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Title

Sub Critical Reactivity Measurements in the AGN-201 Reactor

by

Sultan Mohammad Mahbub Sharif

A thesis

submitted in partial fulfillment

of the requirements for the degree of

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Committee Approval

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Sub Critical Reactivity Measurements in the AGN-201 Reactor

Thesis Abstract -- Idaho State University (2019)

A Californium-252 and Radium-Beryllium neutron source were used in the Idaho State University's AGN 201 nuclear reactor. The main objective of this thesis was to study the competing effects of Californium-252 as an independent source, a parasitic absorber, and a fissile material. Four experiments were conducted with Californium-252 and Radium-Beryllium neutron sources. A linear oscillator was used in the glory hole, and the Californium-252 source was attached to the linear oscillator which was placed in glory hole through the center of the reactor core. Performance Motion Device was used to control linear oscillator. A Fourier analysis was conducted with the obtained experimental data to see the different frequencies in different regions and peak of the oscillations. Lastly, a perturbation and transfer function analysis were done to see the difference between the theoretical perturbation calculation and experimental results.

Key Words: AGN-201, Subcritical Reactivity, Neutron Source, Californium -252, Radium-Beryllium, Perturbation Analysis, Linear Oscillation.

Chapter 1: Introduction

1.1 Introduction to the Thesis:

Oak Ridge National Laboratory (ORNL) is proposing a sub-critical fast assembly with a modest neutron source. In this thesis four experiments were conducted at the Idaho State University's AGN-201 nuclear reactor to see the behavior of the neutron source and absorber oscillation in the slightly sub-critical condition. The interest of this thesis is to study the competing effects of Californium-252 as an independent source, a parasitic absorber, and a fissile material. Experiments commenced with simple in-out measures using RaBe neutron source and a small Californium-252 neutron source. Then more controlled experiments were performed with a linear oscillator.

1.2 The I.S.U. AGN-201 Reactor:

The ISU AGN- 201 Reactor is located in the Lillibridge Engineering Laboratory building (room 20, designated the Reactor Laboratory) on the Idaho State University campus. It is a homogeneous thermal reactor manufactured by Aerojet General Nucleonics Corporation, San Ramon, California. The reactor is licensed by the Nuclear Regulatory Commission (NRC) to operate at a maximum power level of five watts (5 W). The principle uses of this reactor are education and training. The reactor reflector is graphite, the shield is lead and water, and there are two safety rods and two control rods. It is a cylindrical shape reactor, the core being 25.6 cm in diameter and 25.8 cm in height.

1.3 Reactor Fuel:

The reactor fuel consists of a homogeneous mixture of uranium dioxide (UO_2) powder in polyethylene. The uranium dioxide is enriched to just less than twenty weight percent (20 w/o) of

Uranium-235 (U 235). The total mass of Uranium-235 in the reactor is approximately 672.9 g. The ratio of hydrogen atoms to uranium total atoms in the fuel is over 110 to 1. Therefore, the calculations of reactor parameters may be greatly simplified. The actual atom densities are as follows:

Uranium-235: 1.43 X 10^{-4} atoms/b-cm

Uranium-238: 5.71 X 10^{-4} atoms/b-cm

Hydrogen: 7.91 X 10^{-2} atoms/b-cm

1.4 Reactor Assembly:

The reactor assembly and main components are shown in Figure 1. The main reactor components in the Figure 1 from the outside to the center are the reactor vessel, the water shield, the steel support tank, the lead shield, the graphite reflector, and the core tank assembly. Also, Fig. 1 shows the "glory hole" through the center of the reactor and the thermal column at the top of the reactor. External cooling is not required due to the low operating power (5 W) of the reactor.

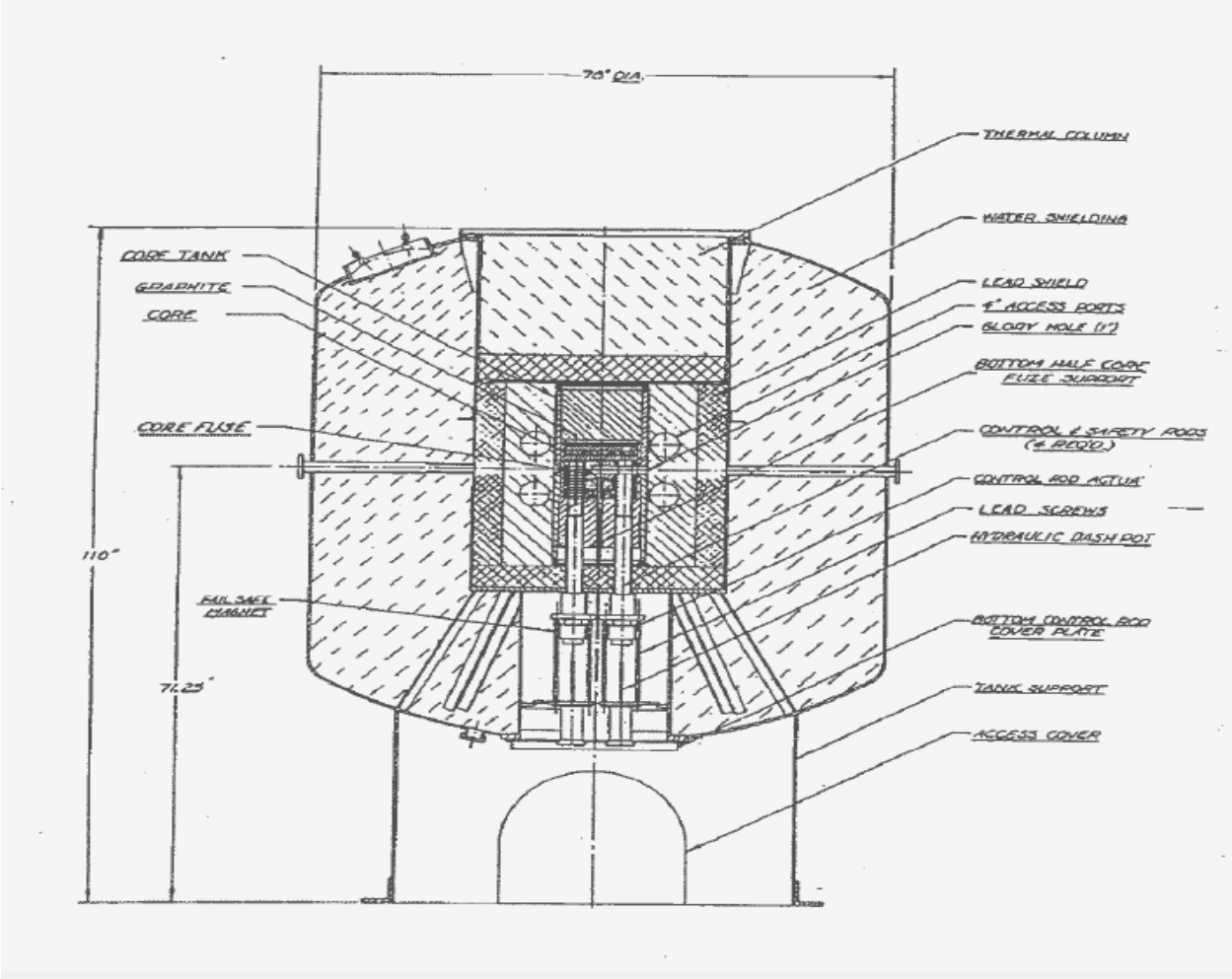

Figure 1: Reactor Assembly

1.5 Shielding:

Shielding of the reactor is contained in the 2-meter diameter steel reactor vessel which is a 0.866-meter diameter steel support tank. The walls of the support tank are approximately 10 millimeters thick. The volume from the inside of the steel reactor vessel to the outside of the steel support tank is filled with water. The water is shown in the Figure 1 with dashed lines. The water is approximately 3800 liters and forms a 550-millimeter-thick shield between the reactor vessel and the support tank.

1.6 Reflector:

Heavy density graphite (1.75 g/cm^3) is used for the reflector and thickness of graphite is 20 cm on all sides of the core. The reflector is shown in Figure 1 as the diagonal lines. The reflector is open at the top and bottom forming a hollow cylinder. The top and bottom reflector is in the core tank, the side reflector surrounds the core tank and contains four 10 cm through-holes running tangentially to the core tank. The core tank also contains some graphite to provide approximately 22.5 cm of reflector above and below the core as well as an additional 2.5 cm around the core for a total radial width of 22.5 cm. The graphite reflector cylinder contains five horizontal holes, four of these holes are for the access ports and the fifth is for the glory hole.

1.7 Source:

A ten milligram Radium-Beryllium (RaBe) mixture source is used to start up the reactor. This source produces approximately 10^5 neutrons per second.

1.8 Control & Safety Rods:

The AGN 201 has two safety and two control rods. Two control rods are fine control rod (FCR) and coarse control rod (CCR). The two safety and CCR are identical in design. Each is composed of 15 grams of U-235 mixed with polyethylene. Safety and coarse control rod reactivity worths are 1.25 %Δk/k or $1.69 each. In addition, fine control rod worth is 0.310 %Δk/k when fuel loaded and 0.155 %Δk/k when polyethylene loaded.

1.9 Nuclear Data:

Fuel loading for the AGN 201 is 672.9 grams of U-235. Excess reactivity of this reactor at 20° C is \$0.24 or 0.18 %Δk/k. The average and peak thermal flux is 1.5×10^8 n cm^{-2} s^{-1} and 2.5×10^8 n cm^{-2} s^{-1} at 5 watts, respectively. Temperature coefficient of reactivity is approximately -0.025 %Δk/k $°C^{-1}$.

Chapter 2: Reactor Theory

2.1 Point Kinetics Equation:

The time dependent diffusion equation based on the assumption of the one thermal energy group is given below [1][3][4],

$$\frac{\partial n(\vec{r},t)}{\partial t} = \frac{1}{\upsilon}\frac{\partial(\varphi(\vec{r},t))}{\partial t} = S(\vec{r},t) - \Sigma_a\ \varphi(\vec{r},t) + D\nabla^2\ \varphi(\vec{r},t) \qquad \textbf{Eq. 1}$$

υ= neutron speed (cm/sec)

n= neutron density (neutron/cm^3)

φ= neutron flux (neutron/cm^2 sec)

Σ_a= macroscopic absorption cross section (cm^{-1})

D= diffusion coefficient (cm)

S= source (neutron/cm^3 sec)

The source term can be written as

$$S(\vec{r},t) = (1-\beta)\nu\Sigma_f\,\varphi(\vec{r},t) + \sum_{i=1}^{6}\lambda_i C_i(\vec{r},t) + q(\vec{r},t) \qquad \textbf{Eq.2}$$

ν= number of neutrons emitted per fission (note the difference between $\boldsymbol{\nu}$ and $\boldsymbol{\upsilon}$ in this thesis, throughout the thesis ν is used for number of neutrons emitted per fission and υ is used for neutron speed)

q(r,t)= external neutron source. For this thesis experiment, the external neutron sources are RaBe and Cf-252.

