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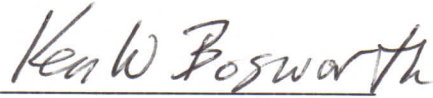
Determination of the Optimal Piping Diameter Through Entropy Generation Minimization

By
Taylor Sandow

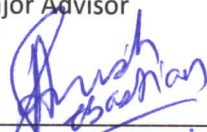
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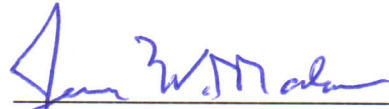
The members of the committee appointed to examine the thesis of Taylor Sandow find it satisfactory and recommend that it be accepted.



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Determination of the Optimal Piping Diameter Through Entropy Generation Minimization

Thesis Abstract – Idaho State University (2019)

A heat exchanger with constant heat flux is selected to apply a method of optimization through entropy minimization to determine the diameter of the system. This entropy minimization is examined for the dependency upon the assumptions required to make during the analysis for the friction factor and Nusselt number. The first approach is to use the differential method proposed by Dr. Adrian Bejan while examining various assumptions. The second approach is to use a direct computation of the entropy generated by analyzing the entire heat exchanger while also examining various assumptions.

These resulted in various diameters that mathematically are the optimum, but had an 85% variation based upon the different assumptions and methods of calculation. This indicates that the method of entropy minimization as an optimization method is too dependent upon the required assumptions for the friction factor and Nusselt number to be a reliable method of design.

Key Words: Entropy Generation, Minimization, Heat Exchanger

Introduction

In determining the optimal piping diameter for heat exchangers, the method of entropy generation minimization has frequently been used. This is a differential approach proposed by Dr. Adrian Bejan to examine the competing effects of the pressure loss within the system and the heat exchanged within the system. However, there is a mathematical discrepancy within the method resulting from finding the minimum through the assumptions placed upon the parameters. The friction factor and Nusselt number are two instances where an approximation equation has to be substituted to then find the optimal diameter mathematically without experimentation. The various approximation equations for these parameters can potentially have a significant effect upon the resulting optimal diameter.

Therefore, various approximation equations will be assumed and then applied to a single heat exchanger system. Once the different optimal diameters have been determined through the differential approach, a direct method of determining the entropy generated will be applied to the system. This will then allow an independent optimal diameter to be determined for the system which will give insight into the validity of the method.

The heat exchanger system that will be used will be a constant linear heat flux heat exchanger. This type of heat exchanger is achieved by placing a constant linear heat flux through various diameters of piping by using electrical heaters. The length of the piping will be fixed due to the typical heat exchanger having a physical limit. Finally, the mass flow rate will be fixed due to pumping restrictions. However, this leaves the diameter of the heat exchanger undetermined, which creates a one dimensional optimization problem.

Operational Definitions

The heat exchanger system will use water as the working fluid. This is due to the use of water in many heat exchanger applications, and that the property tables for water are readily available. The friction factor will be approximated using two different equations. The first is the assumption of smooth pipes which is used by Dr. Adrian Bejan. The second will be the more accurate Haaland equation approximation for the friction factor. Similarly, the Nusselt number will be approximated with two different equations namely the Dittus-Boelter equation and the Gnielinski equation.

The differential approach will be applied to this system. Then the entropy generated will be determined through the fixed inlet temperature and pressure, and then comparing to the outlet temperature and pressure mathematically determined. The entropy generated will be determined through the use of established property tables of the working fluid.

Theory

Entropy generation is simply defined to be the amount of entropy increased in a process due to the second law of thermodynamics and the frictional losses of the system. However, it is easier to conceptualize that entropy generation is directly proportional to the lost available work within the system which is a result of the Guoy-Stodola Theorem [B]. As such it is easy to see that the minimization of lost available work would result in the most efficient system.

Within a heat exchanger, the two sources of entropy generation are due to the heat transfer and the fluid friction. To minimize the entropy generated from heat transfer, a small diameter pipe is desired. Whereas, to minimize the frictional losses, a large diameter pipe is desired. This results in the two competing sources of entropy generation which establishes the optimization problem.

Most often, heat exchangers are run for long periods of time which allows the assumption that a steady state system has been achieved. Applying the second law to such a system results in the entropy generation equation shown in Equation 1 in per unit length form [B].

$$\dot{S}'_{gen} = \frac{32\dot{m}^3 f}{\pi^2 \rho^2 T D^5} + \frac{(q')^2}{\pi T^2 k^* Nu} \quad (1)$$

This allows the entropy generation per unit length to be determined. However, within this equation, there is a dependence upon the properties of the working fluid, the friction factor of the piping system, and the Nusselt number for the heating system. Therefore some assumptions must be made to achieve an equation that can be differentiated. This leads to the potential mathematical discrepancies within the method.

There are various assumptions that can be made for the friction factor. The easiest assumption is to assume smooth pipes which results in an approximation as shown in Equation 2 with Reynold's number bounds of $10^4 < Re < 10^6$ [B]. A more accurate approximation can be assumed using the Haaland Equation as a function of Reynold's number as shown in Equation 3 [M]. There are more possible assumptions for the friction factor. However, these two assumptions have been selected to examine.

$$f = 0.046 Re^{-0.2} \quad (2)$$

$$f = -1.8 \log \left(\frac{\varepsilon \mu Re}{14.8 \dot{m}} \right)^{1.11} + \frac{6.9}{Re}^{-2} \quad (3)$$

Similarly the Nusselt number must also be approximated. The approximation used by Dr. Adrian Bejan is the Dittus-Boelter equation which is shown in Equation 4 with bounds $0.7 < Pr < 160$ and $Re > 10^4$ [B]. The second approximation that will be used is the Gnielinski equation which can be

seen in Equation 5 [C]. It is important to note that the Gnielinski equation also has the bounds $0.5 \leq Pr \leq 2000$ and $3 \times 10^3 < Re < 5 \times 10^6$. This results in the bounds $0.7 \leq Pr \leq 160$ and $10^4 < Re < 10^6$ to ensure all formula are applicable.

