, respectively	Yes. For two days	The authors hypothesized that the MC (multiple components) approach would be more effective for learning that using worked examples to teach algebra ideas in a chemistry context. The four experiments tested different combinations of symbolic equations (symbols, hints, and categorizing statements in the question prompt) as part of the MC method.	They found the MC method to be preferable to worked examples. They found that hints, or the use of strategies to remind the learner to access earlier knowledge plays a critical role in facilitating transfer.	Effect on transfer between MC and WE groups. Experiment 1: $\eta^2 = .18$; Experiment 2: $\eta^2 = .20$; Experiment 3: $\eta^2 = .03$; Experiment 4: $\eta^2 = .05$

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Ngu & Yeung	2013	Paper	Algebra problems in a chemistry context	High school students ($n = 22$) in Malaysia	Yes. For two days	1. Students in the equation worked examples condition would outperform those in the text editing condition on the conceptually more difficult molarity problems. 2. Students in the equation worked examples condition would develop a superior 2-step solution strategy for solving molarity problems.	Worked examples led to better transfer and more common use of 2-step solution or multi-step solutions than the textedit technique	Effect on transfer between two groups $\eta^2 = 0.20$
Nievelstein, Van Gog, Van Dijck, & Boshuizen	2013	eLearning	Law Education	First year law students ($n = 75$)	No	Tested whether the expertise reversal effect was evident in a less-structured domain (solving legal cases). Had 3 independent variables 1) worked examples vs problem solving; 2) process steps vs no problem steps; and 3) first-year vs third year students.	No expertise reversal effect; worked examples were more efficient (better performance after investing less time).	Main effect due to worked examples for first year students: Cohen's $f = 1.24$ and for third year students: Cohen's $f = 1.02$
Quilici & Mayer	1996	Paper	Statistics	College students in three experiments ($n = 81$, $n = 54$, $n = 128$)	No	Experiment 1 compared three groups: no example, one example, and three examples. The hypothesis was that viewing three examples would help with schema construction on what to consider when deciding what statistical test to use. Experiment 2 tried to determine if some examples were more likely to promote schema development than others. Example 3 added to the previous examples by going beyond categorizing examples and rather having the students solve them.	Experiment 1 found that examples helped with schema development - but both the one example and three example groups showed such development. Experiment 2 found that students developed schema better when they viewed examples that differed - using the same situation with different statistical details in the different examples hampered schema acquisition. Experiment 3 found that lower ability students had a harder time categorizing problems.	None reported
Reed, Corbet, Hoffman, Wagner, Hoffman, & MacLaren	2012	Cognitive tutor	Arithmetic and algebra	High School students ($n = 128$)	Yes. Three sessions, separated by several weeks.	Compared four groups 1) Cognitive tutor; 2) Static Table, 3) Static Graphics, 4) Interactive Graphics. Groups 2-4 viewed worked examples alternating with practice problems. Group 1 was the control group and just used the hints and tips in the cognitive tutor.	There was no difference among the four groups on a delayed post-test. The CT group made the fewest mistakes during the learning, but they also took the longest and then there was no difference on the post-test. Worked examples were more efficient.	No significant differences on post-test.

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Richey & Nokes-Malach	2013	Paper	All three experiments: Electricity and Electrical circuits	Experiment 1: Middle school students ($n = 97$) Experiments 2 and 3: Undergrad psychology students ($n = 84$ and $n = 92$)	Experiment 1: Yes - students received class participation points for completing activities. Experiments 2 and 3: No	They did three experiments to compare the results when instructional explanations were provided and when they were withheld.	Students in both experiments did better when instructional explanations were withheld. The result was more distinct in the second example with college students - possibly because it was in a more controlled laboratory setting, rather than in a classroom setting. And also possibly because this type of example is more appropriate for college students than middle school students because they have more experience working through materials on their own. Providing instructional explanations interfered with conceptual understanding more than with procedural.	Providing vs. Withholding effect on conceptual understanding: Experiment 1 - $\eta^2 = .04$; Experiment 2 - $\eta^2 = .07$; Experiment 3 - $\eta^2 = .23$
Rourke & Sweller	2009	Paper	Art and Design	1st year Undergrads	No	Used worked examples to teach characteristics of different designers' work. Considered the difference for expert and novice students. Expected worked examples to help in spite of the ill-defined problem.	Examples were preferable to problem solving - for both near and far transfer. Worked-example effect is evident in an ill-defined domain. No expertise reversal effect.	Main effect due to type of instruction (examples vs problem solving): $\eta^2 = .69, \eta^2 = .54$
Scheiter, Gerjets, & Schuh	2009	Computer based with animations	Algebra word problems	9th grade ($n = 32$)	No	They "tested the assumption that hybrid animations, where a realistic animation of the problem statement is morphed into a more abstract representation of the problem statement and of subsequently carried-out solution steps, can improve problem-solving performance compared to a text-only version of the instructional materials." (p. 492).	Worked example group spent less time	Effect of condition on transfer for similar problems $f = 0.71$

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Schwonke, Renkl, Krieg, Wittwer, Aleven, & Salden	2009	Computer based tutored problem solving with faded worked examples	Geometry	Eighth and Ninth Grade Students ($n = 50$)	No	"Recently it has been argued that the worked-example effect, as postulated by Cognitive Load Theory, might only occur when compared to unsupported problem-solving, but not when compared to well-sup- ported or tutored problem-solving as instantiated, for example, in Cognitive Tutors. In two experiments, we compared a standard Cognitive Tutor with a version that was enriched with faded worked examples" (p. 258).	"In Experiment 1, students in the example condition needed less learning time to acquire a comparable amount of procedural skills and conceptual understanding. In Experiment 2, the efficiency advantage was replicated. In addition, students in the example condition acquired a deeper conceptual understanding. The present findings demonstrate that the worked-example effect is indeed robust and can be found even when compared to well-supported learning by problem-solving" (p. 258).	Experiment 1: Students in example group spent significantly less time Cohen's $d = -0.88$ Experiment 2: Examples better for transfer $d = .73$
Schwonke, Renkl, Salden, & Aleven	2011	Cognitive tutor	Geometry	Ninth grade students ($n = 125$)	No	They investigated the effects of fading sequences differing in the ratio of worked steps and to-be-solved problem steps on cognitive load and learning outcomes. Expected more worked steps to be beneficial for conceptual skills more than procedural skills.	"Problem solving alone had an advantage over higher proportions of example-based learning – yet, only with respect to the acquisition of procedural knowledge. Generally, this finding points to the importance of problem solving opportunities for the development of procedural skills (Trafton & Reiser, 1993). For conceptual knowledge, on the other hand, no ratio of worked steps and to-be-solved steps had an advantage over another" (p. 61).	To-be-solve steps had higher cognitive load that worked steps with effect size $\eta^2 = .27$ for easy steps and .29 for hard steps.

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Sweller & Cooper	1985	Paper	Algebra	Experiment 1: 22 year 9 students, 22 year 11 students, and 18 math ed university students; Experiment 2: year 9 students ($n = 20$) Experiment 3: year 9 students ($n = 22$) Experiment 4: year 8 students ($n = 40$) Experiment 5: year 8 students ($n = 24$)	No	Experiment 1: Tested memory of students and ability to correctly use a procedure they remembered from a previous example problem. Experiment 2: Compared a worked example group with a non-worked example group. Experiment 3: Compared a worked example group with a problem solving group - used more complicated problems than Experiment 2. Experiment 4: Similar to experiment 3, except learners were reminded to make sure they completely understood a worked example before going on to a practice problem. Experiment 5: Similar to experiment 4 except the learners in the two groups were paired based on similar ability and then the conventional problem solver was timed and the paired student in the worked example group was given the same amount of time.	Experiment 1: Year 11 students were more like university students in ability to remember, but more like year 9 students in ability to select correct procedures. They concluded more experienced problem solvers have schemas that link specific skills to specific types of problems. Experiment 2: The worked example group spent less time with "no discernable detriment to their problem solving skills". Experiment 3: Worked examples may help with schema development as evidenced by the fact that the worked example group made fewer errors on the test problems than the traditional problem solving group did. Experiment 4: The benefits of worked examples may be decreased when dissimilar problems are inserted between viewing the example and seeing the results. Example 5: Students in worked example group viewed a lot more examples than the conventional group.	None reported

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Van Gog	2011	Video with expert model solving the problem	Problem solving - Frog Leap problem - moving frogs from one side of the river to another on stones	Undergrads ($n = 32$)	No.	This article looked at the difference in using a worked example before asking a student to work a problem and asking a student to work a problem before providing a worked example. The author noted that prior research had shown that an example-problem pair was more effective than a problem-example pair. However, prior research had looked at pairs of problems that used different types of problem solving techniques. She hypothesized that if using several example-practice pairs for problems that all used the same problem solving technique, then the order of the example and the practice problem would not matter. One group completed an e-Learning lesson that used an example-practice-example-practice sequence the other completed the same lesson but used a practice-example-practice-example sequence.	The author concluded that the order the problems are presented in does not matter as much as the number of opportunities to practice a particular problem solving technique.	Not a significant result
Van Gog & Kester	2012	Paper	Trouble-shooting electrical circuits	Undergrads ($n = 40$)	No	Compared worked examples only to example problem pairs. Hypothesized that there would be no difference in an immediate retention test, but that a test one week later would show the example problem pairs were better, due to the testing effect.	Surprisingly, they saw the opposite of their hypothesis - the students who saw only examples did better on tests of far transfer then those who saw example-problem pairs. The testing effect did not seem to apply for teaching problem solving.	Effect on student performance one week after training: Cohen's $d = .66$
Van Gog, Kester, & Paas	2011	Paper	Trouble-shooting electrical circuits	High School Students ($n = 103$)	No	Compared four groups: 1) worked examples only, 2) examples then problem solving, 3) problem solving then examples, 4) problem solving only.	"Results showed that the problem solving only and problem-example pairs conditions were less effective [an efficient] than the examples only and example-problem pairs conditions."	Main effect on mental effort due to group $\eta^2 = .20$; when included student performance $\eta^2 = .17$

Author	Date	Type of Examples	Content Area	Age of Subjects	In a classroom setting?	Research Questions	Findings	Effect Size
Vogel-Walcutt, Gebrim, Bowers, Carper & Nicholson	2011	eLearning	Military Simulation tasks	Undergrads ($n = 78$)	No	Compared an eLearning lesson using CLT (worked examples) to one using a constructivist approach (problem based learning (PBL)).	Found no difference in the acquisition of procedural, conceptual or declarative knowledge. CLT had a slight advantage in integrated knowledge. But they argued that CLT is more efficient in regards to both time and cost. Creating PBL activities might not be worth the cost.	Effect size (ES) = 0.04 (They didn't report what type of measurement they used).
Wong, Leahy, Marcus, & Sweller	2012	Video/ Static Graphics	Origami	Ten and eleven year old children ($n = 66$)	No	Compared four groups: 1) static graphic short segments, 2) static graphic long segment, 3) animated graphics short segments, 4) animated graphics long segments	They found the animations are effective if they are provided in short segments. They found that providing long segments significantly increased the cognitive load and thus negated the positive effects of using the animation.	Animation superior to static graphics $\eta^2 = .123$; significant interaction effect $\eta^2 = .080$

APPENDIX B

List of Mathematical Problems in Practice Assignments

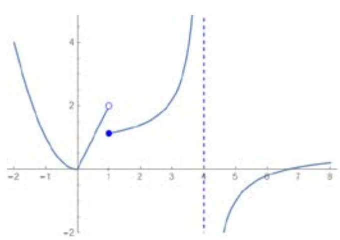
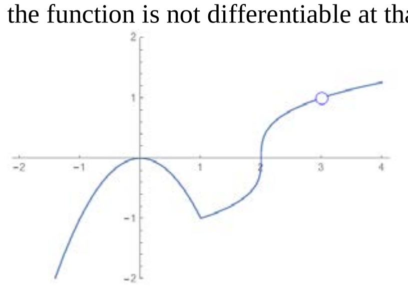
List of Mathematical Problems in Problem Sets

Lesson 1A: Tangent Lines

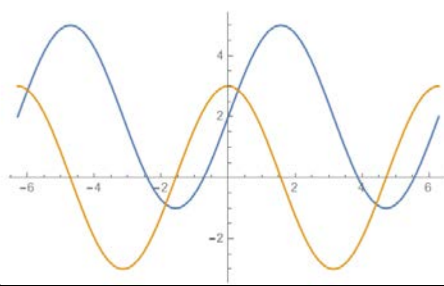
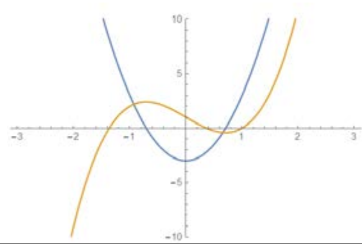
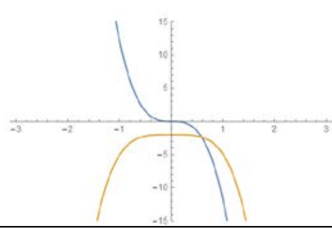
Exercise #	Exercise Prompt	Objective(s)	Problem Type
1A.1	Find the slope of the secant line for the function $f(x) = 3x^2 - 2x + 1$ through the points where $x = 1$ and $x = 3$.	Obj. 1A.3	Fully Worked
1A.2	Find the slope of the secant line for the function $f(x) = \frac{12}{x-5}$ through the points where $x = -1$ and $x = 4$.	Obj. 1A.3	Partially Worked
1A.3	Find the slope of the secant line for the function $f(x) = 2x(x - 3)$ through the points where $x = 2$ and $x = 3$.	Obj. 1A.3	Practice
1A.4	Find the slope of the secant line for the function $f(x) = \sqrt{x^2 + 4x + 4}$ through the points where $x = 1$ and $x = 4$.	Obj. 1A.3	Practice
1A.5	Find the slope of the tangent line for the function $f(x) = \frac{2x}{x-1}$ at the point $x = 2$.	Obj. 1A.4	Fully Worked
1A.6	Find the slope of the line tangent to the curve $f(x) = \sqrt{x - 3}$ at the point $x = 4$.	Obj. 1A.4	Partially Worked
1A.7	Find the slope of the line tangent to the curve $f(x) = 8x^2 - 12x + 1$ at the point $x = 1$.	Obj. 1A.4	Practice
1A.8	Find the slope of the line tangent to the curve $f(x) = \frac{1}{x-2}$ at the point $x = -4$.	Obj. 1A.4	Practice
1A.9	Consider the function $f(x) = 4 - x^2$. a. Graph the function. b. Sketch the secant line to $f(x)$ through the points where $x = 1$ and $x = 2$. c. Sketch the tangent line to $f(x)$ at the point $x = 1$. d. Find the slope of the secant line though the points where $x = 1$ and $x = 2$. e. Find the slope of the tangent line at the point $x = 1$. f. Explain how the slopes you found in parts d and e are related to the graph you drew in parts a, b, and c.	Obj. 1A.1 Obj. 1A.2 Obj. 1A.3 Obj. 1A.4	Fully Worked
1A.10	Consider the function $f(x) = \sqrt{x + 5}$. a. Graph the function. b. Sketch the secant line to $f(x)$ through the points where $x = -1$ and $x = 4$. c. Sketch the tangent line to $f(x)$ at the point $x = -1$. d. Find the slope of the secant line though the points where $x = -1$ and $x = 4$. e. Find the slope of the tangent line at the point $x = -1$. f. Explain how the slopes you found in parts d and e are related to the graph you drew in parts a, b, and c.	Obj. 1A.1 Obj. 1A.2 Obj. 1A.3 Obj. 1A.4	Practice
1A.11	A rock is thrown vertically into the air. The height of the rock at time t (in seconds) is given by the equation $s(t) = -16t^2 + 32t + 4$. a. Find the average velocity of the rock from time $t = 0$ to $t = 2$. b. Find the instantaneous velocity of the rock at time $t = 1$.	Obj. 1A.3 Obj. 1A.4 Obj. 1A.5	Partially Worked