Σ_f = macroscopic fission cross section (cm^{-1})

The precursor equation for each of the delayed groups is

$$\frac{\partial Ci(\vec{r},t)}{\partial t} = \beta_i\nu\Sigma_f\,\varphi(\vec{r},t) - \lambda_i C_i(\vec{r},t), \text{where } \beta \text{ is delayed neutron fraction.} \qquad \textbf{Eq. 3}$$

Assuming the spatial distribution of the precursor population is the same as the spatial distribution of the flux and both are separable in space and time

$$\varphi(\vec{r},t) = \varphi(t)\Psi(\vec{r}) = \upsilon n(t)\psi(\vec{r}) \quad \textbf{Eq. 4}$$

$$C_i(\vec{r},t) = C_i(t)\psi(\vec{r},) \quad \textbf{Eq. 5}$$

Substituting in the above flux and precursor functions into the diffusion equation and precursor equation and doing a separation of variables gives,

$$\frac{dn(t)}{dt} = (1-\beta)\nu\Sigma_f\upsilon n(t) + \sum_{i=1}^{6}\lambda_i C_i(t) - \Sigma_a\upsilon n(t) - DB^2\upsilon n(t) + q \quad \textbf{Eq. 6}$$

$$\frac{dc_i(t)}{dt} = \beta_i\nu\Sigma_f\varphi(t) - \lambda_i C_i(t) \quad \textbf{Eq. 7}$$

Defining,

$$l = \text{neutron lifetime (sec)} = \frac{1}{\upsilon\Sigma_a(1+L^2B^2)}$$

K_{eff} = effective multiplication constant = $\frac{\nu\Sigma_f}{(1+L^2B^2)\Sigma_a}$

L^2 = diffusion area (cm^2) = $\frac{D}{\Sigma_a}$,

Substituting the values of neutron generation time and reactivity,

$$\frac{dn(t)}{dt} = \frac{k(t)(1-\beta)-1}{l}\, n(t) + \sum_{i=1}^{6}\lambda_i C_i(t) + q \quad \textbf{Eq. 8}$$

$$\frac{dc_i(t)}{dt} = \beta_i \frac{k(t)}{l}\, n(t) - \lambda_i C_i(t) \quad \textbf{Eq. 9}$$

$$\Lambda = \text{neutron generation time (sec)} = \frac{l}{k(t)}$$

$$\rho(t) = \text{reactivity} = \frac{k(t)-1}{k(t)}$$

Finally, the point reactor kinetics equations are,

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda}n(t) + \sum_{i=1}^{6}\lambda_i C_i(t) + q \quad \textbf{Eq. 10}$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i(t) \quad (i = 1\ldots..6) \quad \textbf{Eq. 11}$$

2.2 Inverse Kinetics Equation:

The inverse kinetics equation is a solution to the point reactor kinetics equations in which the reactivity is determined by the knowledge of the power "n", and its rate of change dn/dt [1][4].

It is given by,

$$\rho_o = s\Lambda + \beta - \sum_{i=1}^{6} \frac{\lambda_i \beta_i}{(s+\lambda_i)} = s\Lambda + \sum_{i=1}^{6} \frac{\beta_i s}{(s+\lambda_i)} + q \qquad \textbf{Eq. 12}$$

This is known as the inverse kinetics equation.

2.3 Least Square Inverse Kinetics Method (LSKIM):

Least Square Inverse Kinetics Method (LSKIM) can be used for determining the source strength value and the reactivity [1]. The following fitting model is used to find the reactivity and source strength,

$$\rho_d = \rho_j' - \Lambda q / n_j' \qquad \textbf{Eq. 13}$$

$$\rho_j' = \beta eff + \frac{\Lambda}{n_j} . \frac{\Delta n_j}{\Delta t} - \frac{\Lambda}{n_j} \sum_{i=1}^{6} \lambda_i c_{i,j} \qquad \textbf{Eq. 14}$$

Another fitting model is,

$$n_j = \frac{\Lambda}{\rho d - \beta eff} Q_j - \frac{\Lambda S}{\rho d - \beta eff} \qquad \textbf{Eq. 15}$$

where $Q_j = \frac{\Delta nj}{\Delta t} - \sum_{i=1}^{6} \lambda_i C_{i,j}$

The source term (q) was added in the inverse kinetics equation. Then, consider all terms on the right side as ρ_j' except source term and solve for ρ_j'. Hence, two unknowns are left which are reactivity and source. From there by using LSKIM method, source and reactivity can be calculated. In this thesis, the reactivity is considered to be known and constant, which is only partially true since the source also affects the reactivity through absorption. Part of the analysis to follow will attempt to support the effects.

2.4 Perturbation Theory:

Perturbation theory is used to obtain a simple estimate of the reactivity effects of the small source. In this thesis the following equation has been used for perturbation analysis [2].

$$\rho = \frac{\int_{v_{perturb}} [\delta \upsilon_p \Sigma_{f_p} - \delta \Sigma_{a_p}] \varphi_p^2 dv - \int_{v_{perturb}} \delta D(\nabla \varphi_p^2) dv}{\int_{v_{reactor}} \upsilon_r \Sigma_{f_r} \varphi_r^2 dv} \quad \textbf{Eq. 16}$$

In this equation, the numerator is the perturbed part, which is in this thesis experiment the Cf-252 source or the Ra-Be source. The denominator is an integral over the entire reactor core, which is uranium and polyethylene. In order to estimate the reactivity ρ, the values of average number of neutrons per fission υ, the macroscopic fission cross section Σ_f, macroscopic absorption cross section Σ_a, and the flux has to be determined for both the perturbed volume (numerator) and reactor core volume (denominator) separately. Using best estimate values provides the reactivity estimate of the perturbed system. This will be used in a later section.

2.5 Transfer Function:

Rewriting the point reactor kinetics equation with source [1],

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{l} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) + q \quad \textbf{Eq. 17}$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{l} n(t) - \lambda_i C_i(t) \quad \textbf{Eq. 18}$$

Assuming that there are small perturbation of density, reactivity, source, and precursor; about their equilibrium,

$$n(t) = n_0(t) + \delta n(t) \quad \textbf{Eq. 19}$$

$$\rho(t) = \rho_0(t) + \delta \rho(t) \quad \textbf{Eq. 20}$$

$$q(t) = q_0(t) + \delta q(t) \quad \textbf{Eq. 21}$$

$$c(t)= c_0(t) + \delta c(t) \qquad \textbf{Eq. 22}$$

with constant reactivity at equilibrium,

$$\frac{dc_i}{dt} = 0 \text{ and } \frac{dn}{dt} = 0$$

Hence, from equation 18,

$$\lambda_i c_i = \frac{\beta_i}{l} n$$

$$\text{or } c_i = \frac{\beta i}{\ell \lambda_i} n$$

and equation 17 becomes,

$$q = -\frac{n\rho}{\ell}$$

$$\text{or, } \rho = -\frac{q\ell}{n}$$

This just states that the reactivity will be negative if there is a source in the reactor (at equilibrium).

By using equation 18 and 22 and substituting above values, the following equation is found,

$$\frac{d\delta c_i}{dt} = \frac{\beta_i}{\ell} \delta n - \lambda_i \delta c_i \qquad \textbf{Eq. 23}$$

Similarly, for dn(t)/dt,

$$\frac{d\delta n}{dt} = \frac{\rho_0 - \beta}{l} \delta n + \sum_i \lambda_i \delta c_i + \delta q \qquad \textbf{Eq. 24}$$

With equation 22 and 24, the transfer function of the source and reactivity can be obtained by the Laplace transforms.

The transfer function for source perturbation is obtained,

$$\frac{\delta n(s)}{\delta q(s)} = \frac{l}{ls+\beta-\rho_0-\Sigma_i\frac{\lambda_i\beta_i}{s+\lambda_i}} \qquad \textbf{Eq. 25}$$

and the transfer function for the reactivity perturbations is

$$\frac{\delta n(s)}{\delta \rho(s)} = \frac{n_0}{ls+\beta-\rho_0-\Sigma_i\frac{\lambda_i\beta_i}{s+\lambda_i}} \qquad \textbf{Eq. 26}$$

As can be seen, the only difference is the "gain"; the kinetics are given by the equilibrium values of reactor parameters.

Chapter 3: Experimental Setup

3. 1 Experimental Steps:

The first series of experiment was envisioned to be simple in-out measurement of the source effect. Below is the procedure that attempted,

1. Bring the reactor to zero power critical as normal. As normal means 10 mw power
2. After required entries, re-insert the startup source
3. If reactor power decreases, add reactivity to re-establish criticality
4. If reactor power increases, subtract reactivity to re-establish criticality
5. With new critical position, now withdraw source
6. Observe, but do not change rod positions.
7. Re-insert source, but do not change rod positions.
8. After a minute or two, tweak the CR to ensure critical position
9. Now, withdraw enough reactivity to estimate that the reactor is about k=0.98
10. Withdraw source
11. Insert source
12. Withdraw source, then shut down

When the experiment commenced some procedures were changed based on the observation. During all steps, rod positions and time was to be noted. There was a wait time to see what happens during the time of source changes. The main purpose of these experimental steps was to see the response of the reactor to the insertion and removal of the source. Three experiments were performed by single in and out measured insertion of the two sources. The second and third will be analyzed to obtain an estimate of the reactivity of the source. From a dynamics point of view,

results will determine a perturbation of the reactivity and the source strength. Both have similar but not identical transfer functions.

3.2 Linear Oscillator:

A linear oscillator was used for the fourth (and last) experiment using the Cf-252 source. Linear oscillator was controlled by a motor (ION 500 by Performance Motion Devices ™). Independent 24V power supplies were used to supply power to the motor controllers. A picture of the ION 500 motor controller is given below along with a linear actuator. Performance Motion Devices (PMD) software was used to control the motor and a computer program “pro-motion” is used to control the linear oscillator.