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (4)$$

$$Nu = \frac{f(Re-1000)Pr}{8 + 12.7 \frac{f}{8}^{0.5} \frac{Pr^{0.5}}{Pr^{0.5}-1}} \quad (5)$$

It does not matter whether the derivative is taken with respect to the Reynold's number or with respect to the diameter. This can be seen by Equation 6 in which the derivative has been taken with respect to the diameter. However, once the derivative is set equal to zero, the dependence upon the diameter can be factored out due to the derivative of the Reynold's number with respect to the diameter being nonzero. Therefore, due to most approximation formulas being dependent upon the Reynold's number, the derivative will be taken with respect to the Reynold's number.

$$\frac{d\dot{S}'_{gen}}{dD} = \frac{d\dot{S}'_{gen}}{dRe} \frac{dRe}{dD} = 0 \Rightarrow \frac{d\dot{S}'_{gen}}{dRe} = 0 \quad (6)$$

Since, the derivative of the entropy generation will be taken with regards to the Reynold's number, Equation 1 can be written as shown in Equation 7. From here it is easy to see that the derivative will be as shown in Equation 8. However, the Gnielinski equation for the Nusselt number also depends upon the friction factor, as such the derivative will be as shown in Equation 9 when this approximation is used.

$$\dot{S}'_{gen} = \frac{f\pi^3\mu^5}{32\rho^2\dot{m}^2T} Re^5 + \frac{(q')^2}{\pi T^2 k^* Nu} \quad (7)$$

$$\frac{d\dot{S}'_{gen}}{dRe} = \frac{5f\pi^3\mu^5}{32\rho^2\dot{m}^2T}Re^4 + \frac{\pi^3\mu^5}{32\rho^2\dot{m}^2T}Re^5 \frac{df}{dRe} - \frac{(q')^2}{\pi T^2 k^* Nu^2} \frac{dNu}{dRe} \quad (8)$$

$$\frac{d\dot{S}'_{gen}}{dRe} = \frac{5f\pi^3\mu^5}{32\rho^2\dot{m}^2T}Re^4 + \frac{\pi^3\mu^5}{32\rho^2\dot{m}^2T}Re^5 \frac{df}{dRe} - \frac{q'^2}{\pi T^2 k^* Nu^2} \frac{dNu}{dRe} + \frac{dNu}{df} \frac{df}{dRe} \quad (9)$$

From here, the derivatives of the friction factor and Nusselt number with respect to the Reynold's number need to be calculated. The smooth pipe assumption leads to Equation 10. The Haaland friction factor approximation leads to Equation 11. The Dittus-Boelter approximation for the Nusselt number results in Equation 12. Finally, the Gnielinski equation leads to Equation 13.

$$\frac{df}{dRe} = -0.0092 Re^{-1.2} \quad (10)$$

$$\frac{df}{dRe} = \frac{-0.297572 * \frac{\varepsilon^{1.11} \mu^{1.11}}{\dot{m}^{0.11}} Re^{2.11} - 34.728 * \dot{m}}{Re * \frac{\varepsilon^{1.11} \mu^{1.11}}{\dot{m}^{0.11}} Re^{2.11} + 38.5481 * \dot{m} * \log \frac{\varepsilon \mu Re}{\dot{m}}^{1.11} + \frac{38.5481}{Re} - 0.747154} \quad (11)$$

$$\frac{dNu}{dRe} = 0.0184 Re^{-0.2} Pr^{0.4} \quad (12)$$

$$\frac{dNu}{dRe} + \frac{dNu}{df} \frac{df}{dRe} = \frac{0.02784 * f * Pr}{f^{0.5} Pr^{\frac{2}{3}-1} + 0.222711} + \frac{0.01392 f^{0.5} Pr^{\frac{2}{3}-1} + 0.4454 (Re-1000) Pr}{f^{0.5} Pr^{\frac{2}{3}-1} + 0.222711} * \frac{df}{dRe} \quad (13)$$

Substituting the smooth pipe assumption and the Dittus-Boelter assumption into the entropy generation per unit length derivative results in Equation 14.

$$\frac{d\dot{S}'_{gen}}{dRe} = \frac{0.213943\mu^5}{\rho^2\dot{m}^2T}Re^{3.8} - \frac{11.071648 q'^2}{T^2 k Pr^{0.4}} Re^{-1.8} \quad (14)$$

Setting Equation 14 equal to zero and solving for the optimum Reynold's number results in Equation 15.

$$Re_{opt} = \frac{51.7505 * \rho^2 \dot{m}^2 q'^2}{Tk \mu^5 Pr^{0.4}}^{\frac{1}{5.6}} \quad (15)$$

The remaining substitutions are left to the reader due to the complexity and length of the resulting equations. From here either each derivative equation can be set equal to zero to find the Reynold's number which produces the minimum entropy generation, or these other substitutions can have the minimum directly computed through graphing the original entropy generation per unit length function with a computer aided program such as MATLAB rather than solving the differential.

Only the substitution shown involving the smooth pipe friction factor along with the Dittus-Boelter assumptions can easily be explicitly solved for the optimum Reynold's number. The other assumptions were programmed into a MATLAB script to solve the remaining substitutions. However, the MATLAB script finds the minimum directly using the plot of entropy generation per unit length, rather than finding when the derivative is equal to zero.