Exercise #	Exercise Prompt	Objective(s)	Problem Type
1A.12	A particle is moving horizontally along a line. The position of the particle on the line at time t (in seconds) is given by the equation $s(t) = -t^3 - 2t^2$. a. Find the average velocity of the rock from time $t = 0$ to $t = 4$. b. Find the instantaneous velocity of the rock at time $t = 4$.	Obj. 1A.3 Obj. 1A.4 Obj. 1A.5	Practice
1A.13	Explain how slopes of secant and tangent lines are related to average and instantaneous rates of change.	Obj. 1A.5	Practice

Lesson 1B: Derivatives at a Point

Exercise #	Exercise Prompt	Objective(s)	Problem Type
1B.1	Find $f'(2)$ where $f(x) = -x^2 + 4$.	Obj. 1B.2	Fully Worked
1B.2	Find $f'(-1)$ where $f(x) = \frac{1}{3x-4}$.	Obj. 1B.2	Fully Worked
1B.3	Find $f'(0)$ where $f(x) = 3x(x-1)$.	Obj. 1B.2	Partially Worked
1B.4	Find $f'(4)$ where $f(x) = x^2 + 3x$.	Obj. 1B.2	Practice
1B.5	Find $f'(-1)$ where $f(x) = \sqrt{2x+5}$.	Obj. 1B.2	Practice
1B.6	Let $f(x) = x^2 + 9$. a. Sketch the graph of $f(x)$. b. Find $f'(-1)$, $f'(0)$, and $f'(2)$. c. Explain what each of the derivatives you found in part b indicates about the graph of $f(x)$.	Obj. 1B.3	Partially Worked
1B.7	Let $f(x) = x^3 + 3x + 1$. a. Sketch the graph of $f(x)$. b. Find $f'(-1)$ and $f'(1)$. c. If you did part a correctly, you will have found that $f'(-1) = f'(1)$. Explain how that is possible. Use a drawing as part of your explanation.	Obj. 1B.3	Practice
1B.8	State the definition of a derivative for a function $f(x)$ at the point $x = a$. Explain what the derivative tells us about the function $f(x)$.	Obj. 1A.4 Obj. 1B.1	Practice
1B.9	For the function shown in the graph below, please identify each point where the function is not differentiable and explain why the function is not differentiable at that point. 	Obj. 1B.4	Fully Worked
1B.10	For the function shown in the graph below, please identify each point where the function is not differentiable and explain why the function is not differentiable at that point. 	Obj. 1B.4	Practice

Lesson 1C: The Derivative as a Function

Exercise #	Exercise Prompt	Objective(s)	Problem Type
1C.1	Use the definition of the derivative to find the derivative of $f(x) = x^3 - 2x$.	Obj. 1C.1	Fully Worked
1C.2	Use the definition of the derivative to find $\frac{dy}{dx}$ where $y = \frac{x-1}{2x+4}$.	Obj. 1C.1 Obj. 1C.3	Partially Worked
1C.3	Use the definition of the derivative to find $f'(x)$ where $f(x) = \sqrt{x^2 - 2}$.	Obj. 1C.1 Obj. 1C.3	Partially Worked
1C.4	Use the definition of the derivative to find $\frac{dy}{dx}$ where $y = 2x^2 + \sqrt{x}$	Obj. 1C.1 Obj. 1C.3	Practice
1C.5	Use the definition of the derivative to find the derivative of $f(x) = x^{-2}$	Obj. 1C.1	Practice
1C.6	The following graph shows the graph of a function and the graph of its derivative. Determine which graph represents the function and which graph is the derivative. 	Obj. 1C.2	Fully Worked
1C.7	The following graph shows the graph of a function and the graph of its derivative. Determine which graph represents the function and which graph is the derivative. 	Obj. 1C.2	Practice
1C.8	The following graph shows the graph of a function and the graph of its derivative. Determine which graph represents the function and which graph is the derivative. 	Obj. 1C.2	Practice
1C.9	Find the equation of the line tangent to the graph of $y = 2x^2$ at the point $x = 1$.	Obj. 1C.1 Obj. 1C.4	Fully Worked

Exercise #	Exercise Prompt	Objective(s)	Problem Type
1C.10	Find the equation of the line tangent to the graph of $y = 4x^2 - 3x + 2$ at the point $x = 1$.	Obj. 1C.1 Obj. 1C.4	Partially Worked
1C.11	Find the equation of the line tangent to the graph of $y = \frac{1}{x-1}$ at the point $x = 5$.	Obj. 1C.1 Obj. 1C.4	Practice
1C.12	Find the equation of the line tangent to the graph of $y = -3x^2 - 1$ at the point $x = -2$.	Obj. 1C.1 Obj. 1C.4	Practice

Lesson 2A: Basic Differentiation Rules

Exercise #	Exercise Prompt	Objective(s)	Problem Type
2A.1	Find the derivative of $f(x) = 3x^2 - 2x + 1$	Obj. 2A.1	Fully Worked
2A.2	Find the derivative of $f(x) = 2x^{\frac{3}{5}}$	Obj. 2A.2	Fully Worked
2A.3	Find $\frac{dy}{dx}$ where $y = 3x^2(x^3 - 2)$	Obj. 2A.1 Obj. 2A.3	Fully Worked
2A.4	Find $f'(x)$ where $f(x) = \frac{x^2+3x+2}{3x-1}$	Obj. 2A.1 Obj. 2A.4	Fully Worked
2A.5	Find $f'(x)$ where $f(x) = -5x + x^3$	Obj. 2A.1	Practice
2A.6	Find $f'(x)$ where $f(x) = \frac{1}{x} + e^x$	Obj. 2A.5 Obj. 1A.6	Partially Worked
2A.7	Find the derivative of $f(x) = x^{\frac{2}{3}}(x^2 + 5)$	Obj. 2A.6	Partially Worked
2A.8	Find the derivative of $y = -8x^2 + 4x + \pi$	Obj. 2A.1	Practice
2A.9	Find $f'(x)$ where $f(x) = -e^x(x^2 - 2)$	Obj. 2A.5 Obj. 2A.6	Practice
2A.10	Find $\frac{dy}{dx}$ where $y = \frac{3e^x}{x^2+1}$.	Obj. 2A.6	Practice
2A.11	Find the derivative of $\frac{\sqrt{x}(x^2+3x+1)}{3x^2}$	Obj. 2A.6	Practice
2A.12	Find the equation of the line tangent to the graph of $y = -8x^2 - 2x + 1$ at the point $x = -2$.	Obj. 1C.4 Obj. 2A.1	Practice

Lesson 2B: Trigonometric Rules

Exercise #	Exercise Prompt	Objective(s)	Problem Type
2B.1	Find the derivative of $y = 3\tan x$.	Obj. 2B.1	Fully Worked
2B.2	Find the derivative of $y = 4x^2 \cos x$.	Obj. 2B.1	Fully Worked
2B.3	Find the derivative of $y = \sqrt{x}\cot x$	Obj. 2B.1	Fully Worked
2B.4	Find the slope of the line tangent to the curve $y = \frac{1}{2}\sec x$ at the point $x = \pi$.	Obj. 2B.1	Partially Worked
2B.5	Find $\frac{dr}{ds}$ where $r = \frac{3\tan s}{4e^s+4}$.	Obj. 2B.1	Partially Worked
2B.6	Find the derivative of $y = 3\cos x + 4\sin x$.	Obj. 2B.1	Practice
2B.7	Find the derivative of $r = 3\csc t + 4\sec t$.	Obj. 2B.1	Practice
2B.8	Find the derivative of $y = (3x+2)\sin x$ at the point $x = \frac{\pi}{2}$.	Obj. 2B.1	Practice
2B.9	Find the instantaneous rate of change for the function $f(x) = 3\sin x + x\cos x$ at the point $x = 0$.	Obj. 2B.1	Practice
2B.10	Find the derivative of $y = \sec x \tan x$	Obj. 2B.1	Practice
2B.11	Find the first and second derivatives of the function $y = 3x^2 + 2\sin x$.	Obj. 2B.2	Fully Worked
2B.12	Find the first and second derivatives of the function $y = 2x\cos x$.	Obj. 2B.2	Partially Worked
2B.13	Find the first and second derivatives of the function $y = 2\csc x$.	Obj. 2B.2	Practice
2B.14	The position of a particle moving along a horizontal line is given by the position function $s = 2t^3 \sin t$. a. Find the average rate of change of the particle over the interval $1 \le t \le 3$. b. Find the instantaneous rate of change of the particle at the point $t = 1$.	Obj. 1A.5 Obj. 2B.1	Partially Worked
2B.15	The position of a particle moving along a horizontal line is given by the position function $s = (2t-1)\cos t$. c. Find the average rate of change of the particle over the interval $0 \le t \le \pi$. d. Find the instantaneous rate of change of the particle at the point $t = 0$.	Obj. 1A.5 Obj. 2B.1	Practice

Lesson 2C: The Chain Rule

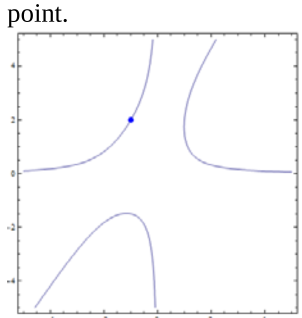
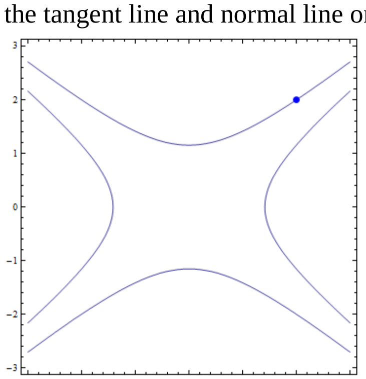
Exercise #	Exercise Prompt	Objective(s)	Problem Type
2C.1	Find the derivative of $y = (3x^2 + 2x)^5$	Obj. 2C.1	Fully Worked
2C.2	Find $\frac{dy}{dx}$ where $y = \sin(2x - 3)$.	Obj. 2C.1	Fully Worked
2C.3	Find the derivative of $y = \sqrt{\tan(x^2)}$.	Obj. 2C.2	Fully Worked
2C.4	Find $f'(x)$ where $f(x) = \cos(e^{3x+2})$.	Obj. 2C.2	Fully Worked
2C.5	Find $f'(x)$ for the function $f(x) = \sqrt{\frac{x-1}{2x+3}}$	Obj. 2C.1	Partially Worked
2C.6	Find the derivative of $y = \left(2\sin\left(\sqrt{x^2 - 3x + 2}\right)\right)^3$	Obj. 2C.2	Partially Worked
2C.7	Find $\frac{dy}{dx}$ for $y = e^{3x}$.	Obj. 2C.1	Practice
2C.8	Find the derivative of $y = -\csc(x^2 + 2)$	Obj. 2C.1	Practice
2C.9	Find the derivative of $y = e^{x^2} + \sec(2x)$.	Obj. 2C.1	Practice
2C.10	Find $f'(x)$ where $f(x) = \sqrt{3 + x^2 + \sin(4x)}$.	Obj. 2C.2	Practice
2C.11	Find the first and second derivative of $y = (x^2 - 3)^{-2}$.	Obj. 2B.2 Obj. 2C.2	Practice
2C.12	Find the equation of the line tangent to the graph of $y = 3xe^{4x^2}$ at the point $x = 0$.	Obj. 1C.4 Obj. 2C.1	Practice

Lesson 3A: Applications

Exercise #	Exercise Prompt	Objective(s)	Problem Type
3A.1	A particle is moving along a horizontal axis. The position of the particle at time t (measured in seconds) is given by the equation $x(t) = t^3 - t^2 - t + 7$. a. Find the position of the particle at time $t = 2$. b. Find the displacement of the particle from time $t = 0$ to $t = 5$. c. Find the times when the particle changes direction. d. Find the total distance traveled by the particle between time $t = 0$ and $t = 5$.	Obj. 3A.1	Fully Worked
3A.2	A particle is moving along a horizontal axis. The position of the particle at time t (measured in seconds) is given by the equation $x(t) = 2t^3 - 14t^2 + 60t + 6$ a. Find the instantaneous velocity of the particle at time $t = 4$. b. Find the speed of the particle when $t = 4$. c. Find the times when the particle changes direction. d. Find the acceleration of the particle at time $t = 3$.	Obj. 3A.1	Partially Worked
3A.3	A particle is moving along a horizontal axis. The position of the particle at time t (measured in seconds) is given by the equation $x(t) = 3t^4 - 16t^3 - 42t^2 + 120t$ a. Find the velocity, speed, and acceleration of the particle at time $t = 3$. b. Find the times when the particle changes direction. c. Find the total distance traveled by the particle from $t = 0$ to $t = 7$.	Obj. 3A.1	Practice
3A.4	Suppose that a stone is thrown vertically upward from the edge of a cliff with an initial velocity of 64 ft/sec from a height of 32 ft above the ground. The height s (in feet) of the stone above the ground t seconds after it is thrown is $s(t) = -16t^2 + 64t + 32$. a. When does the stone reach its highest point? b. When does the stone strike the ground? c. With what velocity will the stone strike the ground?	Obj. 3A.1	Fully Worked
3A.5	Suppose that a stone is thrown vertically upward from the edge of a cliff with an initial velocity of 22 ft/sec from a height of 10 ft above the ground. The height s (in feet) of the stone above the ground t seconds after it is thrown is $s(t) = -16t^2 + 22t + 10$. a. When does the stone reach its highest point? b. When does the stone strike the ground? c. With what velocity will the stone strike the ground?	Obj. 3A.1	Practice
3A.6	Consider the cost function $C(x) = -0.01x^2 + 40x + 100, 0 \leq x \leq 1500$. Find the average and marginal cost functions. Then determine the average and marginal cost when $x = 1000$. Interpret each of these values.	Obj. 3A.2	Fully Worked

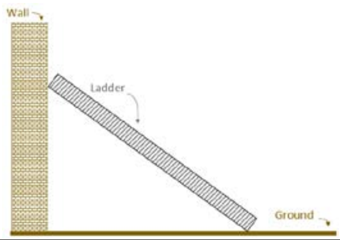
Exercise #	Exercise Prompt	Objective(s)	Problem Type
3A.7	Consider the cost function $C(x) = -0.04x^2 + 100x + 800, 0 \leq x \leq 1000$. Find the average and marginal cost functions. Then determine the average and marginal cost when $x = 500$. Intepret each of these values.	Obj. 3A.2	Practice
3A.8	A drug is injected into a patient's bloodstream. The concentration of the drug in the bloodstream *t* hours after the drug is injected is given by $C(t) = \frac{0.12t}{t^2+t+1}$. Find the rate of change of the concentration 30 minutes after injection.	Obj. 3A.3	Fully Worked
3A.9	The speed of blood flowing along the central axis (in cm/s) of a certain artery is $S(R) = 1.8 \times 10^5 R^2$ where R is the radius of the artery measured in cm. Find the rate of change of the speed with respect to the radius if an artery has a radius of 1.2×10^{-2} cm.	Obj. 3A.3	Fully Worked
3A.10	When a bactericide was added to a nutrient broth in which bacteria were growing, the bacterium population continued to grow for a while, but then stopped growing and began to decline. The size of the population at time t (in hours) is given by $b(t) = 10^6 + 10^4 t - 10^3 t^2$. How long did it take for the population to stop growing?	Obj. 3A.3	Practice
3A.11	The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$. How fast is the water running out at the end of 10 minutes? What is the average rate at which the water flows out during the first 10 minutes?	Obj. 3A.3	Practice