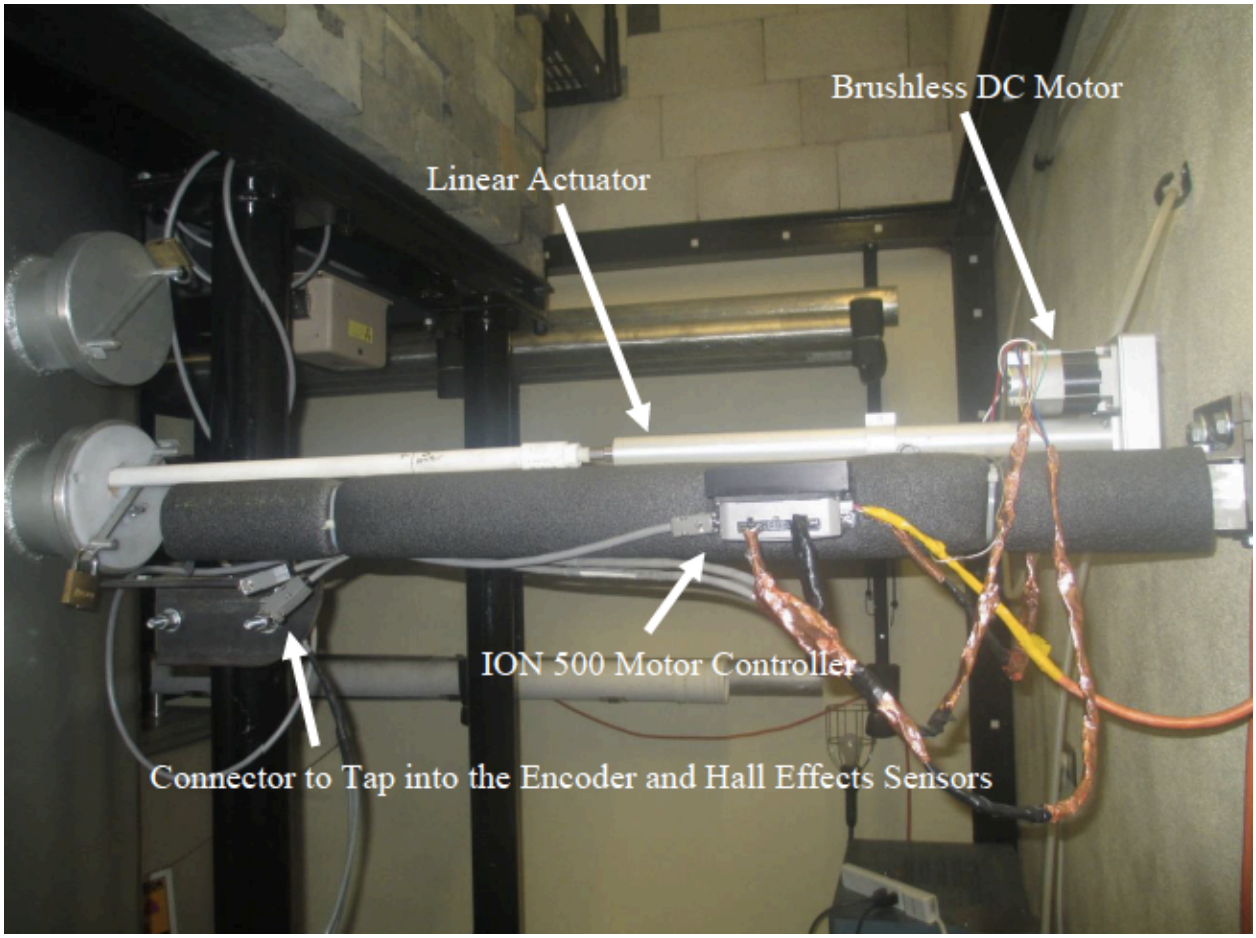

Figure 2: Linear Oscillator and Motor Controller

An aluminum rod is connected with the linear oscillator through the reactor glory hole. An aluminum tube goes through the entire reactor and contains a sample of Californium 252. The aluminum tube is 114.5 inches in length and 5/8 inches in diameter. Inside of the aluminum tube, Cf-252 was placed and secured such that in one oscillator position the source would be at the center and in the other it would be outside the active core. There are connectors on the linear oscillator side and the tube is connected with the linear oscillator.

Figure 3: Aluminum Tube from Reactor Operator Side

3. 3 Californium 252:

The Californium 252 source was manufactured by Frontier Technology Corp. Activity of Cf-252 was 112.3 millicurie or 4.155 GBq, calibrated on 24th June 1993. Total mass of the Cf-252 was 209.4 µg at the time of the calibration.

Chapter 4: Data and Result Analysis

4.1 Experiment 1:

Experiment 1 was conducted on April 9, 2019 Experiment 1 was conducted with the Radium-Beryllium (RaBe) source. The main purpose of this experiment was to observe the effect of the RaBe source. Experiment 1 was the initial experiment and the preparation for the experiment 2,3, and 4. The procedure for experiment 1 was,

1. Reactor was brought to 10 mW power (as it is considered as normal power) and reactor was in critical condition. Start time: Time: 10:38 am, Fine Control Rod position 10.7 cm, Coarse Control Rod position 24 cm,
2. Time: 11:07 am, source was re-inserted and FCR was maintained at position 10.7 cm.
3. At 11:26 am, it was seen that power decreased. Hence, reactivity was added to re-establish the criticality by adjusting FCR rod. Then, critical condition reached, and power is 10 mw. The new FCR position was 20.62 cm (nearly fully inserted).
4. At 11:32 am, the source was withdrawn and FCR position remain unchanged. Again, from the observation power increased and reactor was supercritical.
5. Source re-inserted at time 11:35 am, rod position still remains same as before.
6. After re-inserting source and while waiting for reactor become critical, the operator accidently allowed it to coast down too far (assuming it went to subcritical position). Hence, to re-establish the critical condition FRC is all the way in to get back to 10mW power. Hence, in this process it took some extra time. In addition, in this process when FCR is all the way in, the operator suspected that reactor could go to supercritical from sub critical. Hence, the operator waited and observed. But reactor did not go to supercritical. At 11:47 am FCR is all the way in and operator waited till 12:31 pm. The reactor was

apparently sub-critical at that point. Thus, the operator mistakenly did not withdraw enough reactivity to settle reactor into subcritical condition K=0.98(FCR position should be 23.5 from the chart). So, it was not possible to establish the K=0.98 conditions. Reactor operator then tried to establish critical at 10mW. Then, operator wanted to settle K=0.98 by withdrawing reactivity but cannot reestablish critical position. It was very near to sub critical condition, which was nearly 0.98 and FCR position was 24.01.

7. Source withdrawn at 12:56PM and FCR position was 24.01 cm. Power increased and operator stopped at 3W.
8. Source put back in at 1:00PM and FCR position is 24.01cm.
9. Source removed at 1:10PM and power goes to 3W. Then, reactor shut down at 1:18 AM.

After obtaining data from the experiment 1, the data was analyzed. The data plot of channel 2 (logarithmic scale) and channel 3 (linear scale) is given in the next page. The total data population was obtained 459,001 from the experiment 1 and time interval between the data is 20 milliseconds. Experiment 1 was just a scoping trial to help conduct the following three experiments with more defined purpose and procedure. ***Thus, there was no further analysis of experiment 1.***

Raw data power plot of experiment 1 for linear and log channel is given below,

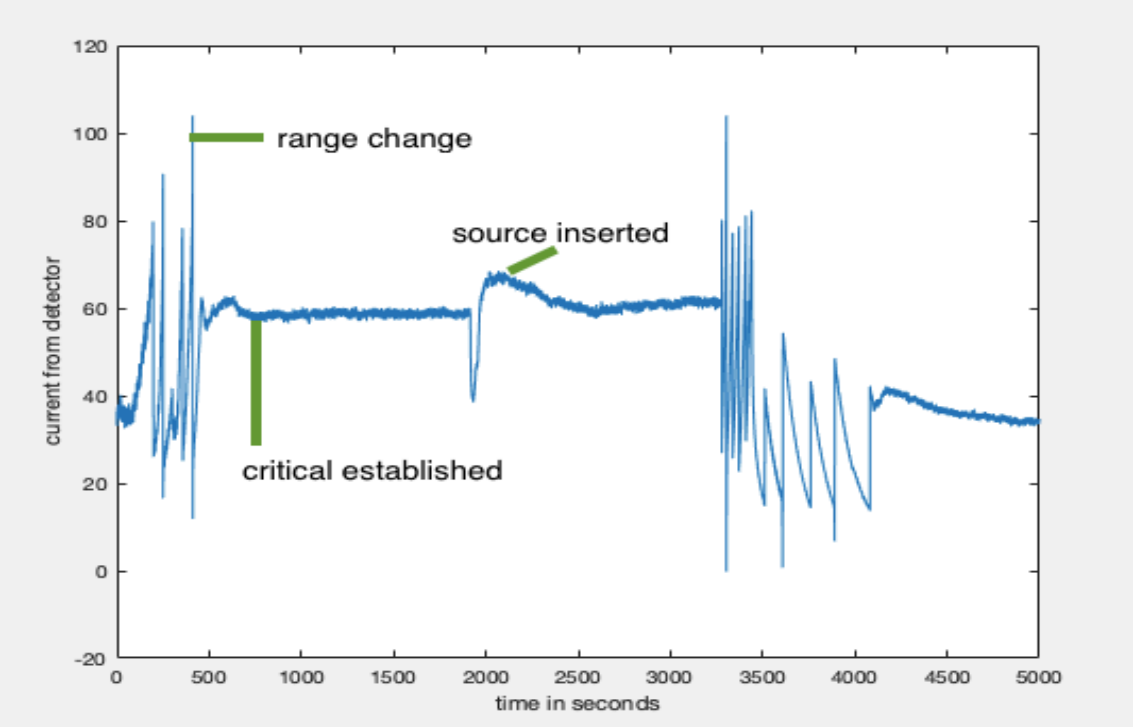

Figure 4: Raw Data Power of Channel 3 (experiment 1, linear)

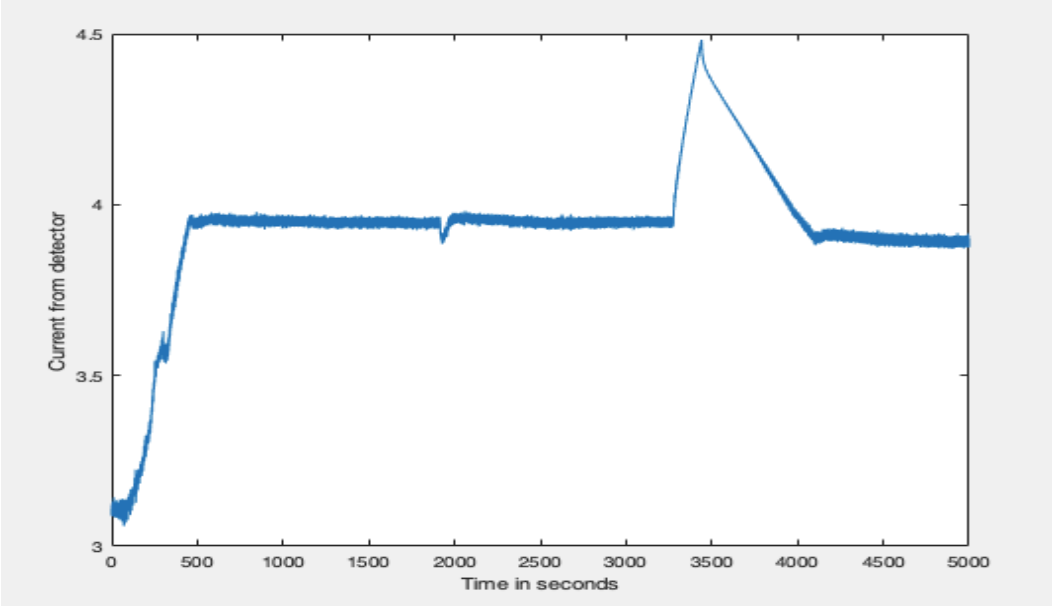

Figure 5: Raw Data Power of Channel 2 (experiment 1, log)

4.2 Experiment 2:

Experiment 2 was conducted on April 23rd, with the Cf-252 source. In this experiment, the Cf-252 source was controlled manually through the glory hole in the nuclear reactor. The data contained 315,001 points with data obtained in 20 millisecond increment. The procedure for experiment 2 was,

1. Reactor was brought to critical at 10:13 am and power was 10 mW. Fine control rod (FCR) position observed and it was 13.6 cm and Coarse Control Rod (CCR) was 24 cm. During the whole experiment CCR remains same.
2. At time 10:17 am, source was re-inserted, and power decreased.
3. After adjusting control rod and current, at time 10:24 am reactor become critical again. Control rod position was 14.6 cm, and reactor power back to 10 mW.
4. During the source withdrawn procedure reactor was scrammed at 10:27 am.
5. Between time 10:27 am and 10:41 am reactor was brought to nearly critical but just subcritical.
6. At 10:41 am, reactor was in sub critical condition and control rod position was 8 cm.
7. At 10:55 am source was removed, and K values was approximately 0.998. Control rod position was 10.1 cm.