The following values shown in Table 1 were used in the MATLAB script as the conditions of the heat exchanger. The equations required to determine the optimal Reynold's number require the properties of water at the bulk mean temperature. Therefore, an iteration must occur to adjust the calculations until the bulk mean temperature is accurate. It was determined to also investigate the dependence of the entropy generation minimization method upon the properties of water.

Table 1: Selected Initial Properties

$\dot{m} = 0.32 \frac{kg}{s}$	$q' = 500 \frac{W}{m}$	$T_i = 20 \text{ }^\circ\text{C}$	$P_i = 650000 \text{ Pa}$	$L = 20 \text{ m}$	$\varepsilon = 0.26 \text{ mm}$
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Therefore three options were selected to determine the bulk mean temperature properties. Two of the methods used the tabulated data in Cengel's *Heat and Mass Transfer Fundamentals and Applications 4th Edition*; a linear interpolation and a spline interpolation between the data points provided [C]. Finally, the third method used was an online properties calculator by peace software called Steam Calculation [P]. This online calculator uses the formulas of IAPWS-IF97 by Dr. Bernhard Spang, however the site gives no guarantee for correctness on the property values calculated [P]. Since this is only to show the dependence of the method upon the property values, this potential error was accepted.

This resulted in the following exit temperatures as well as the bulk mean properties which are shown in Table 2. It can be seen that the values are marginally different amongst the various property determination methods. Therefore, due to the Steam Calculation allowing easy computation of the properties based upon both temperature and pressure, it will be the method used for the direct computation of the entropy generated. Following this determination, the results of the entropy generation minimization were calculated and are shown in Table 3 and the graphs of the entropy generation are shown in Figures 1-3.

Table 2: Properties of Water

	Linear Interpolation	Spline Interpolation	Steam Calculation
T_e (°C)	27.476	27.475	27.475
T_{bm} (°C)	23.738	23.738	23.737
$\rho_{bm} \frac{kg}{m^3}$	997.252	997.248	997.6
$k_{bm} \frac{W}{m * K}$	0.6047	0.6048	0.6053
$\mu_{bm} \frac{kg}{m * s}$	0.000919	0.0091706	0.000916
Pr_{bm}	6.3596	6.3432	6.328
$c_{p-bm} \frac{J}{kg * K}$	4180.5	4180.5	4180.9

Based upon these calculations, it can be seen that the small variations in the water properties did not produce significant changes in the resulting optimal diameter. However, this shows that the optimal diameter can significantly change based upon the assumptions placed upon the friction factor and Nusselt number. Therefore, a verification of the optimum diameter is needed, and hence the need to use the second method of a direct computation of the entropy generated in the heat exchanger by determining the outlet conditions.

It is important to note that the Haaland assumption takes the pipe roughness into account. This increases the contribution to entropy generation due to pressure loss, and therefore results in the optimal diameter increasing as seen in the results.

Table 3: Results of Entropy Minimization Technique

Assumptions	Linear Interpolation		Spline Interpolation		Steam Calculation	
	Reynold's Number	Diameter (m)	Reynold's Number	Diameter (m)	Reynold's Number	Diameter (m)
Smooth pipe and Dittus – Boelter	26040.8	0.0170	26094.2	0.0170	26122.5	0.0170
Smooth pipe and Gnielinski	31177.6	0.0142	31242.3	0.0142	31277.4	0.0142
Haaland and Dittus – Boelter	18380.8	0.0241	18417.6	0.0241	18437	0.0241
Haaland and Gnielinski	18515.0	0.0239	18550.6	0.0240	18569.3	0.0240

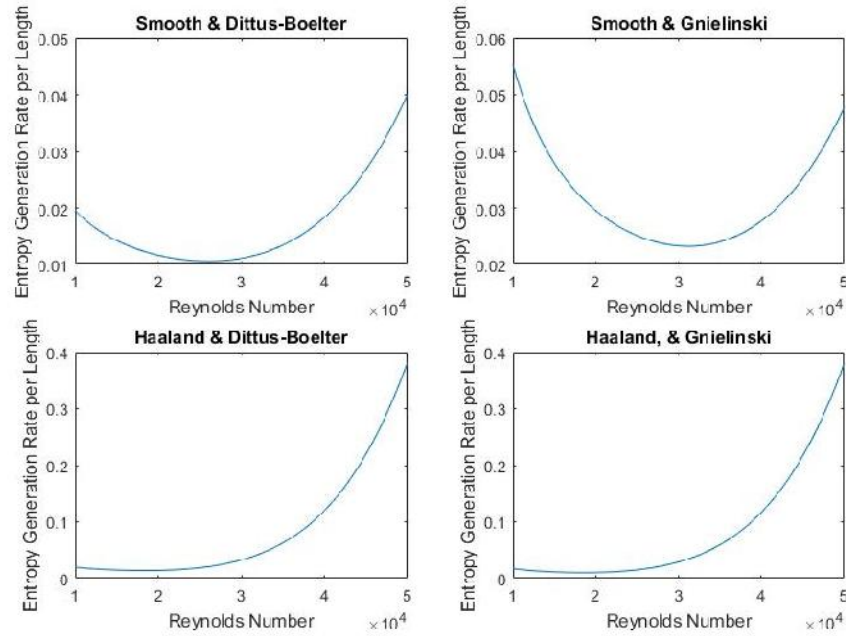


Figure 1: Reynold's Number vs. Entropy Generation – Linear Interpolation Properties