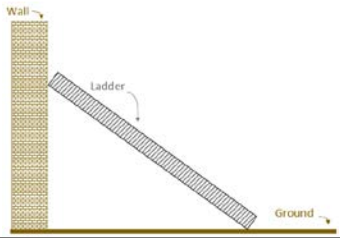
Lesson 3B: Implicit Differentiation

Exercise #	Exercise Prompt	Objective(s)	Problem Type
3B.1	Determine whether $x^2y^3 + 3x \sin y = 2$ is implicitly or explicitly defined. Then find $\frac{dy}{dx}$.	Obj. 3B.1 Obj. 3B.2	Fully Worked
3B.2	Determine whether $y = \frac{3x-1}{4x^2+3x-2}$ is implicitly or explicitly defined. Then find $\frac{dy}{dx}$.	Obj. 3B.1 Obj. 3B.2	Fully Worked
3B.3	Determine whether $2x(x+2y)^3 = 7$ is implicitly or explicitly defined. Then find $\frac{dy}{dx}$.	Obj. 3B.1 Obj. 3B.2	Partially Worked
3B.4	Determine whether $y = \sin(2xy)$ is implicitly or explicitly defined. Then find $\frac{dy}{dx}$.	Obj. 3B.1 Obj. 3B.2	Practice
3B.5	Determine whether $y = \cot^3\left(\sqrt{3x^2}\right)$ is implicitly or explicitly defined. Then find $\frac{dy}{dx}$.	Obj. 3B.1 Obj. 3B.2	Practice
3B.6	Find the equation of the tangent and normal lines to the curve $x^2y + xy^2 = 3x - 8$ at the point $(2, -1)$.	Obj. 3B.3 Obj. 3B.4	Fully Worked
3B.7	Find the equation of the tangent and normal lines to the curve $x^2y + 3xy - 2y^2 = -28$ at the point $(2, -2)$.	Obj. 3B.3 Obj. 3B.4	Partially Worked
3B.8	Find the equation of the tangent and normal lines to the curve $\sin(3x + 2y) = \cos(4x + 2y)$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.	Obj. 3B.3 Obj. 3B.4	Practice
3B.9	For the equation $3x^2y + 4xy - 2xy^2 = 6$, find the equation of the tangent line and the normal line at the point $(-1,2)$. Then sketch the tangent line and normal line on the graph at the given point. 	Obj. 3B.3 Obj. 3B.4 Obj. 3B.5	Partially Worked
3B.10	For the equation $(2x^2 - 3y^2)^2 = 16$, find the equation of the tangent line and the normal line at the point $(2,2)$. Then sketch the tangent line and normal line on the graph at the given point. 	Obj. 3B.3 Obj. 3B.4 Obj. 3B.5	Practice

Exercise #	Exercise Prompt	Objective(s)	Problem Type
3B.11	Find $\frac{d^2y}{dx^2}$ for the following implicitly defined equation: $y^2 - 4x - 5y = 1$	Obj. 3B.2 Obj. 3B.6	Fully Worked
3B.12	Find $\frac{d^2y}{dx^2}$ for the following implicitly defined equation: $x^2 + 2xy + y^2 = 9$	Obj. 3B.2 Obj. 3B.6	Partially Worked
3B.13	Find $\frac{d^2y}{dx^2}$ for the following implicitly defined equation: $xy + 2y^2 = 2$	Obj. 3B.2 Obj. 3B.6	Practice
3B.14	Find $\frac{d^2y}{dx^2}$ for the following implicitly defined equation: $3\sqrt{x} - 4y^2 = 0$	Obj. 3B.2 Obj. 3B.6	Practice
3B.15	Find $\frac{d^2y}{dx^2}$ for the following implicitly defined equation: $x^2y^2 = 4$	Obj. 3B.2 Obj. 3B.6	Practice

Lesson 3C: Related Rates

Exercise #	Exercise Prompt	Objective(s)	Problem Type
3C.1	Assume that $3x^2 - 2xy + 4y^2 = 15$. Find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 3$, $x = 1$, and $y = 2$.	Obj. 3C.1	Fully Worked
3C.2	Assume that $3r^2 + 4s = -1$. Find $\frac{dr}{dt}$ when $r = 1, s = -1$, and $\frac{ds}{dt} = -3$.	Obj. 3C.1	Fully Worked
3C.3	Assume that $L = \sqrt{a^2 + b^2}$. Find $\frac{db}{dt}$ when $\frac{da}{dt} = 2$, $\frac{dL}{dt} = -1$, $a = 3$, and $b = 4$.	Obj. 3C.1	Practice
3C.4	A 26-foot ladder is leaning against a house when its base starts to slide away (see picture). By the time the base is 24 ft from the house, the base is moving at the rate of 3 ft/sec. How fast is the top of the ladder sliding down the wall then? 	Obj. 3C.1	Fully Worked
3C.5	A girl is flying a kite at a height of 200 feet. The wind is carrying the kite horizontally away from her at a rate of 25 ft/sec (the kite stays at the same height). How fast must she let out the string when the string is 250 feet long?	Obj. 3C.1	Practice
3C.6	A spherical balloon is inflated with helium at the rate of 80π ft^3 per minute. How fast is the balloon's radius increasing at the instant the radius of the balloon is 4 ft? How fast is the surface area increasing?	Obj. 3C.1	Fully Worked
3C.7	A piston is seated at the top of a cylindrical chamber with radius 5 cm when it starts moving into the chamber at a constant speed of 3 cm/s (see picture). What is the rate of change of the volume of the cylinder when the piston is 2 cm from the base of the chamber? [insert picture – see Briggs & Stratton pg 179]	Obj. 3C.1	Partially Worked

Exercise #	Exercise Prompt	Objective(s)	Problem Type
3C.8	A 12-foot ladder is leaning against a house when its base starts to slide away (see picture). When the base of the ladder is 4 ft from the house, the base of the ladder is moving at the rate of 3 ft/sec and the top of the ladder is moving down the wall at the rate of 1.06 ft/sec. How fast is the area of the triangle formed by the ladder changing when the base of the ladder is 4 ft from the house? 	Obj. 3C.1	Practice
3C.9	At a sand and gravel plan, sand is falling off a conveyor onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?	Obj. 3C.1	Partially Worked
3C.10	Two boats leave a port at the same time, one travelling west at 20 mi/hr and the other traveling south at 15 miles per hour. At what rate is the distance between them changing 30 minutes after they leave the port?	Obj. 3C.1	Practice
3C.12	A boat is pulled towards a dock by a rope attached to the bow of the boat. The rope is attached to a winch that is on the dock. The winch is 6 feet vertically above the bow of the boat. The rope is hauled in at a rate of 2 ft/sec. How fast is the boat approaching the dock when 10 feet of rope are out?	Obj. 3C.1	Practice

Lesson 4A: Exponential and Logarithmic Functions

Exercise #	Exercise Prompt	Objective(s)	Problem Type
4A.1	Find the derivative of $y = 3^{x^2-2}$	Obj. 4A.1	Fully Worked
4A.2	Find the derivative of $y = \log_2(x^2 - 4x + 1)$	Obj. 4A.2	Fully Worked
4A.3	Find the derivative of $y = \ln(2x) + 4^{-x}$	Obj. 4A.1 Obj. 4A.2	Fully Worked
4A.4	Find the derivative of $y = x^2 + 2^x$	Obj. 4A.1 Obj. 4A.2	Fully Worked
4A.5	Find the derivative of $y = \log_7(\sin(4x - 2))$	Obj. 4A.2	Partially Worked
4A.6	Find the derivative of $f(x) = \ln(5^{2x-4})$	Obj. 4A.1 Obj. 4A.2	Partially Worked
4A.7	The functions $f(x) = \log_3 x$ and $f^{-1}(x) = 3^x$ are inverses of each other. a. Sketch the graph of $f(x)$ and $f^{-1}(x)$. b. Find $f'(9)$ and $(f^{-1})'(2)$. c. Illustrate the derivative you found on part b on the graph you drew for part a. d. The derivative rule for inverses states that $(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}$. Show that the derivatives you found in part b satisfy this inequality.	Obj. 4A.1 Obj. 4A.2	Partially Worked
4A.8	Find the derivative of $f(x) = \sin(\log_3 \sqrt{2x-1})$	Obj. 4A.2	Practice
4A.9	Find the derivative of $f(x) = 7^{5x-\ln(2x)}$	Obj. 4A.1 Obj. 4A.2	Practice
4A.10	Find the derivative of $f(x) = \ln\left(\frac{x-1}{x^2+3}\right)$	Obj. 4A.2	Practice
4A.11	Find the derivative of $f(x) = \sqrt{\sec(2x^5) + 3^x}$	Obj. 4A.1	Practice

Lesson 4B: Logarithmic Differentiation

Exercise #	Exercise Prompt	Objective(s)	Problem Type
4B.1	Use logarithmic differentiation to take the derivative of $y = \frac{3x(x^2-4+2)^2}{x-1}$	Obj. 4B.1	Fully Worked
4B.2	Use logarithmic differentiation to take the derivative of $y = \sqrt{(x^2-4)^5(2x-1)^3}$	Obj. 4B.1	Fully Worked
4B.3	Use logarithmic differentiation to take the derivative of $y = (3x)^{x-1}$	Obj. 4B.1	Fully Worked
4B.4	Use logarithmic differentiation to take the derivative of $y = \frac{(3x-2)^4}{5x^2}$	Obj. 4B.1	Partially Worked
4B.5	Use logarithmic differentiation to take the derivative of $y = (\sin x)^{3x-2}$	Obj. 4B.1	Partially Worked
4B.6	Use logarithmic differentiation to take the derivative of $y = \frac{x^2-4}{(\sin x)^3}$	Obj. 4B.1	Practice
4B.7	Use logarithmic differentiation to take the derivative of $y = (5x^2+3)^{2x}$	Obj. 4B.1	Practice
4B.8	For what types of functions is logarithmic differentiation helpful? When is it required?	Obj. 4B.2	Practice
4B.9	Take the derivative of $y = (3x+4)^2 + \cos x$. Logarithmic differentiation may or may not be useful.	Obj. 4B.2	Fully Worked
4B.10	Take the derivative of $y = (x^2+3)^{x-4}$. Logarithmic differentiation may or may not be useful.	Obj. 4B.2	Partially Worked
4B.11	Take the derivative of $y = (x^2-4x+7)^5$. Logarithmic differentiation may or may not be useful.	Obj. 4B.2	Practice
4B.12	Use logarithmic differentiation to derive the differentiation rule for $y = a^x$.	Obj. 4B.3	Practice

Lesson 4C: Inverse Trigonometric Functions

Exercise #	Exercise Prompt	Objective(s)	Problem Type
4C.1	Find $\frac{dy}{dx}$ for $y = \sin^{-1}(x^2 - 3)$	Obj. 4C.2	Fully Worked
4C.2	Find $\frac{dy}{dx}$ for $y = \tan^{-1}(x) + \cos(3x)$	Obj. 4C.2	Fully Worked
4C.3	Find $\frac{dy}{dx}$ for $y = \tan^{-1}\sqrt{x^2 - 5}$	Obj. 4C.2	Fully Worked
4C.4	Find $\frac{dy}{dx}$ for $y = \sec^{-1} x - \sqrt{x^2 - 1}, x > 1$	Obj. 4C.2	Fully Worked
4C.5	Find $\frac{dy}{dx}$ for $y = (\sin^{-1} x + \tan^{-1} x)^3$	Obj. 4C.2	Partially Worked
4C.6	Find $\frac{dy}{dx}$ for $y = \tan^{-1}(\ln 2x)$	Obj. 4C.2	Practice
4C.7	Find $\frac{dy}{dx}$ for $y = \sec^{-1}(\cos^2 8x)$	Obj. 4C.2	Practice
4C.8	Find $\frac{dy}{dx}$ for $y = \log_2(x^2) + \sin^{-1}(x^2)$	Obj. 4C.2	Practice
4C.9	Find $\frac{dy}{dx}$ for $y = \text{arcsec}(1 + x)$	Obj. 4C.2	Practice
4C.10	Use implicit differentiation to derive the differentiation formula for $y = \sec^{-1} x$. Carefully show all of your work.	Obj. 4C.1	Fully Worked
4C.11	Use implicit differentiation to derive the differentiation formula for $y = \tan^{-1} x$. Carefully show all of your work.	Obj. 4C.1	Practice
4C.12	Use implicit differentiation to derive the differentiation formula for $y = \sin^{-1} x$. Carefully show all of your work.	Obj. 4C.1	Practice

APPENDIX C

Learner Characteristics

Table C1

Learner Characteristics for Calclulus I

Consideration	**Response**
Physical Age Range:	Adults. Approximately 18 – 40 in age.
Educational Range:	Undergraduate students. Many of them will be first semester freshman who are adjusting to the increased demands of a university course. Most learners are majoring in math or science. Most learners will continue to study calculus by taking Math 113 or Math 215 the semester following this course, although FDMath 112 is a terminal course for a small number of students.
Gender:	Learners are predominantly male. Typically only about 5-15% of the learners are female.
Language Skills:	Most of the learners are native English speakers, although there are typically 2-4 non-native English speakers in the course.
Prerequisite Knowledge/Skills:	Students are expected to have a solid foundational understanding of algebraic principles. In particular they should know: the concept of a function, function notation, graphs of functions, polynomial functions, rational functions, exponential functions, logarithmic functions, trigonometric functions, properties of logarithms, properties of exponents, trigonometric identities, and the unit circle. Students will have varying levels of perquisite knowledge/skills. Some students will have taken Math 109: Precalculus in the previous semester while other students will have had a significant amount of time pass between taking Precalculus and Calculus. For some students that length of time is as long as four or five years. Several of the learners will have taken Calculus in high school (either not for college credit or they did not score high enough on the AP test to qualify for college credit). Other learners will be retaking FDMath 112 for the second time after failing the course.
Learner Attitudes:	There are large variations in attitudes related to the course.

Table C1

Learner Characteristics for Calclulus I

Consideration	**Response**
	However, most learners, as math and science majors, are interested in the topics in the course. In particular, many students are interested in how the topics in the course can be used in applications and in how they will use the content in their future courses.
Learning Style Preferences:	Learners are typically most familiar with lecture based math courses. They are familiar with traditional homework assignments given from a textbook. They are accustomed to the answers to the problems being provided in an answer guide in the back of the book.
Attitudinal Factors:	Many students are very busy with responsibilities outside of school such as families and work. Approximately half of the learners are married and many of them have children. This at times might impact their ability and desire to work on assigned coursework.
Environmental Factors:	The homework assignments will be delivered online through the learning management system. Most learners are familiar with the LMS will be familiar with the online environment. Like any other content delivered online, there is a strong possibility that technology issues will occur. Some learners are more equipped to solve technology issues than others. Help may need to be provided for some.