Please note that for experiment 2 source analysis, only the time between 10:42am and 10:55am was analyzed to obtain source reactivity of Cf-252 source.

The raw data plot for channel 3 (linear) and inverse kinetics are given below,

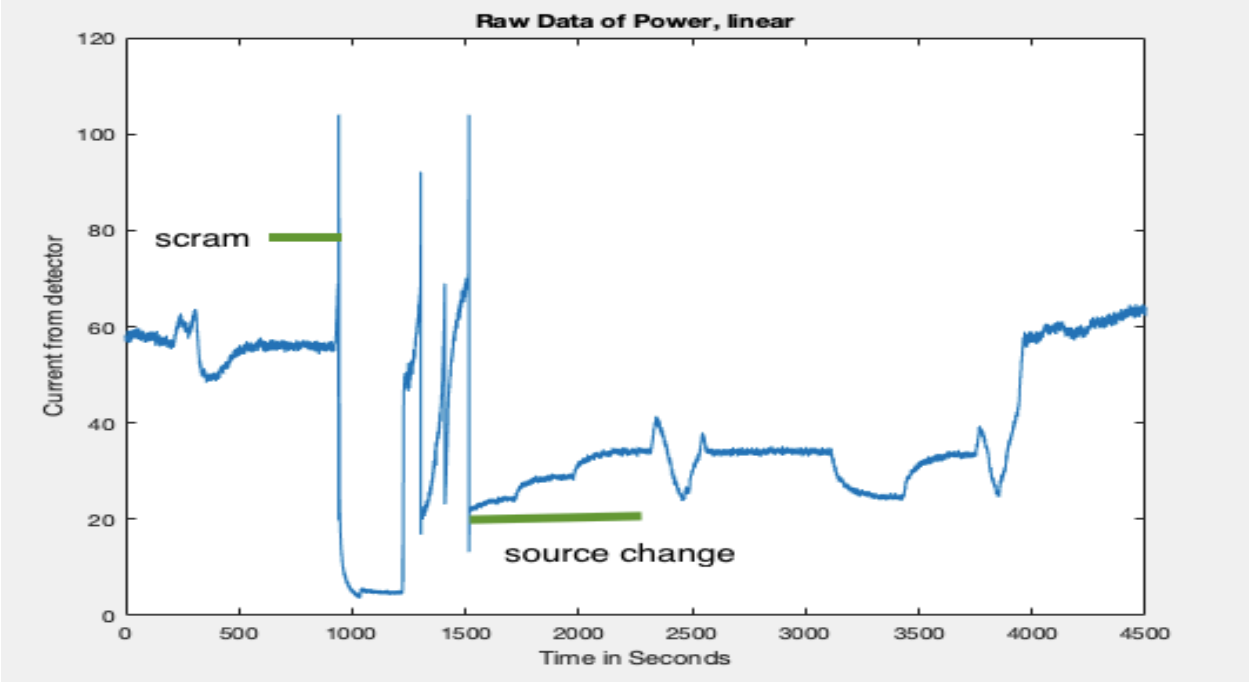

Figure 6: Raw Data Plot of Channel 3 (experiment 2, linear)

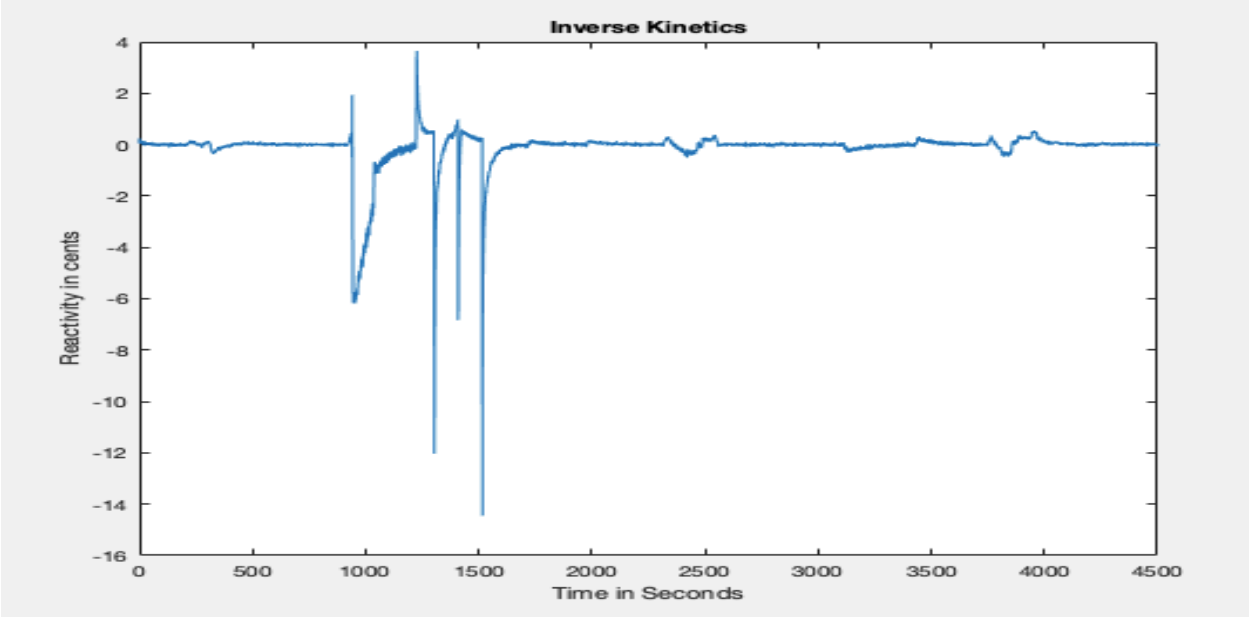

Figure 7: Inverse Kinetics of Channel 3 (experiment 2, linear)

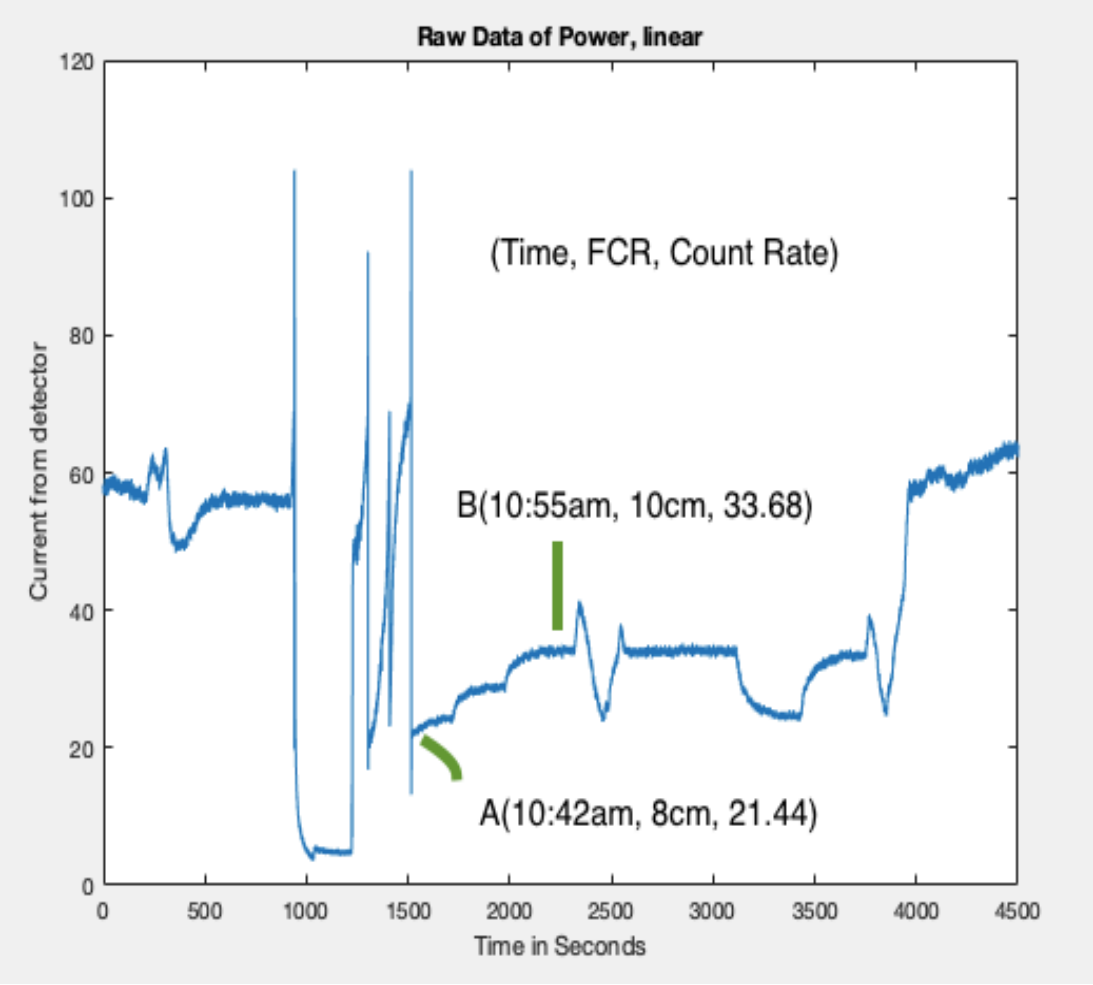

Figure 8: Source Analysis of Experiment 2 (linear)

From the figure 8 above, there are two points determined. The two points are point A (1526,21.34) and point B (2317,33.68). In these points, "x" axis represents the time with 20 milliseconds period , and "y" axis represents the count rate from the current detector. Throughout the thesis count rate is denoted as CR. Considering point A (1526,21.34), here 1526 time with 20 milliseconds period, thus, 1526 x 0.02 sec = 30 minutes (which is 11 am). Add this time (30 min) with the experiment start time (10:30 am), so time is now approximately 10:42 am for point A, and count rate is calculated from Y axis, hence, same process is applicable for the point B. From the experiment 2 description it can be said that at time 10:42 am Cf-252 source was in, and rod position was 8 cm. Therefore, from all of the information and S-curve, the Δk for the source can be calculated.

Consider following parameters for point A,

$CR_A = \frac{S}{1-K_A-\Delta k}$; CR_A is count rate or power n(t) of point A and K_A is reactivity of point A due to control rod position, and Δk is source reactivity.