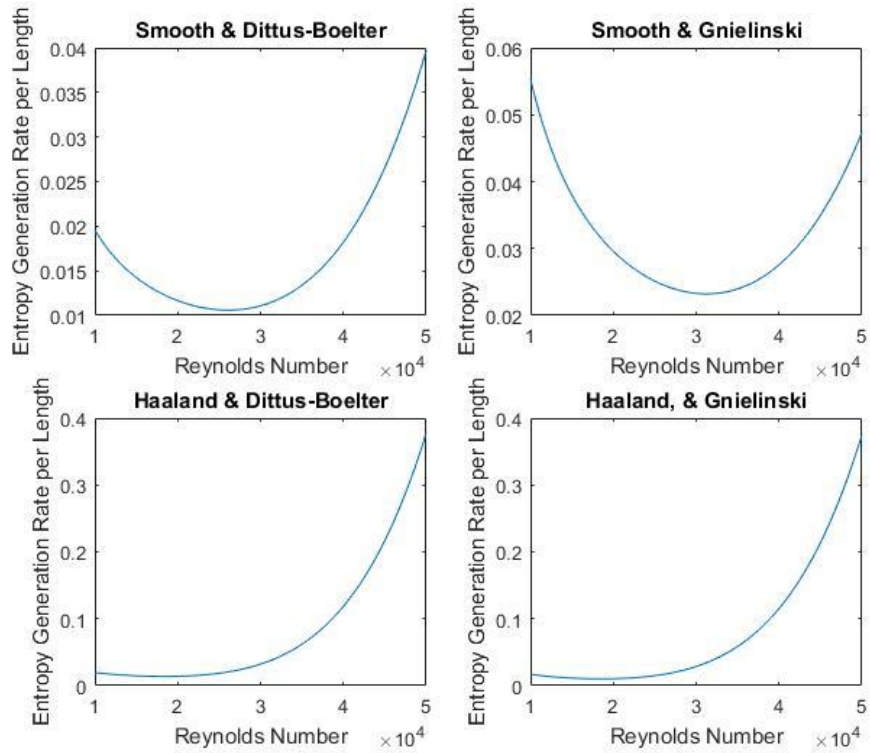


Figure 2: Reynold's Number vs. Entropy Generation - Spline Interpolation Properties

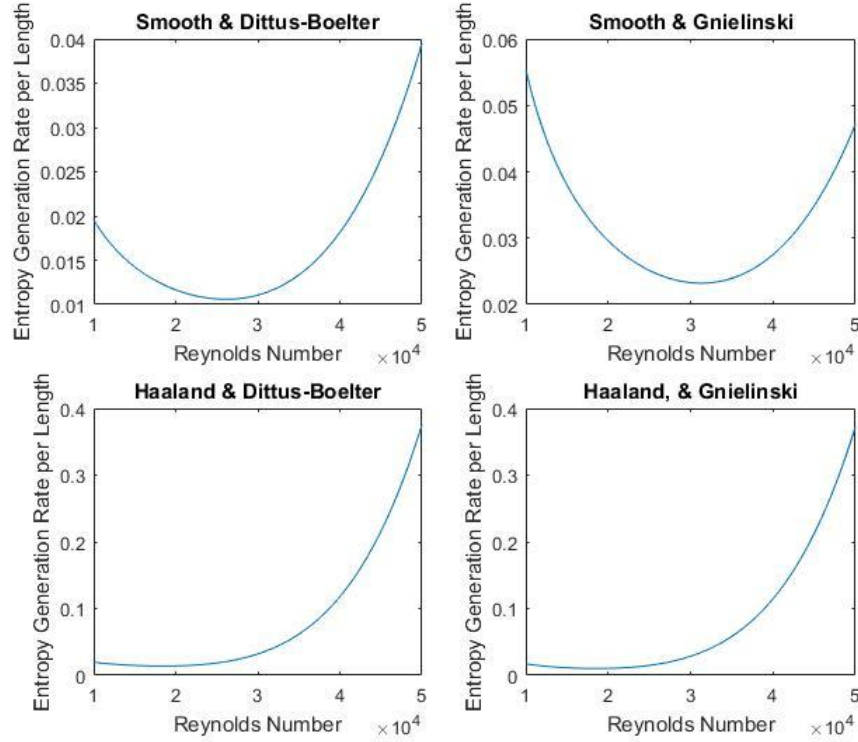


Figure 3: Reynold's Number vs. Entropy Generation - Steam Calculation Properties

Examining Figure 1 indicates that the true entropy generation minimization occurs with the Haaland and Gnielinski assumptions in place. However, it is important to note that when other property values were used different assumptions resulted in the optimum entropy generation rate.

This presents the issue of determining the correct optimal diameter. Do the more accurate friction factor and Nusselt number approximations result in a more accurate optimal diameter, or is the method unstable in regards to the approximations assumed? Thus another mathematical method would be preferred in the verification of which diameter is the optimum. For a steady state single flow system we have Equation 16 for the entropy generation rate which can be applied to the entire heat exchanger [C].

$$\dot{S}_{gen} = \dot{m} s_e - s_i - \frac{L}{0} \frac{\delta \dot{Q}}{T_s} \quad (16)$$

Since the heat exchanger is a constant linear heat flux system, we can substitute for the temperature profile along the pipe surface and the linear heat flux shown in Equations 17 and 18 respectively, resulting in Equation 19 [C]. Substituting Equation 17, the mean temperature profile, and the differential area results in Equation 20.

$$T_s = T_m + \frac{q'}{\pi dh} \quad (17)$$

$$\delta\dot{Q} = \int T_s - T_m dA \quad (18)$$

$$\dot{S}_{gen} = \dot{m} s_e - s_i - \int_0^L \frac{h T_s - T_m dA}{T_m + \frac{q'}{\pi dh}} \quad (19)$$

$$\dot{S}_{gen} = \dot{m} s_e - s_i - \int_0^L \frac{q' d^* dx}{4 T_i + \frac{q' x}{\dot{m} c_p} + \frac{q'}{\pi dh}} \quad (20)$$

From here, it can be seen that all terms within the integral are constant except for the variable of integration. Substituting the relation between the Nusselt number and the heat transfer coefficient shown in Equation 21, performing the integration, and applying the limits results in Equation 22 [C].