APPENDIX D

Instructional Objectives

Instructional Objectives

Calculus I: Unit 2 – Differentiation

Lesson 1: The Concept of the Derivative

Lesson 1A: Tangent Lines

- Objective 1A.1: After completing Lesson 1A, learners will be able to explain the concept of a secant line.
- Objective 1A.2: After completing Lesson 1A, learners will be able to explain the concept of a tangent line.
- Objective 1A.3: After completing Lesson 1A, learners will be able to compute the slope of a secant line when given a function and two points.
- Objective 1A.4: After completing Lesson 1A, learners will be able to use the limit of a difference quotient to compute the slope of a tangent line when given a function and one point.
- Objective 1A.5: After completing Lesson 1A, learners will be able to explain the connection between slopes and rates of change.

Lesson 1B: Derivatives at a Point

- Objective 1B.1: After completing Lesson 1B, learners will be able to state the definition of a derivative at a point.
- Objective 1B.2: After completing Lesson 1B, learners will be able to compute the derivative of a function at a point using the definition of a derivative when given a function and a point.
- Objective 1B.3: After completing Lesson 1B, learners will be able to use a graph to explain why non-linear functions have different derivatives at different points.
- Objective 1B.4: After completing Lesson 1B, learners who are provided with the graph of a function will be able to identify points where a function is not differentiable.

Lesson 1C: The Derivative as a Function

- Objective 1C.1: After completing Lesson 1C, learners will be able to compute the derivative using the definition of a derivative when given a function.
- Objective 1C.2: After completing Lesson 1C, learners who are provided with a graph of a function and its derivative will be able to identify which curve is the function and which curve is the derivative.

- Objective 1C.3: After completing Lesson 1C, learners will be able to use prime notation and Leibnitz notation for the derivative interchangeably.
- Objective 1C.4: After completing Lesson 1C, learners will be able to find the equation of the tangent line for a given function at a given point.

Lesson 2: Differentiation Rules

Lesson 2A: Basic Differentiation Rules

- Objective 2A.1: After completing Lesson 2A, learners will be able to take the derivative of polynomial functions.
- Objective 2A.2: After completing Lesson 2A, learners will be able to take the derivative of power functions.
- Objective 2A.3: After completing Lesson 2A, learners will be able to take the derivative of products of functions.
- Objective 2A.4: After completing Lesson 2A, learners will be able to take the derivative of quotients of functions.
- Objective 2A.5: After completing Lesson 2A, learners will be able to take the derivative of $y = e^x$.
- Objective 2A.6: After completing Lesson 2A, learners will be able to take the derivative of functions involving a combination of polynomial, power, and exponential functions.

Lesson 2B: Trigonometric Rules

- Objective 2B.1: After completing Lesson 2B, the learners will be able to take the derivative of functions involving trigonometric functions.
- Objective 2B.2: After completing Lesson 2B, the learners will be able to find second derivatives of functions using the basic differentiation and/or trigonometric differentiation rules.

Lesson 2C: The Chain Rule

- Objective 2C.1: After completing Lesson 2C, the learners will be able to take the derivative of composed functions using the chain rule.
- Objective 2C.2: After completing Lesson 2C, the learners will be able to take the derivative of functions made up of compositions of more than two functions using repeated applications of the same rule.

Lesson 3: Rates of Change

Lesson 3A: Applications

- Objective 3A.1: After completing Lesson 3A, when given a position function $s(t)$, learners will be able to correctly analyze motion along a line.
- Objective 3A.2: After completing Lesson 3A, given a cost function $C(x)$, learners will be able to use a marginal cost function.
- Objective 3A.3: After completing Lesson 3A, learners will be able to interpret the rate of change for a given pair of independent and dependent variables.

Lesson 3B: Implicit Differentiation

- Objective 3B.1: After completing Lesson 3B, learners will be able to explain the difference between implicitly and explicitly defined equations.
- Objective 3B.2: After completing Lesson 3B, learners will be able to find the first derivative of an implicitly defined equation using implicit differentiation.
- Objective 3B.3: After completing Lesson 3B, learners will be able to find the equation of the tangent line for an implicitly defined function.
- Objective 3B.4: After completing Lesson 3B, learners will be able to find the equation of the normal line for an implicitly defined function.
- Objective 3B.5: After completing Lesson 3B, learners who are given the graph of a curve will be able to sketch the tangent line and the normal line at a given point on the graph.
- Objective 3B.6: After completing Lesson 3B, learners will be able to find the second derivative of an implicitly defined equation using implicit differentiation.

Lesson 3C: Related Rates

- Objective 3C.1: After completing Lessons 3C and 3D, learners will be able to solve related rates problems.

Lesson 4: Derivatives of Inverses

Lesson 4A: Exponential and Logarithmic Functions

- Objective 4A.1: After completing Lesson 4A, learners will be able to take the derivative of exponential functions.
- Objective 4A.2: After completing Lesson 4A, learners will be able to take the derivative of logarithmic functions.

Lesson 4B: Logarithmic Differentiation

- Objective 4B.1: After completing Lesson 4B, learners will be able to use logarithmic differentiation to take the derivative of functions.
- Objective 4B.2: After completing Lesson 4B, learners will be able to identify functions where logarithmic differentiation would be optimal for taking the derivative.
- Objective 4B.3: After completing Lesson 4B, learners will be able to derive the differentiation rule for $y = a^x$ using logarithmic differentiation.

Lesson 4C: Inverse Trigonometric Functions

- Objective 4C.1: After completing Lesson 4C, learners will be able to derive the differentiation rules for the inverse trigonometric functions using implicit differentiation.
- Objective 4C.2: After completing Lesson 4C, learners will be able to take the derivative of functions involving inverse trigonometric functions.

APPENDIX E

Survey Results – Instructional Objectives Survey

Instructional Objectives Survey Results

Lesson 1A				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	0	1	2
Do you have any suggestions for improving the objectives for this sub-lesson?	Is it necessary to delineate so much? It seems that you could combine these down to just one or two considering that this is a prerequisite knowledge.			
Lesson 1B				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	0	1	2
Do you have any suggestions for improving the objectives for this sub-lesson?	None.			
Lesson 1C				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	0	1	1
Do you have any suggestions for improving the objectives for this sub-lesson?	None.			
Lesson 2A				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	0	0	3
Do you have any suggestions for improving the objectives for this sub-lesson?	None.			
Lesson 2B				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	1	0	2
Do you have any suggestions for improving the objectives for this sub-lesson?	I think it might be important to tie the trig derivative rules back to the definition of the derivative. Students should be able to show how to generate these rules from the definition. Also, they could verify the accuracy of these rules graphically.			

<table>
<tr><td colspan="5">Lesson 2C</td></tr>
<tr><td rowspan="2">The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.</td><td>Strongly Disagree</td><td>Disagree</td><td>Agree</td><td>Strongly Agree</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>3</td></tr>
<tr><td>Do you have any suggestions for improving the objectives for this sub-lesson?</td><td colspan="4">Wording on Objective 2C.2: When I first read this I was thinking that you meant 2nd and 3rd derivatives—but I think you mean extending the chain rule to involve more than two functions in the composition. Maybe consider a different wording to make it more clear.</td></tr>
<tr><td colspan="5">Lesson 3A</td></tr>
<tr><td rowspan="2">The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.</td><td>Strongly Disagree</td><td>Disagree</td><td>Agree</td><td>Strongly Agree</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>Do you have any suggestions for improving the objectives for this sub-lesson?</td><td colspan="4">None.</td></tr>
<tr><td colspan="5">Lesson 3B</td></tr>
<tr><td rowspan="2">The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.</td><td>Strongly Disagree</td><td>Disagree</td><td>Agree</td><td>Strongly Agree</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>Do you have any suggestions for improving the objectives for this sub-lesson?</td><td colspan="4">Objective 3B.4 - After reading the tasks, I do not know that this is an important objective. You might consider deleting it. But, if this is considered an application of the derivative, then leave it.

Objective 3B.5 - This objective is very similar to previous objectives, other than the normal part. Is it necessary?</td></tr>
<tr><td colspan="5">Lesson 3C</td></tr>
<tr><td rowspan="2">The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.</td><td>Strongly Disagree</td><td>Disagree</td><td>Agree</td><td>Strongly Agree</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>Do you have any suggestions for improving the objectives for this sub-lesson?</td><td colspan="4">Do you also want to focus on how students are able to set-up the problem?</td></tr>
</table>

Lesson 4A				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	1	0	2
Do you have any suggestions for improving the objectives for this sub-lesson?	I would teach this objective after logarithmic differentiation. These derivative rules can be derived using implicit and logarithmic differentiation. I think that understanding how to use implicit differentiation to do these derivatives is a great connection to implicit and logarithmic differentiation and will help the students understand better how derivative rules are created. I would not just want them to do these derivatives with memorized rules. Also, I like to include exponential functions like x^x at this point.			
Lesson 4B				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	0	0	3
Do you have any suggestions for improving the objectives for this sub-lesson?	None.			
Lesson 4C				
The objectives for this lesson accurately describe what a student should be expected to learn in this lesson.	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
	0	**1**	0	2
Do you have any suggestions for improving the objectives for this sub-lesson?	Objective 4C.1 - Do you need justification as to why you only focus on three of them? I think it is necessary at this point to demonstrate why we need these derivatives, so I would include some practical applications. I really agree with the fact that you expect them to derive these inverse trig derivatives and not just memorize them.			

APPENDIX F

Task Analysis

Task Analysis

This task analysis lists the tasks that are required in order for a learner to meet the objectives for the unit on differentiation in a FDMath 112: Calculus I course. Unit 2, the unit covering differentiation, will comprise 3 weeks of instruction. The instruction has been divided into four lessons, each consisting of four sub-lessons as shown in Table E1. A task analysis was done for each of the twelve sub-lessons. For each objective, a list of tasks was given. Then each task was classified as to knowledge type, difficulty, duration, and importance.

Table F1.

Lessons and Sub-lessons in Unit 2

Unit 2: Differentiation
Lesson 1: The Concept of the Derivative
Lesson 1A: Tangent Lines
Lesson 1B: Derivatives at a Point
Lesson 1C: The Derivative as a Function
Lesson 2: Differentiation Rules
Lesson 2A: Basic Differentiation Rules
Lesson 2B: Trigonometric Rules
Lesson 2C: The Chain Rule
Lesson 3: Rates of Change
Lesson 3A: Applications
Lesson 3B: Implicit Differentiation
Lesson 3C: Related Rates
Lesson 4: Derivatives of Inverses
Lesson 4A: Exponential and Logarithmic Functions
Lesson 4B: Logarithmic Differentiation
Lesson 4C: Inverse Trigonometric Functions

Task Analysis for Lesson 1A: Tangent Lines

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 1A.1: After completing Lesson 1A, learners will be able to explain the concept of a secant line.				
Correctly explain that a secant line is a line connecting two points on a curve.	D	L	L	H
Correctly illustrate a secant line with a drawing.	D	L	L	H
Objective 1A.2: After completing Lesson 1A, learners will be able to explain the concept of a tangent line.				
Correctly describe how to approximate a tangent line by gradually decreasing the distance between two points used to form a secant line.	P	M	L	H
Correctly illustrate a tangent line with a drawing.	D	M	L	H
Explain how a tangent line is different from a secant line.	S	M	L	H
Objective 1A.3: After completing Lesson 1A, learners will be able to compute the slope of a secant line when given a function and two points.				
When given a function $f(x)$ and two points $x = a$ and $x = b$, correctly find $f(a)$ and $f(b)$.	P	L	L	H
After finding $f(a)$ and $f(b)$, correctly compute $\Delta y = f(b) - f(a)$.	P	L	L	H
Correctly compute $\Delta x = b - a$	P	L	L	H
Computing the slope of the tangent line as $\frac{\Delta y}{\Delta x} = \frac{f(\square)-f(a)}{b-a}$	P	M	M	H
Objective 1A.4: After completing Lesson 1A, learners will be able to use the limit of a difference quotient to compute the slope of a tangent line when given a function and one point.				
When given a function $f(x)$ and a point $x = \square$, correctly form the difference quotient $\frac{f(x+h)-f(x)}{h}$.	P	H	M	H
After forming the difference quotient, correctly evaluating the limit $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.	P	M	M	H
Objective 1A.5: After completing Lesson 1A, learners will be able to explain the connection between slopes and rates of change.				
Correctly explain that the slope of a secant line represents an average rate of change.	S	M	L	H
Correctly explain that the slope of a tangent line represents an instantaneous rate of change.	S	M	L	H

Task Analysis for Lesson 1B: Derivatives at a Point

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 1B.1: After completing Lesson 1B, learners will be able to state the definition of a derivative at a point.				
Correctly state that the definition of a derivative is: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$.	D	L	L	H
Correctly state that the derivative gives the slope of the tangent line to the curve at the point $x = a$.	S	L	L	H
Correctly state that the derivative gives the instantaneous rate of change of a function at a point.	S	L	L	H
Correctly state that the derivative gives the slope of the curve at a point.	S	L	L	H
Objective 1B.2: After completing Lesson 1B, learners will be able to compute the derivative of a function at a point using the definition of a derivative when given a function and a point				
When given a function $f(x)$ and a point $x = a$, correctly evaluate the difference quotient $\frac{f(a+h)-f(a)}{h}$	P	H	M	H
Correctly evaluate the limit of the difference quotient as $h \to 0$.	P	M	M	H
Objective 1B.3: After completing Lesson 1B, learners will be able to use a graph to explain why non-linear functions have different derivatives at different points.				
When given a graph of a function and a point, correctly represent the derivative at the point by drawing a tangent line and describing the derivative as the slope of the tangent line.	S	M	L	M
When given a graph of a non-linear function and several points, correctly drawing different tangent lines and explaining that the derivative is different because the slopes are different.	S	M	M	M
When given the graph of a non-linear function and two points that have the same derivative, correctly explaining that the derivative is the same because the slope of the tangent lines are the same (although the tangent lines are different).	S	H	M	M
Objective 1B.4: After completing Lesson 1B, learners who are provided with the graph of a function will be able to identify points where a function is not differentiable.				
Correctly identify cusps as points where a function is not differentiable.	D	L	L	M
Correctly identify sharp corners as points where a function is not differentiable.	D	L	L	M
Correctly identify vertical tangents as points where a function is not differentiable.	D	L	L	M
Correctly identify discontinuities as points where a function is not differentiable.	D	L	L	M