So, $1-K_A-\Delta k = \frac{S}{CR_A}$ **Eq. 27**

For point D,

$CR_B = \frac{S}{1-K_B-\Delta k}$, $S = CR_B(1 - K_B - \Delta k)$

Replace S in Eq.27,

$(1 - K_A - \Delta k)\, CR_A = CR_B(1 - K_B - \Delta k)$

Therefore, $\Delta k = \frac{(1-K_A)CR_A - (1-K_B)\,CR_B}{CR_A - CR_B}$ **Eq. 28**

From S-curve, $1- K_A$ value can be determined based on the rod position. For point A, rod position is 8 cm, and reactivity excess for 8 cm position is 0.26 %. At point B, FCR was 10 cm and reactivity excess are 0.24%. When reactor was critical k=1, FCR position was 13.6 cm and reactivity excess was 0.188%. The negative reactivity of point A is -0.072%, similarly excess reactivity of point D is -0.052%. Therefore, K_A and K_B is 0.99928 and 0.99948. Substituting K_A and K_B values in Eq. 28,

Point	Rod Position	K value	Count Rate (CR)
A	8 cm	0.99928	21.34
B	10 cm	0.99948	33.68

Table 1: Source Reactivity Calculation for Experiment 2

$$\Delta k = \frac{[\{-(1-0.99928)\,(21.34)\} - \{-(1-0.99948)\,(33.68)\}]}{21.34 - 33.68}$$

$$= -\,2.34\ ¢.$$

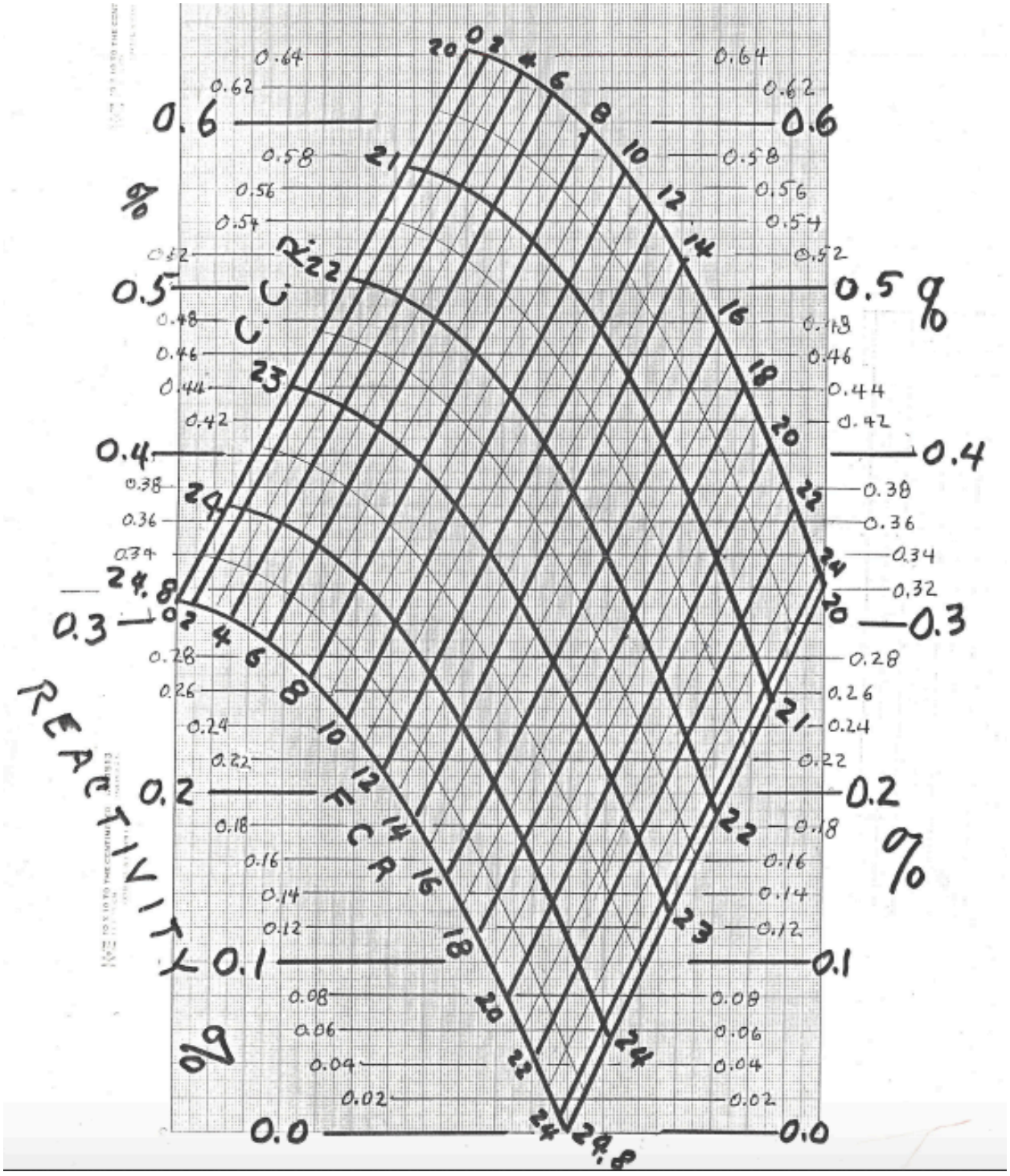

Figure 9: Reactivity Chart for FCR and CCR position

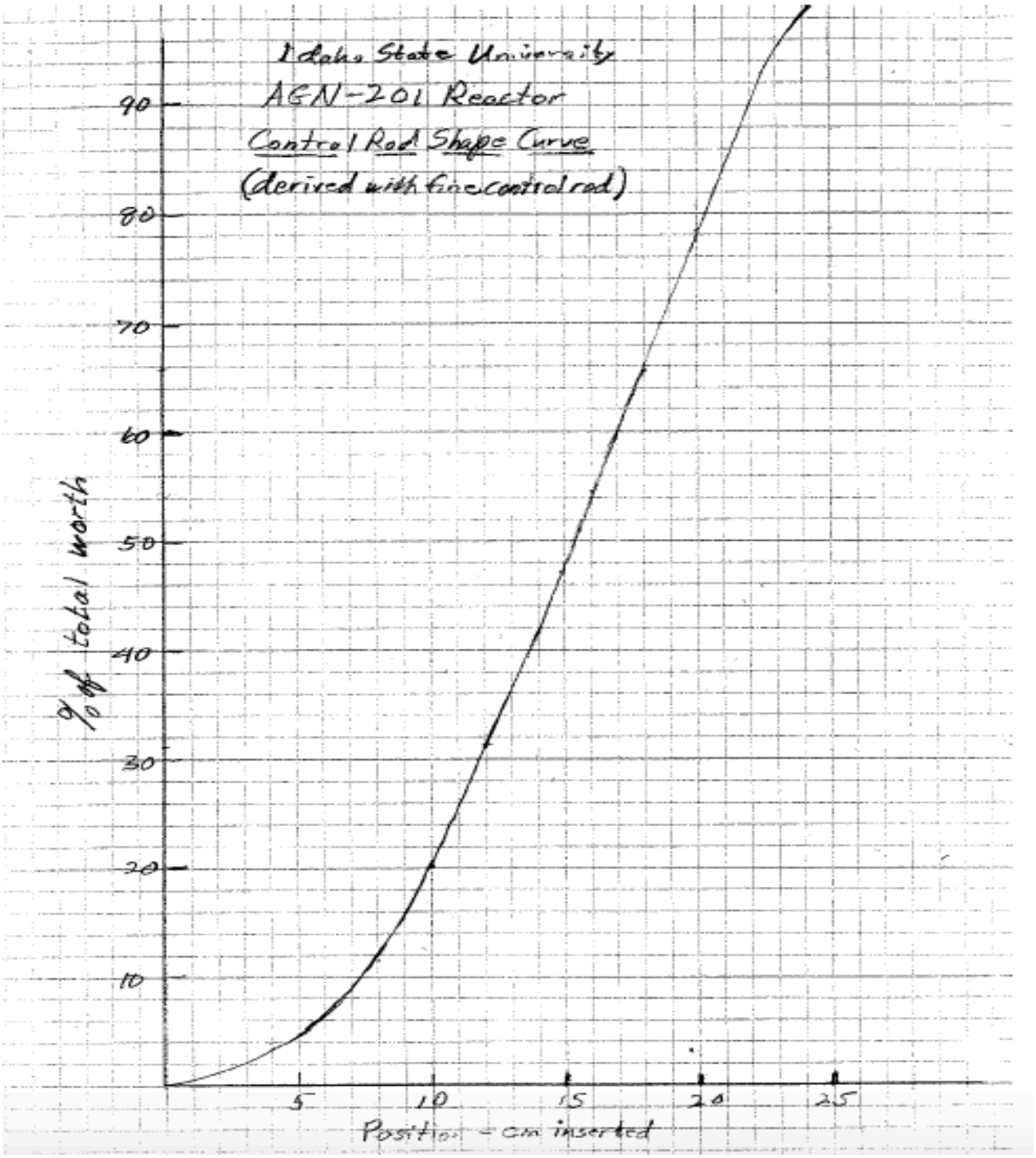

Figure 10: Reactivity Chart for FCR Position

4.3 Experiment 3:

Experiment 3 was conducted on April 25th, with the Radium-Beryllium (RaBe) source. Experiment 3 was similar to experiment 1. In this experiment, RaBe source was also inserted manually through the glory hole in the nuclear reactor. Total data contained 234,001 points time with 20 milliseconds resolution. The procedure for experiment 3 was,

1. At 9:50 am reactor was on.
2. At 9:57 am reactor became critical. FCR and CCR position was 13.8 cm and 24.7 cm, respectively.
3. At 10:05 am, source was re-inserted.
4. Time between 10:12 am and 10:16 am FCR position was 18.81 cm.
5. Time between 10:16 am and 10:18 am FCR position was changed and new position was 19.5 cm.
6. At time 10:18 am FCR position was changed again and new position was 22 cm.
7. Time between 10:22 am and 10:27 am new FCR position was 25.05 cm.
8. At time 10:27 am source was little much out (not entirely) from the core.
9. At 11:08 am reactor was scrammed.

Please note that for experiment 3 source analysis, only the time between 10:16am and 10:22am was analyzed to obtain source reactivity of RaBe source.