$$Nu = \frac{hd}{k} \Rightarrow \int d = Nu * k \quad (21)$$

$$\dot{S}_{gen} = \dot{m} s_e - s_i - \frac{\dot{m} c_p d}{4} \ln \frac{\dot{m} c_p Nu k \pi T_i + \dot{m} c_p q'}{\dot{m} c_p Nu k \pi T_i + \dot{m} c_p q' + Nu k L \pi q'} \quad (22)$$

Substituting the known parameters from Table 1 results in Equation 23. However, it is important to note that a dimensional analysis must be performed prior to substitution to ensure substitutions with base metric units is valid. This results in the expected units of watts per Kelvin, thus for simplicity the units will be left to the reader. From here, a substitution for the Nusselt number is required. We shall use Equation 4 for simplicity. This results in Equation 24 which will place bias towards the results that

used the assumption of Equation 4, but this will still demonstrate the method is inconsistent and depends highly upon the assumptions used throughout the method.

$$\dot{S}_{gen} = 0.32 s_e - s_i - \frac{c_p d}{12.5} \ln \frac{294.71 c_p Nu k + 160 c_p}{294.71 c_p Nu k + 160 c_p + 31415.93 Nu k} \quad (23)$$

$$\dot{S}_{gen} = 0.32 s_e - s_i - \frac{c_p d}{12.5} \ln \frac{6.778 c_p Re^{0.8} Pr^{0.4} k + 160 c_p}{6.778 c_p Re^{0.8} Pr^{0.4} k + 160 c_p + 722.57 Re^{0.8} Pr^{0.4} k} \quad (24)$$

We must now determine the exiting temperature and pressure of the system to determine the specific entropy value of the exiting water. The exiting temperature can easily be computed using Equation 25 [C]. This can be seen to be independent of the selected diameter due to the structure of the problem. Therefore, we will use the online steam calculation properties from Table 2 due to the ability to have variations of the exiting entropy rate based upon both temperature and pressure. This results in Equation 26 for the entropy generation rate with the Reynold's Number substituted in terms of the diameter and the properties of water from Table 2.

$$T_e = T_i + \frac{q' L}{\dot{m} c_p} \quad (25)$$

$$\dot{S}_{gen} = 0.32 s_e - s_i - 334.472 d \ln \frac{4714073 \frac{1}{d}^{0.8} + 668944}{4714073 \frac{1}{d}^{0.8} + 668944 + 120195 \frac{1}{d}^{0.8}} \quad (26)$$

However in determining the exit pressure, Equation 27 will be used which has been written in terms of diameter [M]. This equation is dependent upon the friction factor, and to avoid additional bias towards the previous assumptions, the Moody diagram in conjunction with the Colebrook equation written in terms of diameter shown by Equation 28 will be used [M].

$$P_e = P_i - f \frac{L}{D^5} \frac{8 \dot{m}^2}{\rho \pi^2} \quad (27)$$

$$\frac{1}{f} = -2.0 \log \frac{\varepsilon}{3.7D} + \frac{0.6275\mu\pi D}{\dot{m} f} \quad (28)$$

This will allow the diameter to be iterated to determine which diameter results in the minimal entropy generation. The diameter can be iterated in a simple sectioning scheme. The iteration will begin with the optimal diameter result from the assumptions that produced the minimum entropy generation per unit length as seen in Figure 1. Next the diameter will be perturbed larger and smaller to indicate which direction the minimum exists. Then the diameter will be iterated until the minimum is found. This iteration will use the online calculator Steam Calculation due to the ability to input both the temperature and pressure, whereas the tabulated data uses the saturated water property values.

The specific entropy of the inlet water is $s_i = 296.3709 \frac{J}{kg \cdot K}$ through the property calculator Steam Calculation. The results of the iterations are listed below in Table 4, as well as being graphed in Figure 4. Note, this table has been formatted in decreasing diameter and is not the actual iterations.

Table 4: Diameter Iterations

Diameter (m)	Friction Factor	Exit Pressure (Pa)	Exit Specific Entropy (J/kg K)	Entropy Generation (W/K)
0.016	0.046535	576150.8	401.664	33.83
0.015	0.047477	545963.1	401.673	33.82
0.014	0.048542	499808.4	401.68588	33.81815
0.0135	0.049129	467679.14	401.69492	33.81687
0.0132	0.049501	444455.178	401.70146	33.81645
0.0131	0.049628	435940.567	401.7038565	33.816383
0.0130	0.049757	427001.86797	401.7063723	33.8163527
0.0129	0.049888	417614.3737	401.70901439	33.8163626
0.0128	0.050020	407751.7281	401.711790104	33.8164152
0.0125	0.050429	375021.8698	401.721000895	33.8168555
0.012	0.051152	307928.5	401.7399	33.8187