Task Analysis for Lesson 1C: The Derivative as a Function

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 1C.1: After completing Lesson 1C, learners will be able to compute the derivative using the definition of a derivative when given a function.				
When given a function $f(x)$, correctly evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$	P	H	M	H
Correctly evaluate the limit of the difference quotient as $h \to 0$.	P	M	M	H
Objective 1C.2: After completing Lesson 1C, learners who are provided with a graph of a function and its derivative will be able to identify which curve is the function and which curve is the derivative.				
When provided with the graph a function and the graph of its derivative, correctly connect horizontal tangent lines with zeroes of the derivative.	S	H	M	M
Correctly connect positive values of the derivative with an increasing function.	S	H	M	M
Correctly connect negative values of the derivative with a decreasing function.	S	H	M	M
Correctly identify use the above features to distinguish between the graph of the function and the graph of its derivative.	S	H	M	M
Objective 1C.3: After completing Lesson 1C, learners will be able to use prime notation and Leibnitz notation for the derivative interchangeably.				
Correctly use $f'(x)$ as the notation for the derivative of $f(x)$.	D	L	L	M
Correctly use $\frac{dy}{dx}$ as the notation for the derivative of $y(x)$.	D	L	L	M
Correctly use $f''(x)$ as the notation for the second derivative of $f(x)$.	D	L	L	M
Correctly use $\frac{d^2y}{dx^2}$ as the notation for the second derivative of $y(x)$.	D	M	L	M
Correctly use $\frac{d}{dx}\big(f(x)\big)$ as notation for the derivative of $f(x)$.	D	M	L	M
Objective 1C.4: After completing Lesson 1C, learners will be able to find the equation of the tangent line for a given function at a given point.				
Given a function $f(x)$, correctly compute $f'(x)$ and identify it as the general equation for the slope of the tangent line.	P	M	M	H
After finding $f'(x)$ and given a point $x = a$, correctly compute $f'(a)$ and identify it as the slope of the tangent line at the point $x = a$.	P	M	M	H
Correctly computing the equation of the tangent line using the point-slope equation for a line.	P	L	M	H

Task Analysis for Lesson 2A: Basic Differentiation Rules

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 2A.1: After completing Lesson 2A, learners will be able to take the derivative of polynomial functions.				
Correctly take the derivative of constant functions using the constant rule: $\frac{d}{dx}(c) = 0$.	P	L	L	H
Correctly take the derivative of x^n using the power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$ where n is a positive integer.	P	L	L	H
Correctly take the derivative of constant multiples of functions using the constant multiple rule: $\frac{d}{dx}\left(kf(x)\right) = kf'(x)$.	P	L	L	H
Correctly take the derivative of sums using the sum rule: $\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$.	P	L	L	H
Correctly take the derivative of differences using the difference rule: $\frac{d}{dx}\left(f(x) - g(x)\right) = f'(x) - g'(x)$.	P	L	L	H
Objective 2A.2: After completing Lesson 2A, learners will be able to take the derivative of power functions.				
Correctly take the derivative of functions of the form $y = x^n$, where n is any real number, using the power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$.	P	L	L	H
Objective 2A.3: After completing Lesson 2A, learners will be able to take the derivative of products of functions.				
Correctly identify a function as a product.	S	H	M	H
Correctly take the derivative using the product rule: $\frac{d}{dx}\left(f(x)g(x)\right) = f(x)g'(x) + f'(x)g(x)$.	P	M	M	H
Objective 2A.4: After completing Lesson 2A, learners will be able to take the derivative of quotients of functions				
Correctly identify functions formed by quotients.	S	M	M	H
Correctly take the derivative using the quotient rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$	P	M	M	H
Objective 2A.5: After completing Lesson 2A, learners will be able to take the derivative of $y = e^x$.				
Correctly take the derivative of $y = e^x$ using the rule $\frac{d}{dx}(e^x) = e^x$.	P	M	M	H
Objective 2A.6: After completing Lesson 2A, learners will be able to take the derivative of functions involving a combination of polynomial, power, and exponential functions.				
Correctly identify which differentiation rules are required to take the derivative of a given function.	S	H	M	H
Correctly apply two or more differentiation rules for the same function.	P	M	M	H

Task Analysis for Lesson 2B: Trigonometric Rules

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 2B.1: After completing Lesson 2B, the learners will be able to take the derivative of functions involving trigonometric functions.				
Correctly take the derivative of equations involving the sine function.	P	L	M	H
Correctly take the derivative of equations involving the cosine function.	P	L	M	H
Correctly take the derivative of equations involving the tangent function.	P	M	M	H
Correctly take the derivative of equations involving the cosecant function.	P	M	M	H
Correctly take the derivative of equations involving the secant function.	P	M	M	H
Correctly take the derivative of equations involving the cotangent function.	P	M	M	H
Correctly take the derivative of equations formed from a combination of polynomial, power, e^x, and trigonometric functions.	P	H	H	H
Objective 2B.2: After completing Lesson 2B, the learners will be able to find second derivatives of functions using the basic differentiation and/or trigonometric differentiation rules.				
Correctly take the second derivative of polynomial functions.	P	L	L	H
Correctly take the second derivative of power functions.	P	L	L	H
Correctly take the second derivative of $y = e^x$	P	L	L	H
Correctly take the second derivative of trigonometric functions.	P	M	M	H
Correctly take the second derivative of equations formed from a combination of polynomial, power, e^x, and trigonometric functions.	P	H	M	H

Task Analysis for Lesson 2C: The Chain Rule

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 2C.1: After completing Lesson 2C, the learners will be able to take the derivative of composed functions using the chain rule.				
Correctly identify an equation as the composition of two functions.	S	H	H	H
Correctly take the derivative of composed functions using the chain rule: $\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)g'(x)$.	P	M	H	H
Objective 2C.2: After completing Lesson 2C, the learners will be able to take the derivative of functions made up of compositions of more than two functions using repeated applications of the same rule.				
Correctly identify an equation as the composition of three or more functions	S	H	H	H
Correctly use repeated applications of the chain rule to take the derivative of equations that are the composition of three or more functions.	P	H	H	H
Correctly identify which of the differentiation rules learned so far are required to take the derivative of an equation formed by a combination or composition of polynomial, power, e^x, and trigonometric functions.	S	H	H	H
Correctly take the derivative of an equation formed by a combination or composition of polynomial, power, e^x, and trigonometric functions.	P	H	H	H

Task Analysis for Lesson 3A: Applications

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 3A.1: After completing Lesson 3A, when given a position function $s(t)$, learners will be able to correctly analyze motion along a line.				
Given a position function $s(t)$ and a time interval $a \leq t \leq b$, correctly find the displacement of the object over the given time interval.	P	L	L	M
Given a position function $s(t)$, and a time interval □ $\leq t \leq b$, correctly find the average velocity of the object over the given time interval.	P	M	L	M
Given a position function $s(t)$, correctly find and interpret the velocity function.	P	M	M	H
Given a position function $s(t)$, correctly find and interpret the acceleration function.	P	M	M	H
Given a position function $s(t)$, correctly find and interpret the jerk function.	P	M	L	L
Given a position function $s(t)$, correctly find and interpret the speed function.	P	M	M	H
Correctly evaluate the velocity, acceleration, jerk, or speed functions at a given point in time.	P	M	M	H
Objective 3A.2: After completing Lesson 3A, given a cost function $C(x)$, learners will be able to use a marginal cost function.				
Given a cost function $C(x)$, correctly compute the cost of producing $x = a$ items.	P	L	L	L
Given the cost function $C(x)$, correctly compute the average cost of producing $x = a$ items.	P	M	L	L
Given a cost function $C(x)$, correctly compute the marginal cost at a production level of $x = a$ items.	P	M	M	L
Correctly explain that the marginal cost is roughly the cost of producing one more item at that particular production level.	S	H	M	L
Objective 3A.3: After completing Lesson 3A, learners will be able to interpret the rate of change for a given pair of independent and dependent variables.				
Given an independent variable, x, and a dependent variable, y, correctly interpret the derivative as the instantaneous rate of change of y with respect to x.	D	M	L	M
Correctly interpret the meaning of instantaneous rate of change when provided with a real-world context for the independent variable, x, and the dependent variable, y.				

Task Analysis for Lesson 3B: Implicit Differentiation

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Objective 3B.1: After completing Lesson 3B, learners will be able to explain the difference between implicitly and explicitly defined equations.				
Correctly define explicitly defined equations as equations that are written in the form $y = f(x)$.	D	L	L	M
Correctly define implicitly defined equations as equations that are written in the form $f(x, y) = g(x, y)$.	D	L	L	M
Correctly identify a given equation as being explicitly or implicitly defined.	D	L	L	M
Objective 3B.2: After completing Lesson 3B, learners will be able to find the first derivative of an implicitly defined equation using implicit differentiation.				
Correctly differentiate each side of an implicitly defined equation.	P	H	M	H
Correctly use algebra to solve for $\frac{dy}{dx}$.	P	M	M	H
Objective 3B.3: After completing Lesson 3B, learners will be able to find the equation of the tangent line for an implicitly defined function.				
Given an implicitly defined function and a point, correctly find $\frac{dy}{dx}$.	P	M	M	H
Evaluate $\frac{dy}{dx}$ at the given point to find the slope of the tangent line.	P	M	M	H
Correctly use the point-slope equation of a line to find the equation of the tangent line.	P	M	M	H
Objective 3B.4: After completing Lesson 3B, learners will be able to find the equation of the normal line for an implicitly defined function.				
Correctly use the fact that the normal line is the negative reciprocal of the slope of the tangent line to find the slope of the normal line.	P	L	L	L
Correctly use the point-slope equation of a line to find the equation of the normal line.	P	M	M	L
Objective 3B.5: After completing Lesson 3B, learners who are given the graph of a curve will be able to sketch the tangent line and the normal line at a given point on the graph.				
Correctly sketch the tangent line at the given point, either by hand or using technology.	S	L	L	H
Correctly sketch the normal line at the given point, either by hand or using technology.	S	L	L	L
Objective 3B.6: After completing Lesson 3B, learners will be able to find the second derivative of an implicitly defined equation using implicit differentiation.				
After finding $\frac{dy}{dx}$, correctly differentiate both sides of the equation to find $\frac{d^2y}{dx^2}$.	P	H	M	M
Substitute $\frac{dy}{dx}$ into the equation for $\frac{d^2y}{dx^2}$.	P	M	M	M
Correctly simplify the equation for $\frac{d^2y}{dx^2}$.	P	H	M	M

Task Analysis for Lessons 3C: Related Rates

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importanc e (Low, Medium, or High)
Objective 3C.1: After completing Lesson 3C, learners will be able to solve related rates problems.				
Correctly solve related-rates problems that involve standard geometric formulas (such as area, volume, surface area, etc.)	P	M	H	M
Correctly solve related-rates problems that involve the Pythagorean Theorem.	P	M	H	M
Correctly solve related-rates problems that involve trigonometric functions.	P	H	H	M
Correctly solve related-rates problems that involve similar triangles	P	H	H	M
Correctly identify the correct approach to solving a given related-rates problem.	S	H	H	M

Task Analysis for Lesson 4A: Exponential and Logarithmic Functions

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Lesson 4A.1: After completing Lesson 4A, learners will be able to take the derivative of exponential functions.				
Correctly take the derivative of exponential functions of the form $y = a^x$ using the rule $\frac{d}{dx}(a^x) = a^x \ln a$.	P	M	M	H
Correctly take the derivative of functions that are a combination or composition of polynomial, power, exponential, and trigonometric functions.	P	H	M	H
Lesson 4A.2: After completing Lesson 4A, learners will be able to take the derivative of logarithmic functions.				
Correctly take the derivative of the natural log function using the rule $\frac{d}{dx}(\ln x) = \frac{1}{x}$.	P	M	M	H
Correctly take the derivative of the general logarithmic functions using the rule $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$.	P	H	M	H
Correctly take the derivative of functions that are a combination or composition of polynomial, power, exponential, logarithmic, and trigonometric functions.	P	H	H	H

Task Analysis for Lessons 4B: Logarithmic Differentiation

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Lesson 4B.1: After completing Lesson 4B, learners will be able to use logarithmic differentiation to take the derivative of functions				
Correctly take then natural log of both sides of an equation.	P	L	M	M
Correctly use properties of logarithms to rewrite the equation as the sum and difference of multiples of logarithms.	P	M	M	M
Correctly use implicit differentiation to find $\frac{dy}{dx}$.	P	M	M	M
Correctly substitute the original equation for y in for the y in the equation in $\frac{dy}{dx}$.	P	M	M	M
Lesson 4B.2: After completing Lesson 4B, learners will be able to identify functions where logarithmic differentiation would be optimal for taking the derivative.				
Correctly identify functions with quotients of products as being functions where logarithmic differentiation may be beneficial.	S	M	M	M
Correctly identify functions with a variable base raised to a variable power as ones where logarithmic differentiation is necessary for taking the derivative.	S	H	M	M
Lesson 4B.3: After completing Lesson 4B, learners will be able to derive the differentiation rule for $y = a^x$ using logarithmic differentiation.				
Correctly use logarithmic differentiation to find the derivative of $y = a^x$.	P	M	M	M

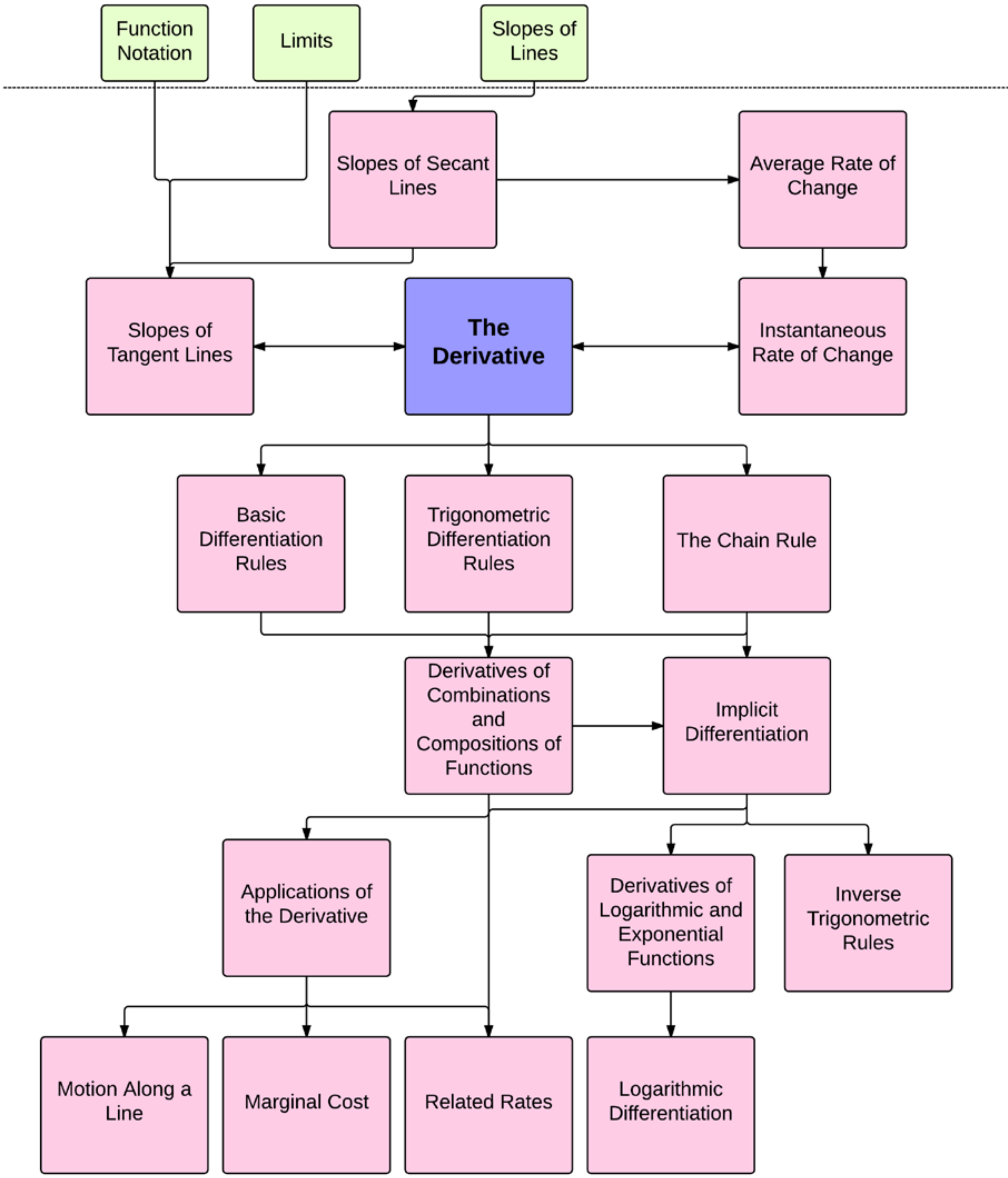
Task Analysis for Lessons 4C: Inverse Trigonometric Functions