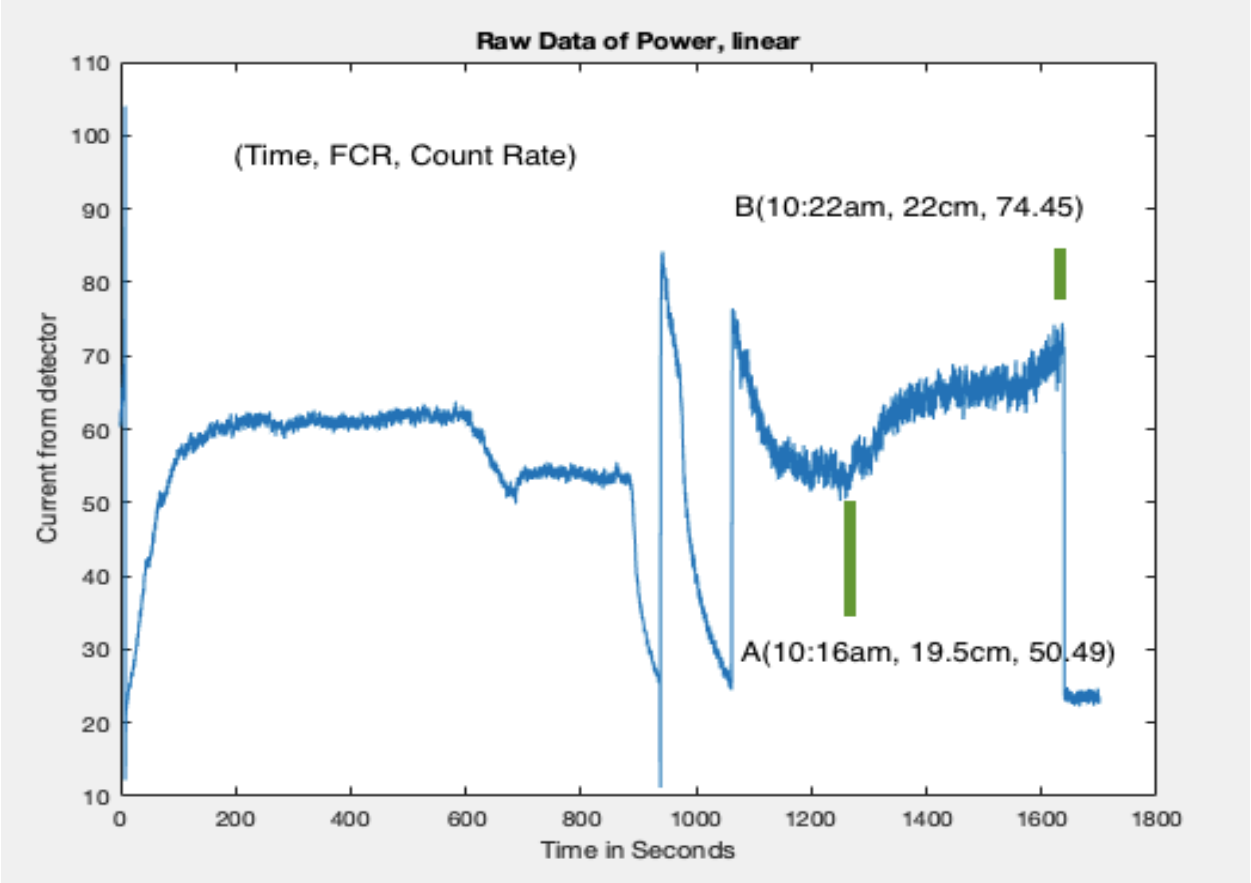

Figure 11: Source Analysis of Experiment 3 (channel 3, linear)

Point	Rod Position	K value	Count Rate (CR)
A	19.5 cm	0.9991	50.49
B	22 cm	0.9996	74.45

Table 2: Source Reactivity Calculation for Experiment 3

$$\Delta k = \frac{(1-0.9991)(50.49) - (1-0.9996)(74.45)}{50.49 - 74.45} = -8.7\ ¢$$

The raw data plot and inverse kinetics for channel 3 is given below,

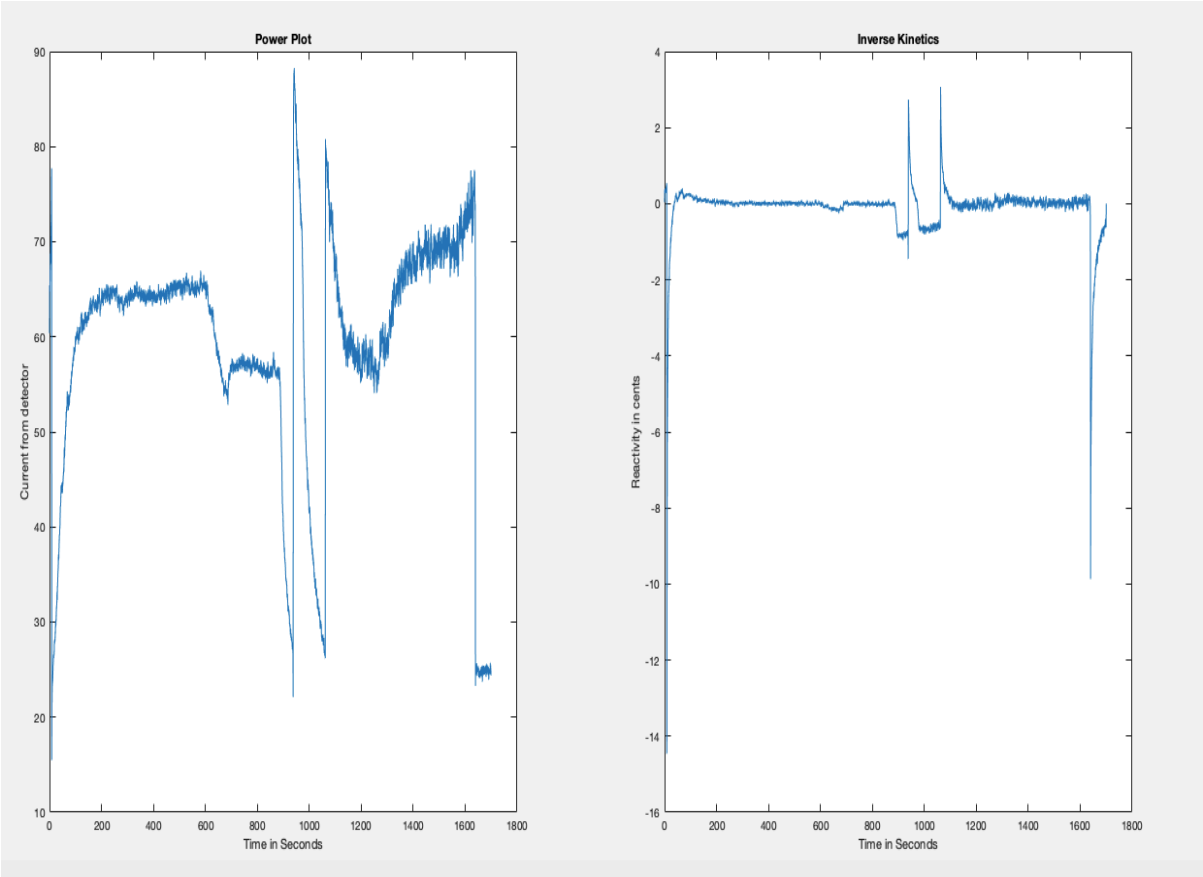

Figure 12: Raw Data Plot and Inverse Kinetics of experiment 3 (channel 3, linear)

4.4 Perturbation Analysis:

The perturbation equation is given below, [2]

$$\rho = \frac{\int_{v_p}[\delta\nu_p\Sigma_{f_p} - \delta\Sigma_{a_p}]\varphi_p^2 dv - \int_{v_p}\delta D(\nabla\varphi_p^2)dv}{\int_{v_r}\nu_r\Sigma_{f_r}\varphi_r^2\, dv}$$

Here, numerator is an integral over perturbation sample which is Cf-252. The subscript "p" stands for the perturbation, and "r" stands for the reactor. The core of the reactor is comprised of U-235 and polyethylene. Thus, the denominator is an integral of the reactor (U-235 and polyethylene). In addition, in the numerator ν_p represents the average number of neutrons per fission of Cf-252 and in the denominator part ν_r represents the average number of neutrons per fission of U-235, and v represents volume. The macroscopic cross section values of Cf-252 and U-235 was calculated and presented in appendix A. The Cf-252 source is encapsulated by stainless steel (304L), which contains approximately 18% Cr, 8 % Ni, and 74% Fe. Therefore, absorption cross section for stainless steel was also calculated and added with Cf-252 (perturbation) absorption cross-section. Since, the reactor core shape is cylindrical, the flux $\varphi(r,z)$ is

$$\varphi(r,z) = Aj_0(\frac{2.405r}{R})\cos(\frac{\pi z}{H})$$

It can be assumed that the diffusion coefficient is not perturbed by the source, hence, the term $\delta D(\nabla\varphi_p^2)$ is zero. In the center of the core $\varphi(0,0) = Aj_0(0) = A$ {since $j_0(0)=1$}

Flux for the reactor calculated by using Bessel function (Appendix B), $\varphi = 1.7893 \times 10^3\ cm^{-2}s^{-1}$

For simplification all values are listed below,

Macroscopic Cross Section (Fission and Absorption)	Calculated Value
Fission Cross-section of Cf-252 (Σ_{f_p})	6.85 x 10^{-6} cm^{-1}
Absorption Cross Section of Cf-252 (Σ_{a_p})	4.42 x 10^{-5} cm^{-1}
Fission Cross-section of U-235 (Σ_{f_r})	0.0753 cm^{-1}
Absorption Cross Section of Fe, (Σ_{Fe})	0.162 cm^{-1}
Absorption Cross Section of Cr (Σ_{Cr})	0.0496 cm^{-1}
Absorption Cross Section of Ni (Σ_{Ni})	0.0287 cm^{-1}
Stainless Steel Absorption Cross Section ($\Sigma_{Fe}+\Sigma_{Cr}+\Sigma_{Ni}$)	0.2403 cm^{-1}

Table 3: Macroscopic Cross Section of Elements

Also, ν_p and ν_r are 3.768 and 2.4, respectively.

Therefore, after simplifying perturbation equation and adding stainless steel absorption cross section with Cf-252 absorption cross section,

$$\rho = \frac{[(3.768 \times 6.85 \times 10^{-6})] - [(4.42 \times 10^{-5}) + (0.2403)]}{2.4 \times 0.0735 \times 1.789 \times 10^{3}}$$

$$= -9.92\ ¢$$

Note that we are very likely overestimating the amount of stainless steel in the source, but this calculation does give an idea of the order of magnitude of the effect.

4.5 Experiment 4:

Experiment Number 4 was conducted on April 30th, 2019. This analysis is based on channel 3 of the reactor console. The total experiment time was from 9:8:0:0 to 10:49:48:500. This time format is based on "Hour:Minute:Second:Millisecond". Data were analyzed by Mathematica and MATLAB® and code is provided in appendix B. The linear channel plot is provided below and divided into two region, region 1 and region 2, respectively. Region 1 was divided further into four different sub regions, sub region 1 (SR1), sub region 2 (SR2), sub region 3 (SR3), and sub region 4 (SR4). During this experiment in each region the frequencies were varied.

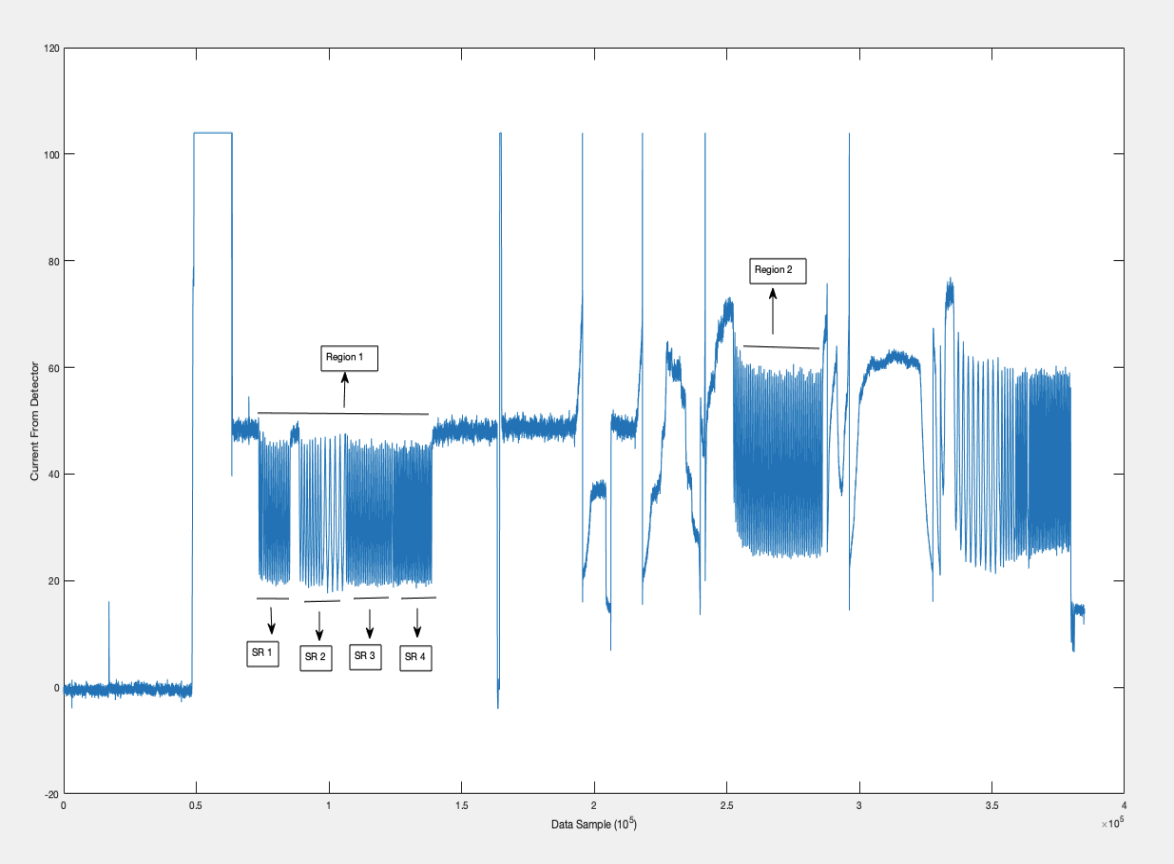

Figure 13: Raw Data of Experiment 4 (linear channel)