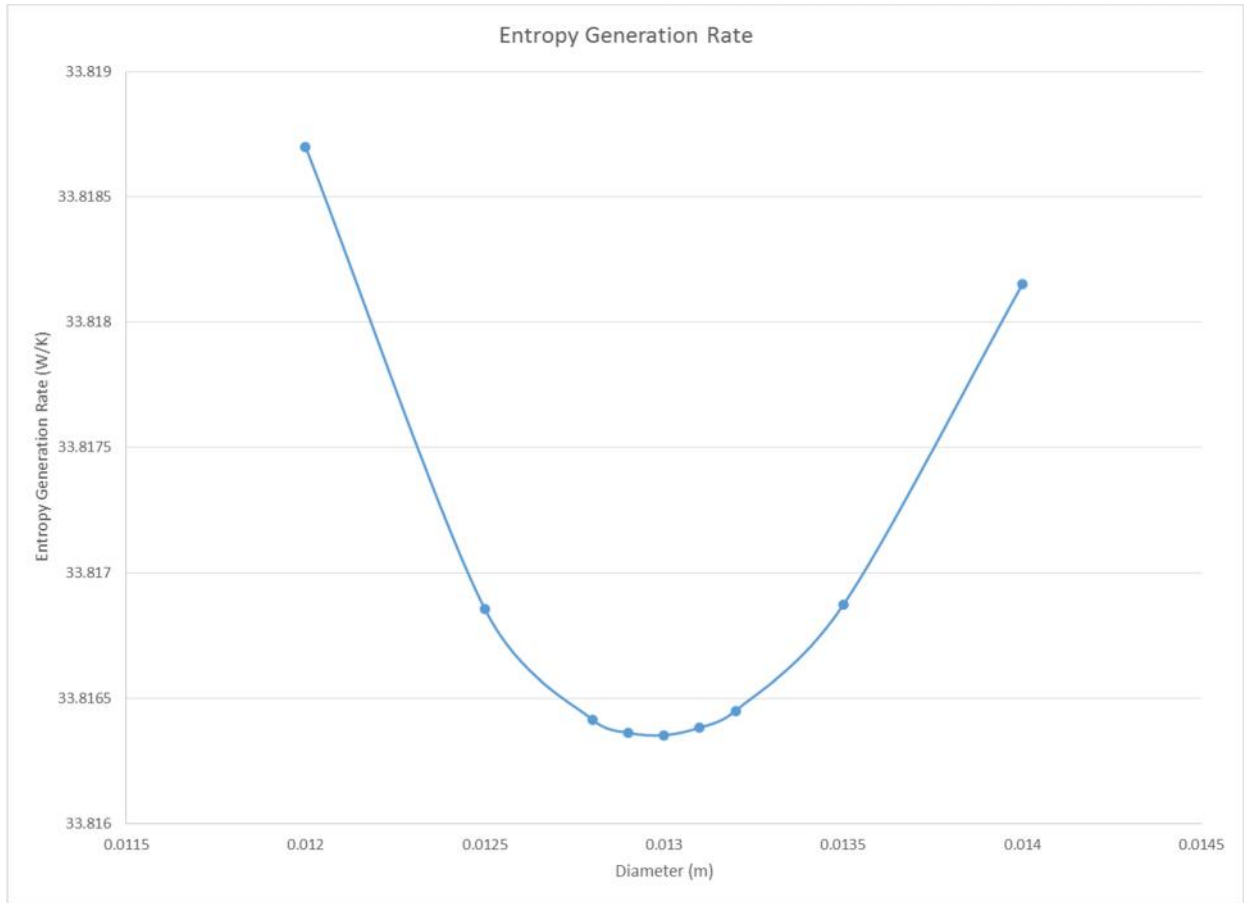


Figure 4: Entropy Generation Rate

From here we see that the optimal diameter is 0.013 m. This is yet another method producing a different claimed optimum diameter that still appears to achieve the goal of minimizing entropy generation. However, it is important to note that although some bias is introduced towards the selected Nusselt number approximation, this method also included the pressure losses due to a rough pipe. That would lead one to expect that a higher optimal diameter closer to the 0.0241 m result would be found. Instead, an optimal diameter smaller than the previous four results was found. Furthermore, the values in Table 4 could not be determined with the precision indicated, but this was needed to fine tune the diameter to 0.013 m. Otherwise, a range of optimal diameters from 0.016 m to 0.012 m would have resulted in the minimal entropy generation rate of 33.8 W/K. However, this range of optimal diameters does include the result of 0.0142 m from the differential approach with the smooth pipe and Gnielinski

assumptions. This indicates that the smooth pipe and Gnielinski assumptions may result in the most accurate optimal diameter.

A final comparison between the various methods is to compare the actual friction factor and Nusselt number approximations for each purported optimal diameter to see the variations amongst each approximation method. These comparisons are shown in Table 5, which is set up so that the main diagonal contains the corresponding method's claimed optimal diameter. This shows the instability of the method of entropy generation minimization lies within the approximations due to no discernible pattern amongst the friction factor and Nusselt number corresponding to the asserted optimal diameters. One particular example to consider is the direct calculation method with the Colebrook equation and the Dittus-Boelter approximation compared to the Haaland and Dittus-Boelter approximations. These two approximations are very similar in the friction factor for all diameters and have the same assumption for the Nusselt number, but the two result in vastly different optimal diameters.

Table 5: Friction Factor and Nusselt Numbers

Diameter	0.013 m		0.0142 m		0.017 m		0.024 m		0.0241 m	
Reynold's Number	34215.4		31323.9		26164.7		18533.3		18456.4	
Method	f	Nu	f	Nu	f	Nu	f	Nu	f	Nu
Direct Colebrook and Dittus-Boelter	0.049757	203.99	0.04832	190.08	0.04570	164.59	0.04182	124.91	0.04179	124.50
Smooth and Gnielinski	0.00570	82.250	0.00580	76.11	0.00602	65.00	0.00644	47.70	0.00645	47.55
Smooth and Dittus-Boelter	0.00570	203.99	0.00580	190.08	0.00602	164.59	0.00644	124.91	0.00645	124.50
Haaland and Gnielinski	0.04983	382.05	0.04838	342.19	0.04573	273.78	0.04177	179.80	0.04174	178.93
Haaland and Dittus-Boelter	0.04983	203.99	0.04838	190.08	0.04573	164.59	0.04177	124.91	0.04174	124.50

Conclusion

The method of entropy generation minimization proposed by Dr. Adrian Bejan was analyzed against a heat exchanger with constant heat flux. The method requires approximations of the friction factor and Nusselt number to be made during the analysis. Two assumptions for each term were made, and inserted into the method. This resulted in four different optimal diameters ranging from 0.0241 m to 0.0142.