Objectives Tasks	Knowledge Type (Declarative, Procedural, Structural)	Difficulty (Low, Medium, or High)	Duration (Low, Medium, or High)	Importance (Low, Medium, or High)
Lesson 4C.1: After completing Lesson 4C, learners will be able to derive the differentiation rules for the inverse trigonometric functions using implicit differentiation.				
Correctly derive the differentiation rule for $\frac{d}{dx}(\sin^{-1} x)$.	S	M	M	M
Correctly derive the differentiation rule for $\frac{d}{dx}(\tan^{-1} x)$.	S	M	M	M
Correctly derive the differentiation rule for $\frac{d}{dx}(\sec^{-1} x)$.	S	H	M	M
Lesson 4C.1: After completing Lesson 4C, learners will be able to take the derivative of functions involving inverse trigonometric functions.				
Correctly take the derivative of inverse trigonometric functions.	P	M	M	H
Correctly take the derivative of functions that are a combination or composition of polynomial, power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.	P	H	H	H

APPENDIX G

Learning Hierarchy

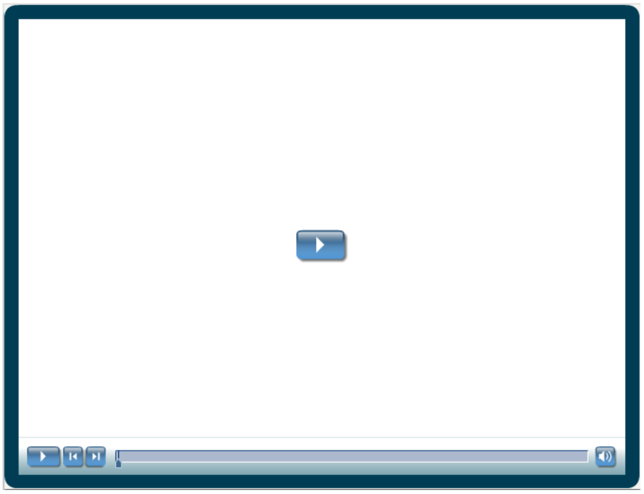
Learning Hierarchy



APPENDIX H

Worked Example Wireframe

Worked Example Wireframe



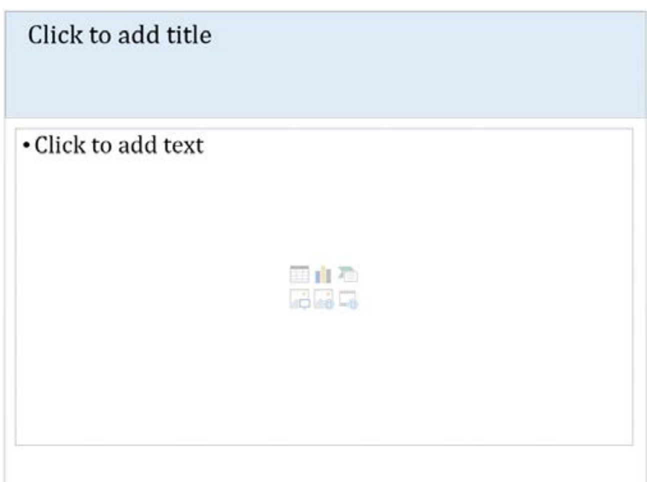
CAPTIVATE SKIN
Size: 640 x 480
Skin: Cool Blue (modified)
Playback Controls: On

- Playbar overlay
- Play
- Forward
- Back
- Mute
- Progress Bar

Table of Contents: Off
Borders: Show borders

- All sides
- Rounded corners
- Color #003D54
- Width: 15

Auto Play: Off



POWERPOINTCONTENT SLIDE:
Slide Size: Standard (4:3)
Title:

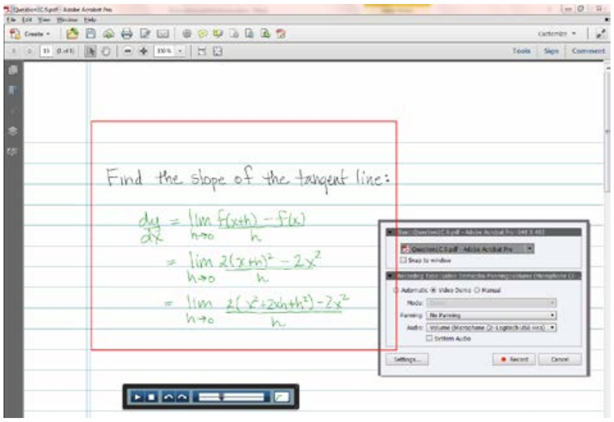
- Font: Cambria
- Font Size: 30
- Blue background
- Blue, Accent 1, 80% lighter

Body:

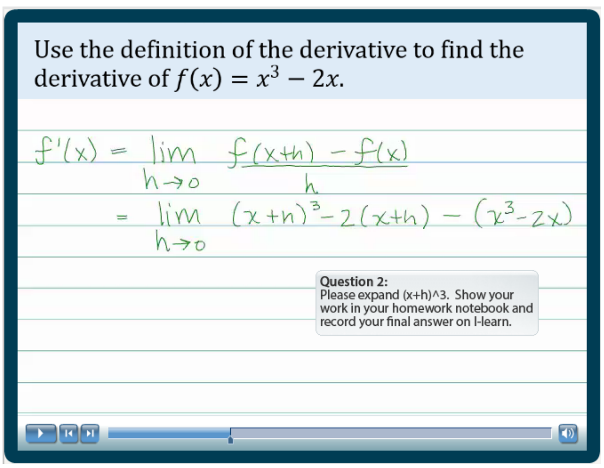
- Font: Cambria
- Font Size: 28
- White background

Math Equations:

- Font: Cambria Math
- Font Size: 28

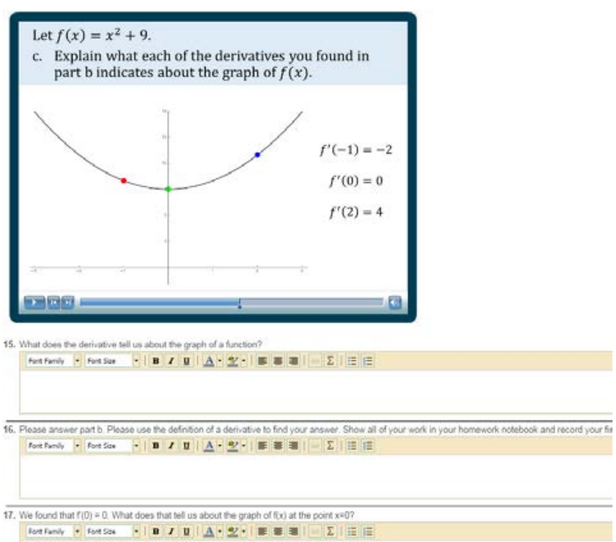


SCREEN CAPTURE OF SMARTPEN SCREENCASTS
Recorded using Captivate
Size: 640 x 480
Zoom: 150%
Margins: Leave 2 lines at top and one line at bottom to accommodate question prompt and playback controls.



CONTENT WITHIN SKIN
Question Prompt: Added as an image on the Master Slide – Width: 640
Question Pop-up Boxes:

- **Style:** Default Caption Style
- **Caption:** Halo
- **Font:** Myriad Pro
- **Style:** Regular
- **Size:** 15 pt
- **Color:** Black



IMPORT INTO LMS
Embedded: swf file
Size: 656 x 492
Question Type: Composite

APPENDIX I

Survey Results – Worked Examples Design Survey

Worked Examples Survey Results

Survey Demographics:

- Total Number of Respondents: 10
  - 4 - Subject Matter Experts
  - 6 - Students
- Type of computer used
  - 5 - PC
  - 4 - MAC
  - 0 - Other
- Internet Browser used (1 respondent used two browsers)
  - 1 - Internet Explorer
  - 3 - Mozilla Firefox
  - 4 - Google Chrome
  - 3 - Safari
  - 0 - Other

Example #1: Smartpen Screencast

Refer back to the first example (the Smartpen screencast). Please indicate your level of agreement with each of the following statements.

#	Question	Strongly Disagree	Disagree	Agree	Strongly Agree
1	This example was easy to navigate.	0	0	5	5
2	This example was easy to use.	0	0	5	5
3	The explanation provided was clear and easy to follow.	0	0	4	6
4	The SmartPen ScreenCast was an effective tool for creating an example.	0	0	5	5
5	I did not have any technology problems while viewing this example.	0	2	2	6

If you disagreed with any of the above statements, please explain why.

Response	**Changes made to address this comment**
I had to click the image to restart the audio several times. I am not sure if this was a technical difficulty or simply the method for proceeding.	Changed the way videos were imported to the Captivate file in order to eliminate the need to click the image to restart the audio.
I could not scroll up and down the page, except for using my arrow buttons on the keyboard (seemed to happen with all of them).	This problem occurs only in the Safari web browser. It is a problem with the LMS. The Faculty Technology Center suggested that students be advised to use Firefox.

Do you have any other suggestions or comments related to this example?

Response	**Changes made to address this comment**
The writing with the SmartPen was a little bit small and thin. Also, the green color is not the best for clear visibility. One other item--you might want to change the wording from "take the derivative" to "find the derivative" or something like that. This is a technicality, but there is better mathematical language. :)	Increased the size of the Smartpen screencasts. It is not possible to change the color of the ink. Revised mathematical language.
Before starting the next part it says: "Click on the image to restart the audio" I feel like it should say: "Click on the image to continue"	Changed the way videos were imported to the Captivate file in order to eliminate the need to click the image to restart the audio.
The timing of the question boxes in the animation was a little confusing.	Added an explanation of the question boxes in the introductory training video.
If there was some way to get around "click on the image to continue with the audio" I would use it. Could the next button continue with the audio without having to click on the image?	Changed the way videos were imported to the Captivate file in order to eliminate the need to click the image to restart the audio.
I appreciated the interaction and explanation of the videos. It was clean and clear.	
The pencast was small and hard to see and there was no full screen option. I would suggest having a full screen option, or using a smaller page for the penchant so that it is easier to see. I liked having a pause button and a progress bar so that I can pause and also so that I know how much longer there is in each section. I knew the answers to the questions, but that was because I have taken your class, the material in the pencast did not seem to cover the answers to the questions.	Increased the size of the Smartpen screencasts. There is no full screen option when the example is embedded in the LMS. Added improved learner controls, including a pause button and a progress bar.
I had a little trouble with switching from video back to the powerpoint. If was doing similarly prepared examples frequently, it would be a problem. The writing in the videos was smaller than the powerpoint; I would have preferred the writing in the video to be larger.	Changed the way videos were imported to the Captivate file in order to eliminate the need to click the image to restart the audio. Increased the size of the Smartpen screencasts.
There was one time when I didn't know how to restart the video to watch you take the derivative again. So maybe you could have it written somewhere that you need to click on the image again to have it replay.	Changed the way videos were imported to the Captivate file in order to eliminate the need to click the image to restart the audio.

Example #2: Narrated PowerPoint

Refer back to the second example (the narrated PowerPoint). Please indicate your level of agreement with each of the following statements.

#	Question	Strongly Disagree	Disagree	Agree	Strongly Agree
1	This example was easy to navigate.	0	2	3	5
2	This example was easy to use.	0	0	7	3
3	The explanation provided was clear and easy to follow.	0	2	4	4
4	The Narrated PowerPoint was an effective tool for creating an example.	0	1	6	3
5	I did not have any technology problems while viewing this example.	0	1	3	6

If you disagreed with any of the above statements, please explain why.

Response	**Changes made to address this comment**
Navigation was not as good as in exercise 1 because it was impossible to pause the narration. Also, another technicality, you said that "since the function is implicitly defined, we have to use implicit differentiation." This was true for this example, but it is not always true. Sometimes you can solve for y in terms of x and use explicit differentiation. :)	Added improved learner controls, including a pause button and a progress bar. Revised mathematical language.
Working through the problems with the steps just appearing doesn't show the process as well as I would like.	Added a pause between each step of the process. Increased use of color and animation to help make steps more obvious. Used Smartpen pencasts on very complicated problems with a lot of steps.
The navigation was a little difficult because the back and forward buttons only allow jumping from one part to another rather than being able to scroll to a specific point in the explanation.	Added improved learner controls, including a pause button and a progress bar.
Again, I could not scroll with my mouse, I had to use the arrow keys on my keyboard.	This problem occurs only in the Safari web browser. It is a problem with the LMS. The Faculty Technology Center suggested that students be advised to use Firefox.
I feel like the power point was not as effective in teaching as the smartpen screencast. The steps of how to take the derivative are there but how you go to those points was not as easily understood as the smartpen screencast.	Used Smartpen pencasts on very complicated problems with a lot of steps.

Do you have any other suggestions or comments related to this example?

Response	**Changes made to address this comment**
This one is easier to read (no small green writing), but the lack of a "pause button" is detrimental.	Added improved learner controls, including a pause button and a progress bar.
Because you can change the color of the text it might be good for teaching things like notation.	
The volume of your voice was higher in this example than in the smart pen video.	Found a method for calibrating the audio to ensure the volume is consistent throughout the example.
The color made a nice touch to correlate the derivative steps with the term.	
I loved the colors! It helped SO much! I also appreciated the explanation on how to take the derivative of the colored parts. I could easily follow and check my work.	
I did not like that I could not pause or rewind. I like having a progress bar, but it isn't as important as a pause feature and the ability to rewind. I really liked the colored breakdown of the derivation. Again, I knew the answers to the questions, but that was because I have taken your class, the material in the pencast did not seem to cover the answers to the questions.	Added improved learner controls, including a pause button and a progress bar.
It is also possible to put handwriting on a PowerPoint using the pen tool (assuming you have a pad and a pen that works as a mouse.)	

Example #3: Combined Narrated PowerPoint and Smartpen Screencast

Refer back to the third example (the combined narrated PowerPoint and Smartpen screencast). Please indicate your level of agreement with each of the following statements.

#	Question	Strongly Disagree	Disagree	Agree	Strongly Agree
1	This example was easy to navigate.	0	1	4	5
2	This example was easy to use.	0	0	6	4
3	The explanation provided was clear and easy to follow.	0	0	6	4
4	The Combined Narrated PowerPoint and SmartPen ScreenCast was an effective tool for creating an example.	0	0	6	4
5	I did not have any technology problems while viewing this example.	0	1	4	5

If you disagreed with any of the above statements, please explain why.