To smooth the data, a 19-point moving average was performed and figure is shown below,

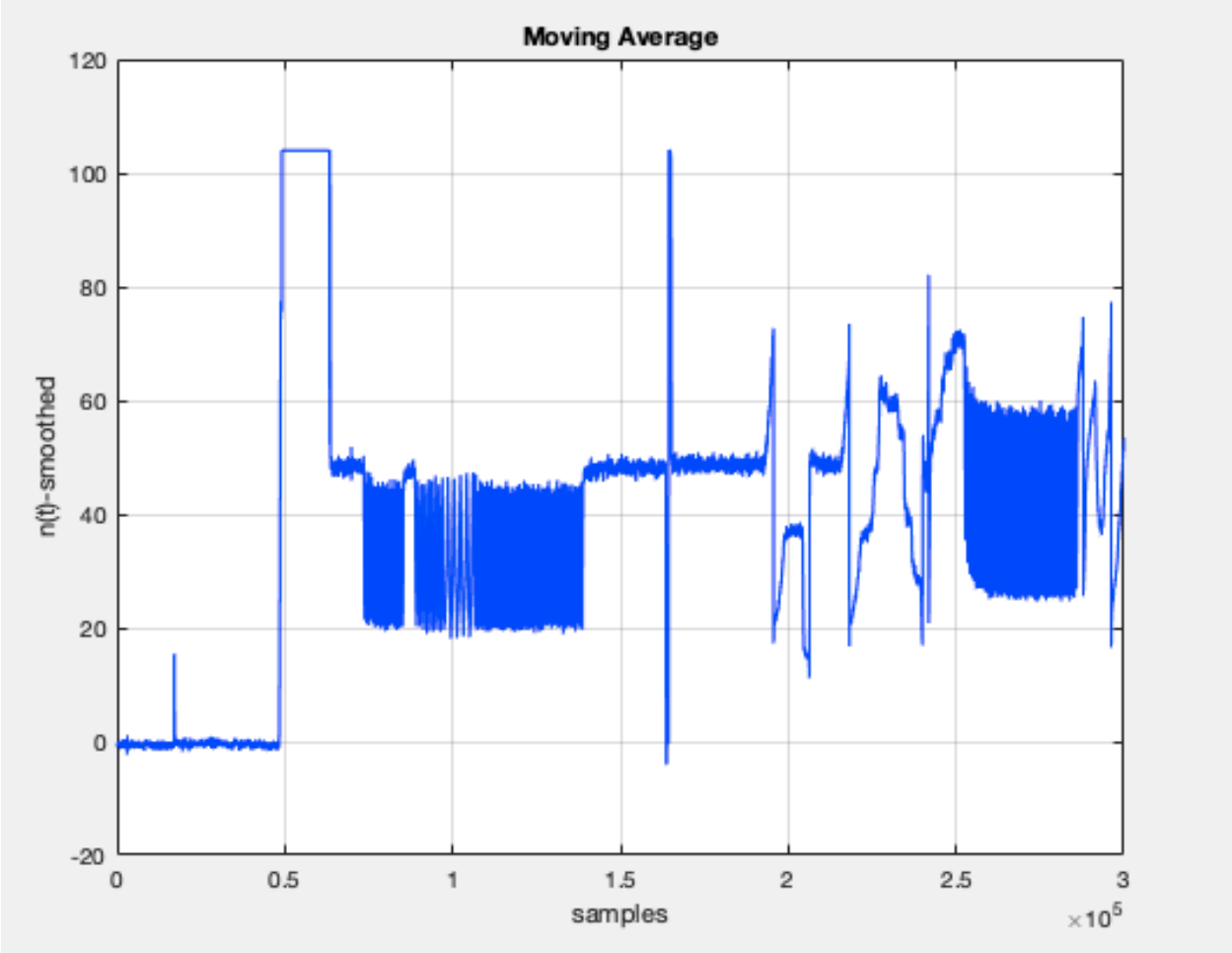

Figure 14: Moving Average of Experiment 4 (linear channel)

Below is the region 1 data plot and power spectral density (PSD) of the region 1. In all of these sub region PSD graphs, the X and Y axes represents the frequency in hertz and power in arbitrary units. Region 1 data are between 80,000 and 120,000 samples, and region 2 data are between 278,000 and 286,000 samples. The main objective of Fourier analysis is to obtain verify the stability of the oscillator with respect to frequency, that is frequency analysis of the oscillation shows the reproducibility of the source oscillation. The goal is to obtain a clean and single frequency oscillation without much noise.

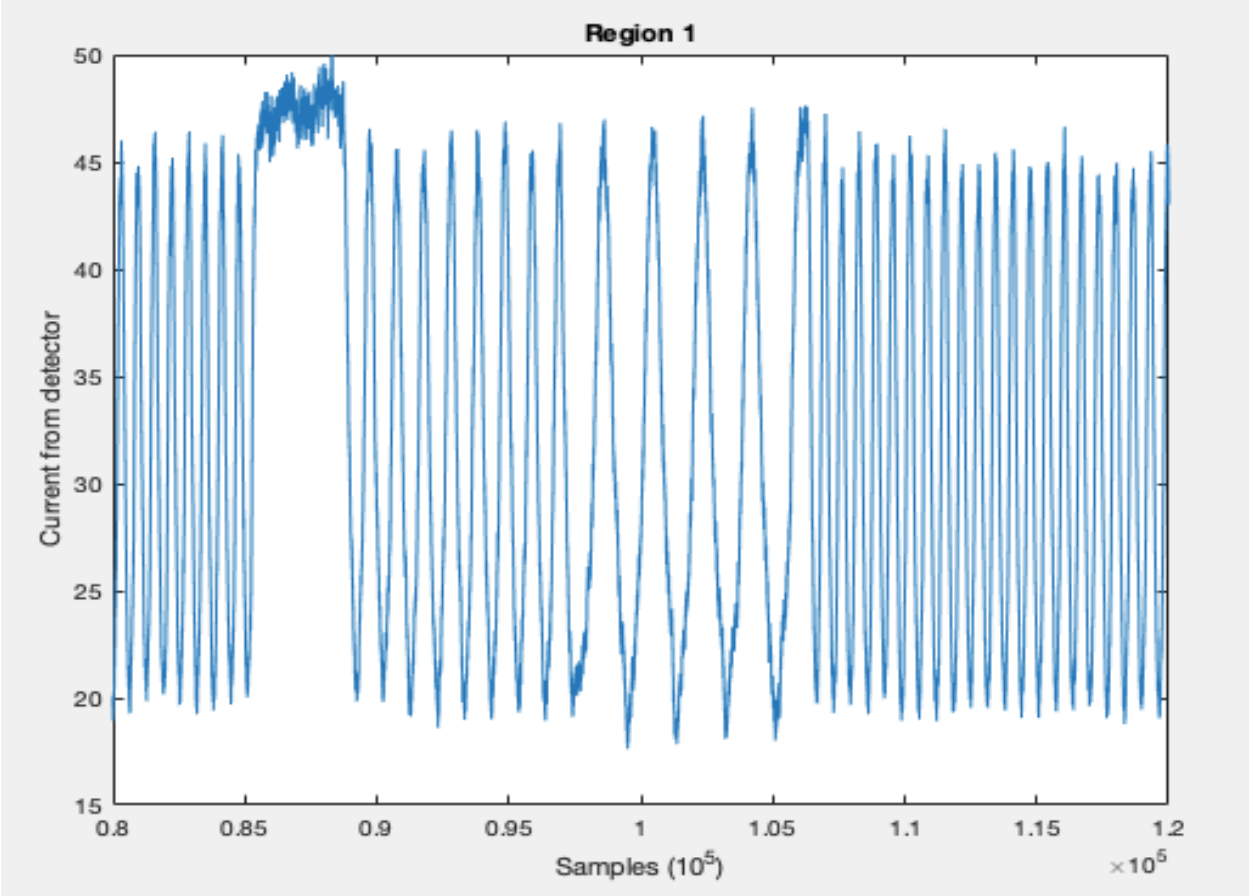

Figure 15: Raw Data Plot of Region 1 of Experiment 4 (linear)

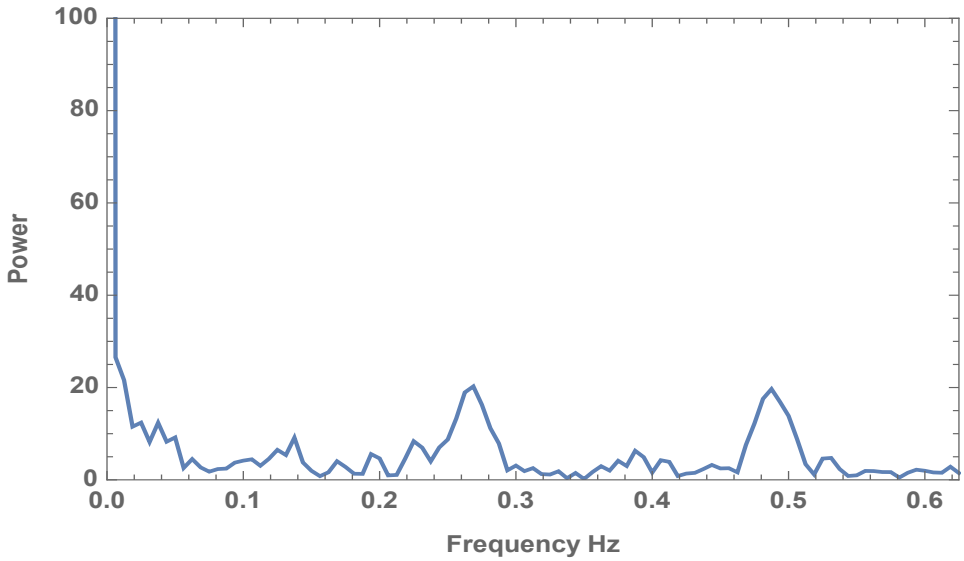

Figure 16: Power Spectral Density of Region 1

Power unit here is an arbitrary unit. Peak is at 0.27 Hz and frequency is divided by 100/160.