An independent method of determining the minimal entropy generation rate through directly calculating the entropy generation rate based upon the heat transferred throughout the system and the inlet and outlet properties. This also required approximations of the friction factor and Nusselt number. The more accurate implicit Colebrook formula for the friction factor was used, which cannot be used in the method proposed by Dr. Adrian Bejan, along with the Dittus-Boelter equation for the approximation of the Nusselt number was used. This resulted in a fifth optimal diameter of 0.013 m, giving a range of optimal diameters from 0.013 m to 0.0241 m. This results in an 85% variation based upon the different assumptions and methods of calculation.

This shows that the method of entropy generation minimization is a mathematically unstable problem with a high dependence upon the approximations chosen for the friction factor and Nusselt number. This results in the method being inadequate for obtaining results to base the design of systems upon, and it should not be used in place of other analyses in the optimization of design.

Future Work

With the conclusion that the entropy generation minimization is unstable with regards to the assumptions used within the approach, it brings into question which assumptions lead to the most accurate results for implementation into design. Was the smooth pipe and Gnielinski assumptions the most accurate optimal diameter since the assumptions resulted in the closest diameter to the direct computation method? To determine this, an experimental validation is required. The specific problem

presented here is a simplified cooling system that would be required for large data centers. Various electronic components require a fixed wattage to operate. This in turn results in a fixed heat generation.

Although the analysis was simplified, this also demonstrates how to design an experiment to test the results. The experiment would require flowing water through various pipe diameters with electronically controlled heat tape wrapped carefully around the length of pipe. This would allow the heat flux into the pipe to be controlled, but insulation should be placed around the pipe to minimize losses.

At the inlet of the pipes, a flow meter, pressure gauge, and temperature gauge would be required to monitor inlet properties. The outlet of the pipe would require a second pressure gauge and temperature gauge. This would allow the specific entropy of the inlet and outlet water to be determined. The final component is a watt meter to determine the exact amount of energy being transferred within the heat exchanger.

An assumption that the insulation on the pipe was sufficient to isolate the heat exchanger from external heat loss or gain would be required for this particular experiment. However with this assumption, the exact entropy generated for each various pipe diameter can be calculated based upon the data above. This would then allow the actual optimal diameter that minimizes entropy generation to be determined. This result would then be compared against the mathematical results presented above. The mathematical/numerical model which best fits the experimental results would give an indication of which assumptions should be used in other optimization problems. However there is no guarantee the assumptions can transfer to other problems and more experimentation will be warranted.

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Appendix 1: Matlab Code EntropyMin.m

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```

%This program determines the optimum Reynold's number for a constant heat
%flux heat exchanger. The program uses two different friction factor
%assumptions and two different Nusselt number equations. The program uses
%a plotting method to find the minimum to allow for plotting the graphs if desired
%Some properties need to be taken at the bulk mean temperature so the program is set up ✓
to
%easily redefine only the bulk mean temperature properties if desired.

clc
disp('Warning: This program requires compatible units and absolute temperatures' )
def = input('Would you like to define all the variables? Y or N: ' , 's');
if def == 'Y'
    mdot = input('Please enter the mass flow rate: ');
    qlinflux =input('Please enter the linear heat flux: ');
    Tin = input('Please enter the initial absolute temperature: ');
    Pin = input('Please enter the initial pressure: ');
    Tbm = input('Please enter the bulk mean temperature: ');
    row = input('Please enter the bulk mean density: ');
    thermcond = input('Please enter the thermal conductivity at the bulk mean ✓
temperature: ');
    dynvisc = input('Please enter the dynamic viscosity at the bulk mean temperature: ');
    Pr = input('Please enter the Prandtl number at the bulk mean temperature: ');
    Cp = input('Please enter the specific heat at the bulk mean temperature: ');
    rough = input('Please enter the pipe roughness: ');
    L = input('Please enter the length of pipe: ');
else
    def = input('Would you like to enter the bulk mean properties? Y or N: ' , 's');
    if def == 'Y'
        Tbm = input('Please enter the bulk mean temperature: ');
        row = input('Please enter the bulk mean density: ');
        thermcond = input('Please enter the thermal conductivity at the bulk mean ✓
temperature: ');
        dynvisc = input('Please enter the dynamic viscosity at the bulk mean temperature: ✓
');
        Pr = input('Please enter the Prandtl number at the bulk mean temperature: ');
        Cp = input('Please enter the specific heat at the bulk mean temperature: ');
    end
end

def = input('Would you like to enter the Reynolds number range? Y or N: ' , 's');
if def == 'Y'
    Remin = input('Please enter the smallest Reynolds number to consider: ');
    Remax = input('Please enter the highest Reynolds number to consider: ');
    Retol = input('Please enter the tolerance for the Reynolds number: ');
    Rerange=[Remin:Retol:Remax];
end

%Create friction factor and Nusselt number ranges
%f1 - smooth, f2 - Haaland, Nul - Dittus-Boelter, Nu2# - Gnielinski with
%corresponding friction factor
f1 = 0.046.*Rerange.^(-0.2);

```

```

f2 = (-1.8.*log10((rough*pi*dynvisc.*Rerange./(14.8*mdot)).^1.11+6.9./Rerange)).^-2;
Nu1 = 0.023*Pr^0.4.*Rerange.^0.8;
Nu21 = Pr*f1.*(Rerange-1000)./(8*(1+12.7.*(f1./8).^0.5.*(Pr^(2/3)-1)));
Nu22 = Pr*f2.*(Rerange-1000)./(8*(1+12.7.*(f2./8).^0.5.*(Pr^(2/3)-1)));