Response	Changes made to address this comment
Same trouble as in exercise 2, because there is no way to pause the narration for the parts without the SmartPen. I realize this is not a huge problem, because there is a "back" button and you can hear complete steps again--it's just that the ability to pause without having to repeat whole steps is nice.	Added improved learner controls, including a pause button and a progress bar.
In your directions above (in this survey), did you mean to say "Refer back to the third partially worked example" I'm not sure what the problem was, but I could not find a place to put the answer for question 11 on my first try. It said "Record your final answer in I-Learn, but I couldn't find a place to put it. Nor did I see the radio buttons that asked me to rate the mental work needed. After I submitted the third question the first time, I saw that I hadn't answered those questions. That is why you will see that I did problem 3 a second time.	This problem occurs only in the Safari web browser. It is a problem with the LMS. The Faculty Technology Center suggested that students be advised to use Firefox.

Do you have any other suggestions or comments related to this example?

Response	Changes made to address this comment
The audio was a bit quiet in general in all the exercises. I had to crank my volume control.	Found a method for calibrating the audio to ensure the volume is consistent throughout the example.
SmartPen audio seems louder gives you a little shock when you start it. I think this is the best format. allows moving fast through content that should already be known then the "new" concept can be done slowly by hand.	Found a method for calibrating the audio to ensure the volume is consistent throughout the example.
Because you used both a narrated PPT and a smart pen video in this example, I had to change the volume setting. Not a big deal.	Found a method for calibrating the audio to ensure the volume is consistent throughout the example.
I admit, this one was challenging. I got lost in my math at several places. I thought the video was very well done, but I personally needed more explanation to solve it. If a student was taking the class, I think they would be able to figure it out without further explanation.	
For whatever reason I could [not] scroll with my mouse. I liked the close up of the pencast better this time.	This problem occurs only in the Safari web browser. It is a problem with the LMS. The Faculty Technology Center suggested that students be advised to use Firefox.
The writing in the video seemed larger this	Increased the size of the Smartpen screencasts.

Response	Changes made to address this comment
time. One problem was that the video went black at the end. I had to rewind and pause to get the expression for y" in order to simplify	Changed the way videos were imported to the Captivate file in order to eliminate the need to click the image to restart the audio. This will also avoid the problem experienced by this respondent.
I thought that the combination of the powerpoint and the smartpen were the best teaching tool yet. I also think that it would be good to put the correct answer to the second derivative within the powerpoint at the end.	

Example #4: Narrated PowerPoint with Mathematica Screen Capture

Refer back to the fourth example (the narrated PowerPoint with Mathematica screen capture). Please indicate your level of agreement with each of the following statements.

#	Question	Strongly Disagree	Disagree	Agree	Strongly Agree
1	This example was easy to navigate.	0	1	5	4
2	This example was easy to use.	0	0	7	3
3	The explanation provided was clear and easy to follow.	0	1	4	5
4	The Narrated PowerPoint with Mathematica Screen Capture was an effective tool for creating an example.	0	0	5	5
5	I did not have any technology problems while viewing this example.	0	1	4	5

If you disagreed with any of the above statements, please explain why.

Response	Changes made to address this comment
The Mathematica graphing explanation was a bit quick for me; I couldn't follow it fast enough while I was making notes. This is due in part to the fact that, once again, there was no pause button. The narration kept moving ahead and I couldn't stop it to make sure I understood as it went along.	Added improved learner controls, including a pause button and a progress bar.
During the Mathematica section there's no way to fast forward or rewind.	Added improved learner controls, including a pause button and a progress bar.
At first I could not scroll with my mouse, but for whatever reason, scrolling began working after I used the equation editor. (?)	This problem occurs only in the Safari web browser. It is a problem with the LMS. The Faculty Technology Center suggested that students be advised to use Firefox.

Do you have any other suggestions or comments related to this example?

Response	**Changes made to address this comment**
Portions of the audio track for this one faded in and out. That was distracting and made parts of it hard to hear. One other small detail, which is not a huge deal. Once Mathematica generated a graph, you called it "the equation of the graph" rather than "the graph of the equation." :)	Found a method for calibrating the audio to ensure the volume is consistent throughout the example. Revised mathematical language.
The keystroke audio can be a little annoying. In Khan Academies "Intro to JS and Animation" they have it set up to where a video is watched where the presenter will type code then afterwards the user can edit the code themselves. This would probably be a pretty cool feature for Mathematica examples. https://www.khanacademy.org/computing/cs/programming/animation-basics/p/intro-to-animation (I would just skip to the last 10 seconds, note how text can only be changed when paused and it changes back when the video is replayed.)	Deleted the keystroke audio.
For all four of the questions, when I hit the submit button, it showed me the answers that I had typed in and it showed (with the red X) that I had missed the question asking me to rate the mental requirements for the problem. When students see that red X they panic. Would it be possible to have that not come up after submitting?	Changed settings in the LMS so that all answers are marked as correct.
This was very well put together.	
I really like having a pause button and the ability to rewind, both of which are missing from this form.	Added improved learner controls, including a pause button and a progress bar.
One thing I noticed when looking through the example again was that you can move backward and forward through the scenes, but you can't rewind or fast forward within a scene. So, if you're looking for a particular point in a scene you have to rewind, then play, and then wait for that point to come up while you watch.	Added improved learner controls, including a pause button and a progress bar.
For viewing complex graphs in a presentation I agree that mathmatica is the best way to do it. It was a very effective visual aid.	

Overall Comments

Do you have any general comments or suggestions regarding the design of the worked examples?

Response	**Changes made to address this comment**
For me, there were three keys that overshadowed everything else: 1. Having the ability to pause the narration is important. 2. Size of typing makes a difference in visibility. 3. Audio needs to be clear, loud enough, and not too fast.	Added improved learner controls, including a pause button and a progress bar. Increased font size. Found a method for calibrating the audio to ensure the volume is consistent throughout the example.
I loved all 4 types of examples. If a student misses a question in the partially worked examples, is there somewhere that he can go to see how to do the problem?	
Maybe it would be better to use more than one example.	
It isn't a commonly used format, but I thought that it served well and was able to keep clarity. As with any recorded tutorials it could be too slow/too fast according to the viewer so it could be nice to have speed manipulation. They were rather short though and navigation was clear enough that I believe the quality is sufficient for general satisfaction of a group.	Added improved learner controls, including a pause button and a progress bar.
I felt like the steps shown were adequate for a typical calculus student. (You might need to add more detail if the student came in unprepared or had problems with algebra.) On the partially worked examples I presumed that you intended for the students to fill in the missing steps as they worked the problem.	
Again, I loved the colors. The mathamatica explanation was very useful. I believe this is a very modern way of doing homework and will be appreciated by several students. I also appreciated the visual aid of the last example, even though I forgot how to correctly answer the problem. I believe the combination of the powerpoints, mathamatica, and pen were perfectly used.	

Response	**Changes made to address this comment**
The pencast has a nice look (being able to see the letters form) but is harder to see than typed examples. I've always found having the ability to pause and rewind recorded lectures VERY helpful. The volume level of your videos seems to be just right, I could easily hear it with the volume on my computer at a medium setting.	Increased the size of the Smartpen screencasts. Added improved learner controls, including a pause button and a progress bar.
I liked the "next" button that allows you to pause and interact with the video. This is similar to some of the videos I've watched on Undacity.	
They all worked and were easy to use. I also believe if these were used as homework assignments there should always be a correct answer given at the end. So that the students can check to see if they go it or not, and if they didn't then they can go back and rework it until they get the right answer.	
I didn't understand why I missed the problems that asked me about the level of difficulty. Since it was my opinion, it should have been correct. I think you can fix that by making it a multiple answer problem instead of multiple choice.	Changed settings in the LMS so that all answers are marked as correct.

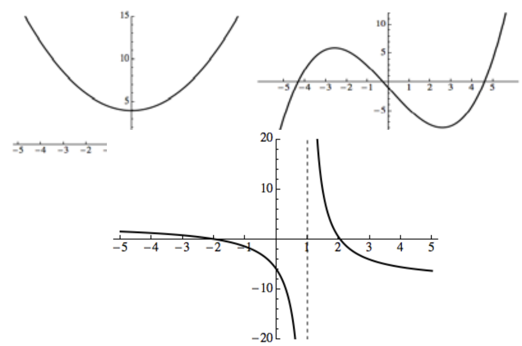
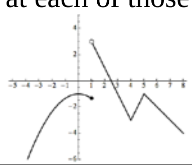
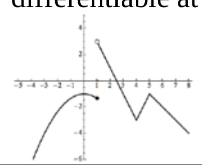
APPENDIX J

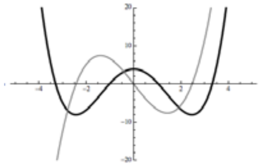
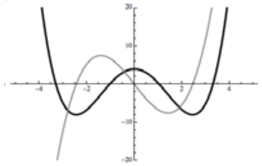
Performance Assessments

Performance Assessments

Exam Questions – Linked to Objectives

Question #	**Pre-test Question**	**Post-test Question**	**Lesson**	**Objective(s)**
1a	Please state the definition of the derivative.	Please state the definition of the derivative.	Lesson 1	Objective 1B.1
1b	Explain in your own words what a derivative tells us about a function.	Explain how the definition of the derivative is connected to the slope of a secant line and to the slope of a tangent line. Please use a drawing as part of your explanation.	Lesson 1	Objective 1A.1 Objective 1A.2 Objective 1A.3
2	Use the definition of the derivative to find $f'(x)$ where $f(x) = x^2 + 3$.	Use the definition of the derivative to find $f'(1)$ where $f(x) = x^2 - 1$.	Lesson 1	Objective 1B.2
3		Use the definition of the derivative to find $f'(x)$ where $f(x) = \frac{x}{2x-3}$.	Lesson 1	Objective 1A.4 Objective 1C.1 Objective 1C.3
4	Find the equation of the tangent line for the curve $y = 5x^2 - 3x + 1$ at the point $x = 2$.	Find the equation of the tangent line for the curve $y = 2\sqrt{x-1} + 3$ at the point $x = 5$.	Lesson 1	Objective 1A.4 Objective 1C.4 Objective 2A.2 Objective 2C.1

Question #	Pre-test Question	Post-test Question	Lesson	Objective(s)
5		Each of the following graphs shows the graph of a function, $f(x)$. For which of the graphs is it true that $f'(-2) = f'(2)$? Carefully explain your answer. 	Lesson 1	Objective 1B.3
6	The graph of $f(x)$ is shown below. Find any points where $f(x)$ is not differentiable and explain why the function is not differentiable at each of those points. 	The graph of $f(x)$ is shown below. Find any points where $f(x)$ is not differentiable and explain why the function is not differentiable at each of those points. 	Lesson 1	Objective 1B.4

Question #	Pre-test Question	Post-test Question	Lesson	Objective(s)
7	The following figure shows the graph of a function and the graph of its derivative. Determine which graph is the function and which is the derivative. Carefully justify your answer. 	The following figure shows the graph of a function and the graph of its derivative. Determine which graph is the function and which is the derivative. Carefully justify your answer. 	Lesson 1	Objective 1C.2
8a	Find the derivative of the following function: $f(x) = 2x^2e^x$	Find the derivative of the following function: $f(x) = -2e^{2x}(x^2+3)$	Lesson 2	Objective 2A.1 Objective 2A.3 Objective 2A.5 Objective 2A.6 Objective 2C.1 Objective 4A.1
8b	Find the derivative of the following function: $f(x) = \cos(2x)$	Find the derivative of the following function: $f(x) = \sin\sqrt{\frac{1}{x}}$	Lesson 2	Objective 2A.2 Objective 2B.1 Objective 2C.2
8c	Find the derivative of the following function: $f(x) = \frac{4x+2}{x-1}$	Find the derivative of the following function: $f(x) = \frac{\sec(2x)}{1+4x}$	Lesson 2	Objective 2A.1 Objective 2A.4 Objective 2A.6 Objective 2B.1 Objective 2C.1
9	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ where $y = 2\sin(x^3)$	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ where $y = -3\csc(x^2)$	Lesson 2	Objective 2A.1 Objective 2A.3 Objective 2B.1 Objective 2B.2 Objective 2C.1

Question #	Pre-test Question	Post-test Question	Lesson	Objective(s)
10	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ where $2x^2 + y^2 = 3$	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ where $\ln(xy) = 3$	Lesson 3	Objective 2A.3 Objective 3B.2 Objective 3B.6 Objective 4A.2
11	Find the equation of the tangent and normal lines to the graph of the equation $4x^2y + 3xy^2 = 7$ at the point (1,1).	Find the equation of the tangent and normal lines to the graph of the equation $x^2 - 3xy + y^2 = -1$ at the point (1,2).	Lesson 3	Objective 2A.3 Objective 3B.2 Objective 3B.3 Objective 3B.4
12	A particle is moving along a horizontal axis. The position of the particle at time t (measured in seconds) is given by the equation $s(t) = 2t^3 - 4t$. Find the particle's position, velocity, speed, and acceleration when $t = 3$. Please include units for each answer.	A particle is moving along a horizontal axis. The position of the particle at time t (measured in seconds) is given by the equation $s(t) = -2t^2 + 4t - 9$. Find the particle's position, velocity, speed, and acceleration when $t = 3$. Please include units for each answer.	Lesson 3	Objective 3A.1
13	A tank of water in the shape of a cone (with the point at the bottom) is leaking water at a constant rate of 2 ft^3/hour. The radius of the tank is 5 ft and the height of the tank is 14 feet. At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft? (Hint: The volume of a cone can be found using the equation $V = \frac{1}{3}\pi r^2 h$).	A tank of water in the shape of a cone (with the point at the bottom) is leaking water at a constant rate of 2 ft^3/hour. The radius of the tank is 5 ft and the height of the tank is 14 feet. At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft? (Hint: The volume of a cone can be found using the equation $V = \frac{1}{3}\pi r^2 h$).	Lesson 3	Objective 3C.1
14a	Find the derivative of the function $y = \log_2(x - 1)$	Find the derivative of the function $y = 3^{8x-1} + \log_5(x - 1)$	Lesson 4	Objective 3C.1 Objective 4A.1 Objective 4A.2
14b	Find the derivative of the function $y = \sin^{-1}(2x)$.	Find the derivative of the function $y = \sin^{-1}\sqrt{1 - x^2}\ , 0 < x < 1$	Lesson 4	Objective 4C.1 Objective 3C.2

Question #	Pre-test Question	Post-test Question	Lesson	Objective(s)
14c	Find the derivative of the function $y = (3x)^{x-1}$	Find the derivative of the function $y = (2 + x)^{3-x}$	Lesson 4	Objective 3A.3 Objective 4A.2 Objective 4B.1 Objective 4B.2
15		Derive the differentiation rule for $y = \tan^{-1} x$.	Lesson 4	Objective 4C.1 Objective 4C.2

APPENDIX K

Post-test Scoring Rubric

Unit 2 Exam Rubric

1. **Please state the definition of the derivative.**

   4: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

   3: Missing the limit, but have the correct difference quotient

   2: Limit with an incorrect difference quotient (but still close to the difference quotient) OR they define it as the slope of the tangent line, or instantaneous rate of change, with no equation

   1: Incorrect difference quotient or limit OR they refer to “slope” or “change” without a complete explanation

   0: Not a correct equation without a correct explanation

   Explain how the definition of the derivative is connected to the slope of a secant line and to the slope of a tangent line. Please use a drawing as part of your explanation.

   4: Correctly use a drawing to explain that the derivative is the slope of the tangent line and we find it by finding slopes of secant lines and taking the limit as the two points used to form the secant line get closer and closer to each other.

   3: Give a correct explanation without a drawing OR give an explanation that is mostly correct, but missing some details

   2: Gives an incomplete explanation without a drawing OR gives an explanation that is mostly correct without a valid drawing

   1: Makes some connections between slopes and the derivative

   0: Does not make any connection between the derivative and slopes

2. **Use the definition of the derivative to find $\boldsymbol{f'(1)}$ where $\boldsymbol{f(x) = x^2 - 1}$.**

   4: Correctly show the work of using the limit of the difference quotient to find the derivative $f'(x) = 2x$. Must show correct work leading to the correct answer.

   3: Use the correct definition of the derivative, but make arithmetic/algebra mistakes. May or may not have answer of $f'(x) = 2x$, although they do have an answer.

   2: Have the correct answer, but did not use the correct difference quotient OR set up the definition correctly, but were not able to compute the limit and did not get an answer.

   1: Set up the definition incorrectly (but on the right track) and did not get right answer

   0: None of the above

3. Use the definition of the derivative to find $f'(x)$ where $f(x) = \frac{x}{2x-3}$.

4: Correctly show the work of using the limit of the difference quotient to find the derivative $f'(x) = \frac{-3}{(2x-3)^2}$. Must show correct work leading to the correct answer.

3: Use the correct definition of the derivative, but make arithmetic/algebra mistakes. May or may not have answer of $f'(x) = \frac{-3}{(2x-3)^2}$, although they do have an answer.

2: Have the correct answer, but did not use the correct difference quotient OR set up the definition correctly, but were not able to compute the limit and did not get an answer.

1: Set up the definition incorrectly (but on the right track) and did not get right answer

0: None of the above

4. Find the equation of the tangent line for the curve $y = 2\sqrt{x-1} + 3$ at the point $x = 5$.

$$\frac{dy}{dx} = \frac{1}{\sqrt{x-1}}; \left.\frac{dy}{dx}\right|_{x=5} = \frac{1}{2}$$
$$(y-7) = \frac{1}{2}(x-5) \text{ OR } y = \frac{1}{2}x + \frac{9}{2}$$

4: Found the correct derivative and equation of the tangent line (did not have to solve for y)

3: Found the correct derivative, but due to a minor arithmetic mistake, missed one of the three numbers 7, 5, and ½ in the equation $(y-7) = \frac{1}{2}(x-5)$.

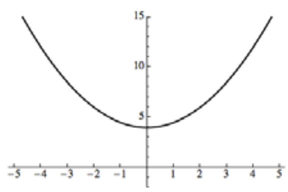
2: Got the answer $(y-7) = \frac{1}{\sqrt{x-1}}(x-5)$ OR found the correct derivative, but missed two of the three numbers: 7, 5, and ½

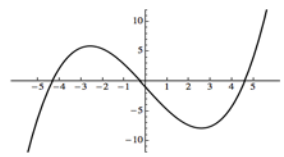
1: Found the correct point on the line with the wrong slope OR found the derivative $\frac{dy}{dx} = \frac{1}{\sqrt{x-1}}$ and didn't go any further

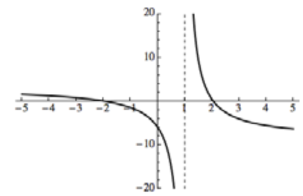
0: Did not find the slop for the tangent line or the correct point on the curve

5. **Each of the following graphs shows the graph of a function, $f(x)$. For which of the following graphs is it true that $f'(-2) = f'(2)$? Carefully explain your answer.**

Graph #1 Graph #2 Graph #3







Correct answer: Graph #2 is the only one where $f'(-2) = f'(2)$ because it is the only graph where the slope of the tangent line at the point $x = -2$ is the same as the slope of the tangent line at the point $x = 2$.

4: They correctly select Graph #2 as the answer, and give a clear explanation of why

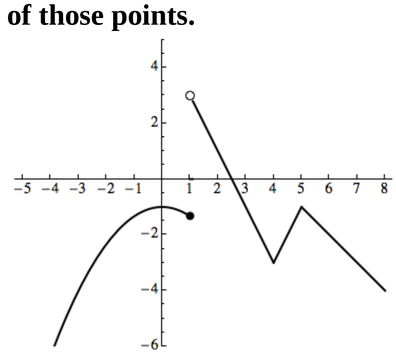
3: They correctly select Graph #2 but give an inadequate explanation

2: They correctly select Graph #2 as the answer, but offer no explanation OR they select both graphs #2 and #3 with a correct explanation (even though the slopes are not the same for graph #3)

1: They select Graph #1 and #3 because those graphs have the same y-values at x = 2 and x = -2.

0: None of the above.

6. **The graph of $f(x)$ is shown below. Find any points where $f(x)$ is not differentiable and explain why the function is not differentiable at each of those points.**



Not differentiable at x = 1 (not continuous) x= 4 (sharp corner), or x = 5 (sharp corner)

4: Correctly identifies all three points and the correct reasons

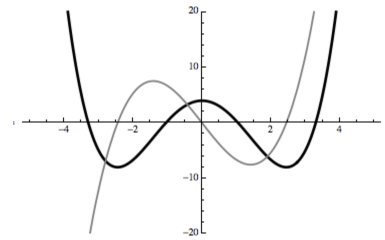
3: Correctly identifies at least 2 points and gives the correct reason for both

2: Correctly identifies at least one point and gives the correct reason

1: Correctly identifies at least one point, but does not give the correct reason for any of the points

0: Does not give any correct points or reasons

7. The following figure shows the graph of a function and the graph of its derivative. Determine which graph is the function and which is the derivative. Carefully justify your answer.



The darker curve is the original function. The lighter curve is the derivative.

4: Correctly identifies each curve and clearly explains their answer.

3: Correctly identifies each curve, but gives a weak or unclear explanation

2: Correctly identifies each curve, but does not give a correct explanation.

1: Does not correctly identify the curves, but gives some explanations that are at least on the right track.

0: Does not correctly identify each curve and does not give any reasonable explanation

8. Find the derivative of each of the following functions

a. $\boldsymbol{f(x) = -2e^{2x}(x^2+3)}$ $\quad f'(x) = -4e^{2x}(x^2+3) - 2e^{2x}(2x)$

4: Correctly found the derivative (do not need to simplify)

3: Used the correct differentiation rules, but made minor arithmetic or algebra errors or simplified incorrectly.

2: Used the product rule incorrectly or with major arithmetic or algebra errors.

1: Did not use the product rule, but took the derivative of each term correctly.

0: Did not use the product rule and did not take any derivatives correctly

b. $\boldsymbol{f(x) = \sin\sqrt{\frac{1}{x}}}$ $\quad f'(x) = \left(\cos\sqrt{\frac{1}{x}}\right)\left(\frac{1}{2}\left(\frac{1}{x}\right)^{-\frac{1}{2}}\right)\left(-\frac{1}{x^2}\right) = \dfrac{-\cos\sqrt{\frac{1}{x}}}{2x^2\sqrt{x}}$

4: Correctly found the derivative (do not need to simplify)

3: Used the correct differentiation rules, but made minor arithmetic or algebra errors or simplified incorrectly.

2: Used the chain rule incorrectly or with major arithmetic or algebra errors.

1: Did not use the chain rule, but took the derivative of some of the terms correctly.

0: Did not use the chain rule and did not take any derivatives correctly

c. $\boldsymbol{f(x) = \frac{\sec(2x)}{1+4x}}$ $f'(x) = \frac{(1+4x)\,2\sec 2x\tan 2x - 4\sec 2x}{(1+4x)^2}$

4: Correctly found the derivative (do not need to simplify)

3: Used the correct differentiation rules, but made minor arithmetic or algebra errors or simplified incorrectly.

2: Used the quotient rule incorrectly or with major arithmetic or algebra errors or forgot to use the chain rule.

1: Did not use the quotient rule, but took the derivative of each term correctly.

0: Did not use the product rule and did not take any derivatives correctly

9. **Find $\boldsymbol{\frac{dy}{dx}}$ and $\boldsymbol{\frac{d^2y}{dx^2}}$ where $\boldsymbol{y = -3\csc(x^2)}$**

$$\frac{dy}{dx} = 6x\csc(x^2)\cot(x^2)\,;$$

$$\frac{d^2y}{dx^2} = -12x^2\csc^3(x^2) + 6\csc(x^2)\cot(x^2) - 12x^2\csc(x^2)\cot^2(x^2)$$

4: Found the correct 1st and 2nd derivatives (The 2nd derivative does not necessarily need to be simplified, so the correct answer could take several different forms).

3: Found the correct 1st derivative and used the product rule on the 2nd derivative, but made minor algebra/arithmetic errors

2: Made minor errors finding the 1st derivative but used the right technique to find the second derivative (based on incorrect 1st derivative) OR found the correct 1st derivative, but did not use the product rule or made other major errors on the 2nd derivative.

1: Although there are some correct pieces, neither derivative is correct.

0: No correct pieces.

10. **Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ where $\ln(xy) = 3$**

$$\frac{dy}{dx} = -\frac{y}{x};\ \frac{d^2y}{dx^2} = \frac{-x\frac{dy}{dx} + y}{x^2} = 0$$

4: Found the correct 1st and 2nd derivatives using implicit differentiation

3: Found the correct 1st derivative and found the correct 2nd derivative, but didn't plug $\frac{dy}{dx}$ into the second derivative and simplify to get 0 OR used the correct procedure, but made minor arithmetic/algebra errors.

2: Made minor errors finding the 1st derivative but used the right technique to find the second derivative (based on incorrect 1st derivative) OR found the correct 1st derivative, but did not use the quotient rule or made other major errors on the 2nd derivative.

1: Although they used implicit differentiation, neither derivative is correct.

0: No correct pieces.

11. **Find the equation of the tangent and normal lines to the graph of the equation $x^2 - 3xy + y^2 = -1$ at the point $(1, 2)$.**

$$\frac{dy}{dx} = \frac{2x - 3y}{3x - 2y};\ \left.\frac{dy}{dx}\right|_{(1,2)} = 4$$

Tangent line: $(y - 2) = 4(x - 1)$

Normal line: $(y - 2) = -\frac{1}{4}(x - 1)$

4: Found all correct answers

3: Found the correct derivative and one of the lines

2: Found the correct derivative and one of the lines OR made minor arithmetic errors to get the incorrect slope, but used the right process to find the lines

1: Made major errors in finding the derivative, but found equations of two lines with opposite reciprocal slopes

0: Incorrect derivatives and equations of lines do not have opposite reciprocal slopes

12. **A particle is moving along a horizontal axis. The position of the particle at time t (measured in seconds) is given by the equation $x(t) = -2t^2 + 4t - 9$. Find the particle's position, velocity, speed, and acceleration when $t = 3$. Please include units for each answer.**

    Position = -15 units; Velocity = -8 units/second; Speed = 8 units/second; Acceleration = -4 units/sec^2

    4: Has all the correct answers and correct units

    3: Has 3 of the four correct answers with correct units OR all 4 correct answers with only 2 or 3 having correct units

    2: Has 2 of the four correct answers with correct units OR 3 correct answers with 1 or 2 having correct units

    1: Has 1 correct answer with correct units OR 2 correct answers with no correct units

    0: Has at most 1 correct answer with no correct units.

13. **A tank of water in the shape of a cone (with the point at the bottom) is leaking water at a constant rate of $2\ \mathrm{ft}^3/\mathrm{hour}$. The radius of the tank is 5 ft and the height of the tank is 14 feet. At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft? (Hint: The equation for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$)**

$$\text{Equation: } V = \frac{1}{3}\pi r^2 h \ \rightarrow \ V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$$

$$V = \frac{\pi}{27}h^3 \ \rightarrow \ \frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dr}{dt} \ \rightarrow \ -2 = \frac{\pi}{9}(6)^2\frac{dh}{dt} \ \rightarrow \ \frac{dh}{dt} = -\frac{2}{\pi} \text{ units per second}$$

    4: found the correct answer (either exact or as a decimal)

    3: Found the correct derivative $\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dr}{dt}$, but made arithmetic errors after that

    2: Found the equation for V in terms of h: $V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$ OR $V = \frac{\pi}{27}h^3$

    1: Drew the correct picture; Took the derivative of V, but not correctly.

    0: Did not try to take the derivative of V

14. Find the derivative of each of the following functions. Please simplify your answers.

a. $\boldsymbol{y = 3^{8x-1} + \log_5(x-1)}$ $\frac{dy}{dx} = 8 * 3^{8x-1}\ln 3 + \frac{1}{(x-1)\ln 5}$

4: Correctly found the derivative and simplified correctly (they don't really need to do anything to simplify, though)

3: Used the correct differentiation rules, but made minor arithmetic or algebra errors or simplified incorrectly.

2: Used one of the chain, log, or exponential rules incorrectly or with major arithmetic or algebra errors.

1: Used two of the chain, log, or exponential rules incorrectly

0: Did not take any derivatives correctly

b. $\boldsymbol{y = \sin^{-1}\sqrt{1-x^2}\ , 0 < x < 1;}$ $\frac{dy}{dx} = \frac{1}{\sqrt{1-(1-x^2)}} * \frac{1}{2\sqrt{1-x^2}} * (-2x) = \frac{-1}{\sqrt{1-x^2}}$

4: Correctly found the derivative and **simplified correctly**

3: Used the correct differentiation rules, but made minor arithmetic or algebra errors or simplified incorrectly. Or did not simplify completely.

2: Used one of the chain and inverse sine rules incorrectly or with major arithmetic or algebra errors.

1: Had mistakes in both the chain and inverse sine rules

0: Did not take any derivatives correctly

c. $\boldsymbol{y = (2+x)^{3-x};}$ $\frac{dy}{dx} = (2+x)^{3-x}\left(\frac{3-x}{x} - \ln(2+x)\right)$

4: Correctly found the derivative using logarithmic differentiation and simplified correctly

3: Used logarithmic differentiation with the correct differentiation rules, but made minor arithmetic or algebra errors or simplified incorrectly.

2: Used logarithmic differentiation but used one of the required differentiation rules incorrectly or with major arithmetic or algebra errors OR did not correctly use the properties of logarithms to rewrite the function

1: Took the derivative without using logarithmic differentiation.

0: Did not take any derivatives correctly

15. Derive the differentiation rule for $y = \tan^{-1} x$.

4: Derived the rule correctly with clear explanations

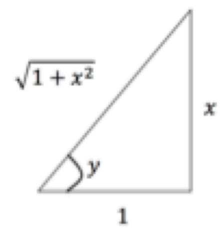
3: Showed correct math computations, without any explanation

2: Drew the correct triangle, but took the derivative incorrectly OR took the derivative correctly but drew the incorrect triangle

1: Had some correct parts, but neither the derivative nor the triangle were correct.

0: Not on the right track

Solution
In order to derive the differentiation rule for $y = \tan^{-1} x$, we first rewrite the equation as $x = \tan y$. Then, we can use this equation to draw the following triangle that tells us how x and y are related.



Now, we take the derivative of $x = \tan y$ using implicit differentiation:

$$x = \tan y$$

$$1 = \sec^2 y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

We can now us the triangle that we drew above to find that $\sec y = \sqrt{1+x^2}$. Substituting this expressions into the equation for $\frac{dy}{dx}$ gives us the following:

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{x^2+1}$$