%Compute the different entropy generation per unit length rates
Sgenprime11 = pi^3*dynvisc^5/(32*row^2*mdot^2*Tbm).*f1.*(Rerange.^5)+qlinflux^2./ ✓
(pi*Tbm^2*thermcond.*Nu1);
Sgenprime12 = pi^3*dynvisc^5/(32*row^2*mdot^2*Tbm).*f1.*(Rerange.^5)+qlinflux^2./ ✓
(pi*Tbm^2*thermcond.*Nu21);
Sgenprime21 = pi^3*dynvisc^5/(32*row^2*mdot^2*Tbm).*f2.*(Rerange.^5)+qlinflux^2./ ✓
(pi*Tbm^2*thermcond.*Nu1);
Sgenprime22 = pi^3*dynvisc^5/(32*row^2*mdot^2*Tbm).*f2.*(Rerange.^5)+qlinflux^2./ ✓
(pi*Tbm^2*thermcond.*Nu22);

%Determine the minimum
[~,I]=min(Sgenprime11);
Rel1 = Rerange(I);
fprintf('\n');
fprintf('The optimum Reynolds number for smooth pipes and Dittus - Boelter assumptions ✓
is: %g \n', Rel1);
%Compute the optimal diameter
Dopt11 = 4*mdot/(Rel1*pi*dynvisc);
fprintf('This results in an optimum diameter of %g \n', Dopt11);
%Compute the exit pressure
Pexit11 = Pin-f1(I)*L*Rel1^2*dynvisc^2/(Dopt11^3*2*row);
fprintf('with an exiting pressure of %g \n', Pexit11);
%Compute the exit temperature
Texit11 = Tin+qlinflux*L/(mdot*Cp);
fprintf('and an exit temperature of %g \n', Texit11);
%Compute the total heat transfer rate
Q11 = qlinflux*L;
fprintf('The total heat transfer rate is %g \n', Q11);
%Compute the entropy generation rate (assumed average over length,
%but this seems to allways be off when trying to do the verification)
Sgen11=Sgenprime11(I)*L;
fprintf('This resulted in an entropy generation rate of %g \n\n', Sgen11);

[~,I]=min(Sgenprime12);
Rel2 = Rerange(I);
fprintf('The optimum Reynolds number for smooth pipes and Gnielinski assumptions is: %g ✓
\n', Rel2);
Dopt12 = 4*mdot/(Rel2*pi*dynvisc);
fprintf('This results in an optimum diameter of %g \n', Dopt12);
Pexit12 = Pin-f1(I)*L*Rel2^2*dynvisc^2/(Dopt12^3*2*row);
fprintf('with an exiting pressure of %g \n', Pexit12);
Texit12 = Tin+qlinflux*L/(mdot*Cp);
fprintf('and an exit temperature of %g \n', Texit12);
Q12 = qlinflux*L;
fprintf('The total heat transfer rate is %g \n', Q12);
Sgen12=Sgenprime12(I)*L;

```

```

fprintf('This resulted in an entropy generation rate of %g \n\n' , Sgen12);

[~,I]=min(Sgenprime21);
Re21 = Rerange(I);
fprintf('The optimum Reynolds number for Haaland and Dittus - Boelter assumptions is: %g \n' , Re21);
Dopt21 = 4*mdot/(Re21*pi*dynvisc);
fprintf('This results in an optimum diameter of %g \n' , Dopt21);
Pexit21 = Pin-f2(I)*L*Re21^2*dynvisc^2/(Dopt21^3*2*row);
fprintf('with an exiting pressure of %g \n' , Pexit21);
Texit21 = Tin+qlinflux*L/(mdot*Cp);
fprintf('and an exit temperature of %g \n' , Texit21);
Q21 = qlinflux*L;
fprintf('The total heat transfer rate is %g \n' , Q21);
Sgen21=Sgenprime21(I)*L;
fprintf('This resulted in an entropy generation rate of %g \n\n' , Sgen21);

[~,I]=min(Sgenprime22);
Re22 = Rerange(I);
fprintf('The optimum Reynolds number for Haaland and Gnielinski assumptions is: %g \n' , Re22);
Dopt22 = 4*mdot/(Re22*pi*dynvisc);
fprintf('This results in an optimum diameter of %g \n' , Dopt22);
Pexit22 = Pin-f2(I)*L*Re22^2*dynvisc^2/(Dopt22^3*2*row);
fprintf('with an exiting pressure of %g \n' , Pexit22);
Texit22 = Tin+qlinflux*L/(mdot*Cp);
fprintf('and an exit temperature of %f \n' , Texit22);
Q22 = qlinflux*L;
fprintf('The total heat transfer rate is %g \n' , Q22);
Sgen22=Sgenprime22(I)*L;
fprintf('This resulted in an entropy generation rate of %g \n\n' , Sgen22);

%Generate plots if desired
def = input('Would you like to see the plots of the four methods? Y or N: ' , 's');
if def == 'Y'
    figure
    subplot(2,2,1)
    plot(Rerange,Sgenprime11)
    title('Smooth & Dittus-Boelter')
    xlabel('Reynolds Number')
    ylabel('Entropy Generation Rate per Length')
    subplot(2,2,2)
    plot(Rerange,Sgenprime12)
    title('Smooth & Gnielinski')
    xlabel('Reynolds Number')
    ylabel('Entropy Generation Rate per Length')
    subplot(2,2,3)
    plot(Rerange, Sgenprime21)
    title('Haaland & Dittus-Boelter')
    xlabel('Reynolds Number')
    ylabel('Entropy Generation Rate per Length')

```

```
subplot(2,2,4)
plot(Rerange, Sgenprime22)
title('Haaland, & Gnielinski')
xlabel('Reynolds Number')
ylabel('Entropy Generation Rate per Length')
end
```