Most importantly, region 1 is divided into another four sub regions. From the raw data plot, it can be seen that sub region 1 data are between 90,000 and 98,000 ; sub region 2 data are between 98,000 and 106,000; sub region 3 data are between 108,000 and 116,000 ; sub region 4 data are between 130,000 and 138,000.

Sub region 1 (90,000 to 98,000) plot is given below,

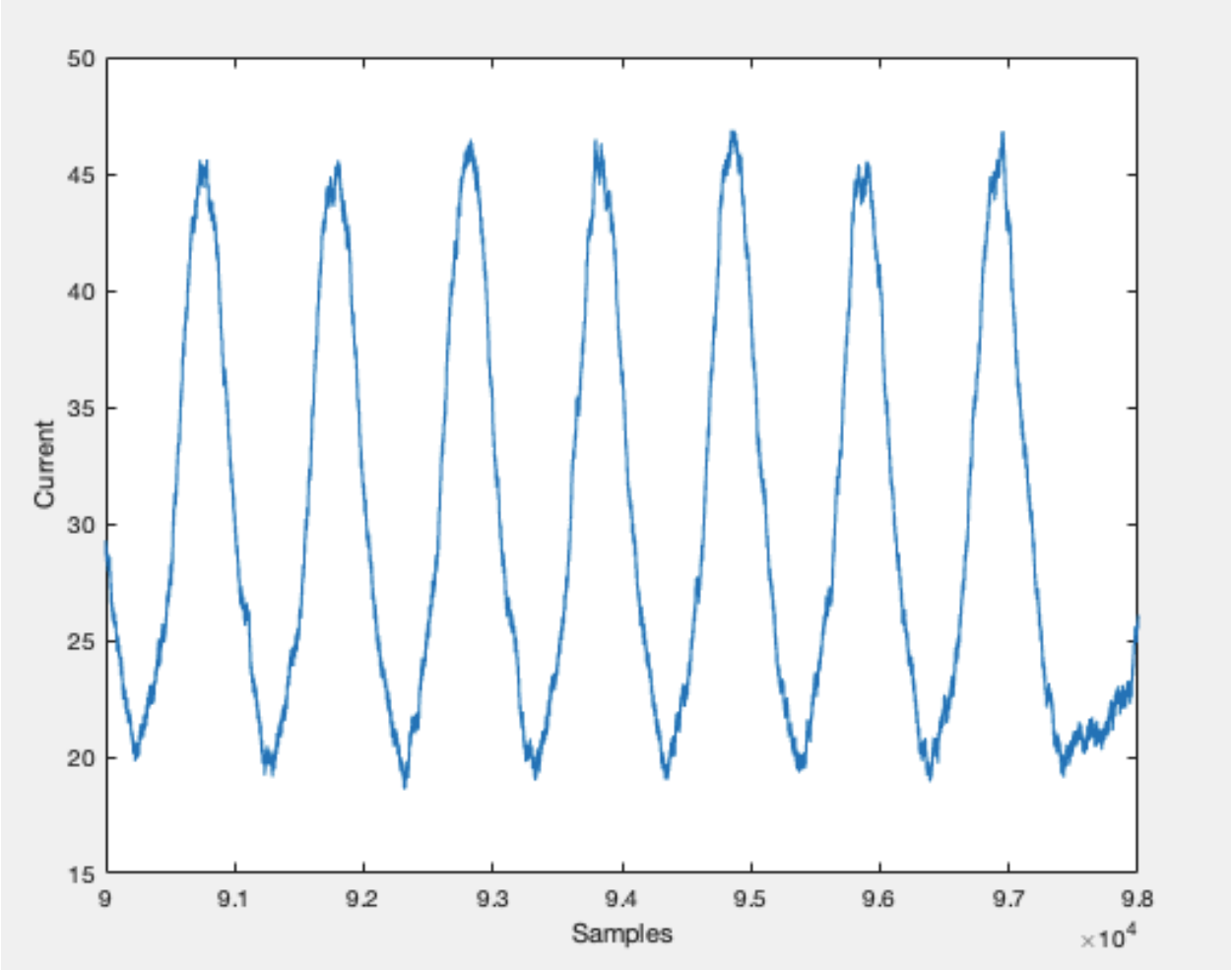

Figure 17: Raw Data of Sub Region 1 (experiment 4, linear)

From the plot it is noted that total points are 8000 and sample period is 20 milliseconds, and 8000 points represents 160 seconds.

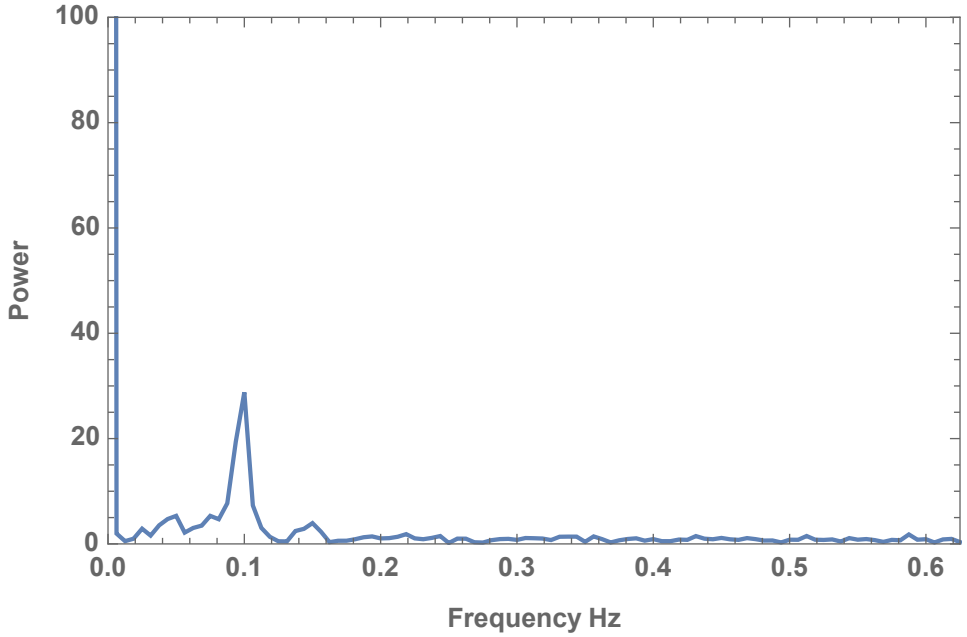

Figure 18: Power Spectral Density of Sub Region 1

Peak is at 0.1 Hz.

Sub region 2 (98,000 to 106,000) plot is given below,

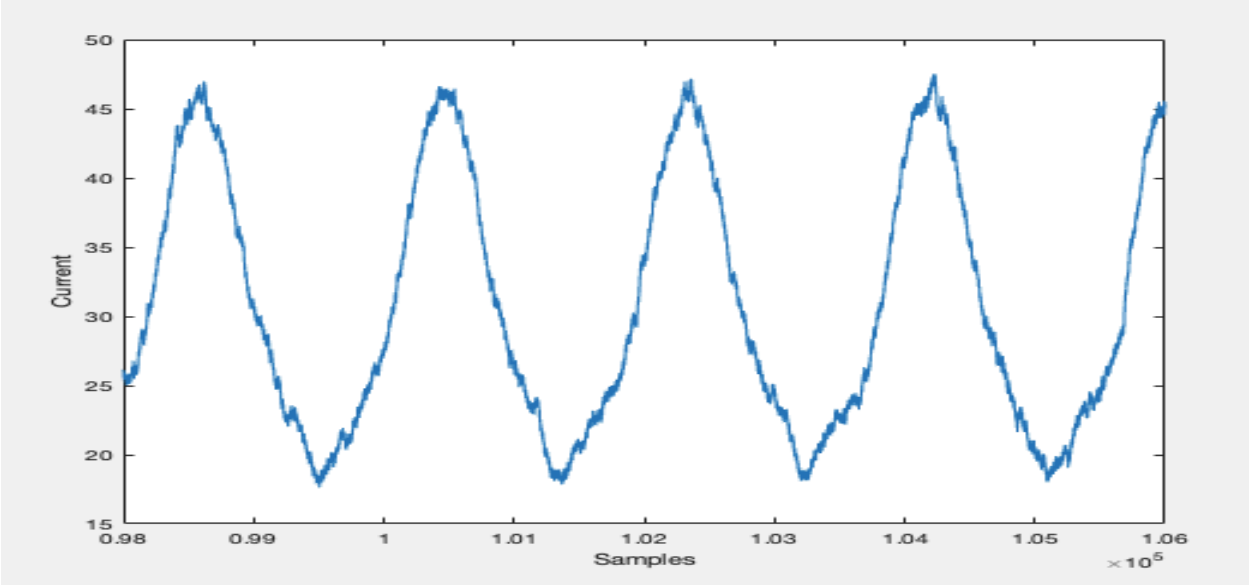

Figure 19: Raw Data Graph

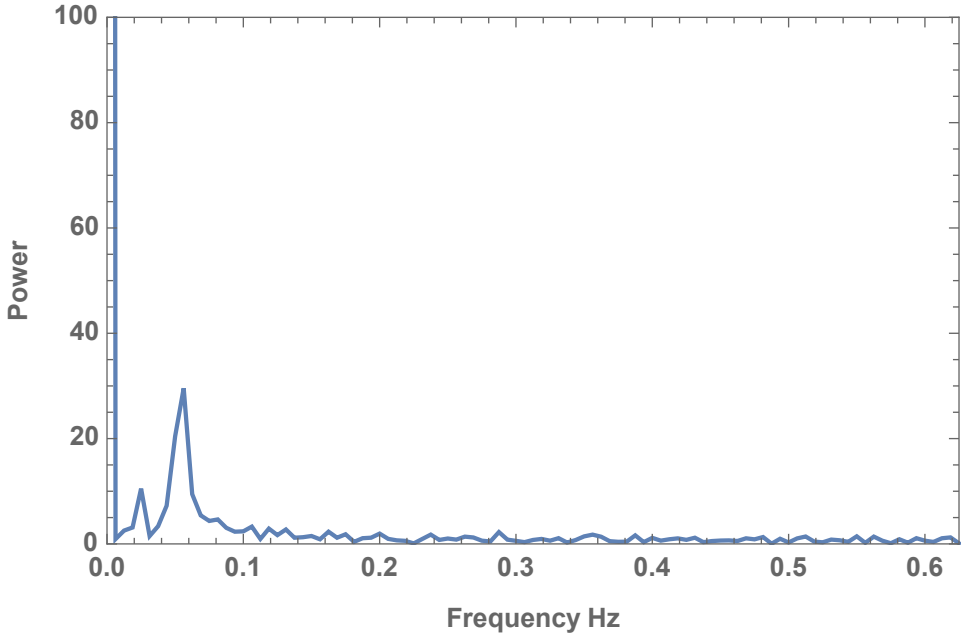

Figure 20: Power Spectral Density of Sub Region 2

Peak is at 0.05 Hz.

Sub region 3 (108,000 to 116,000) plot is given below,

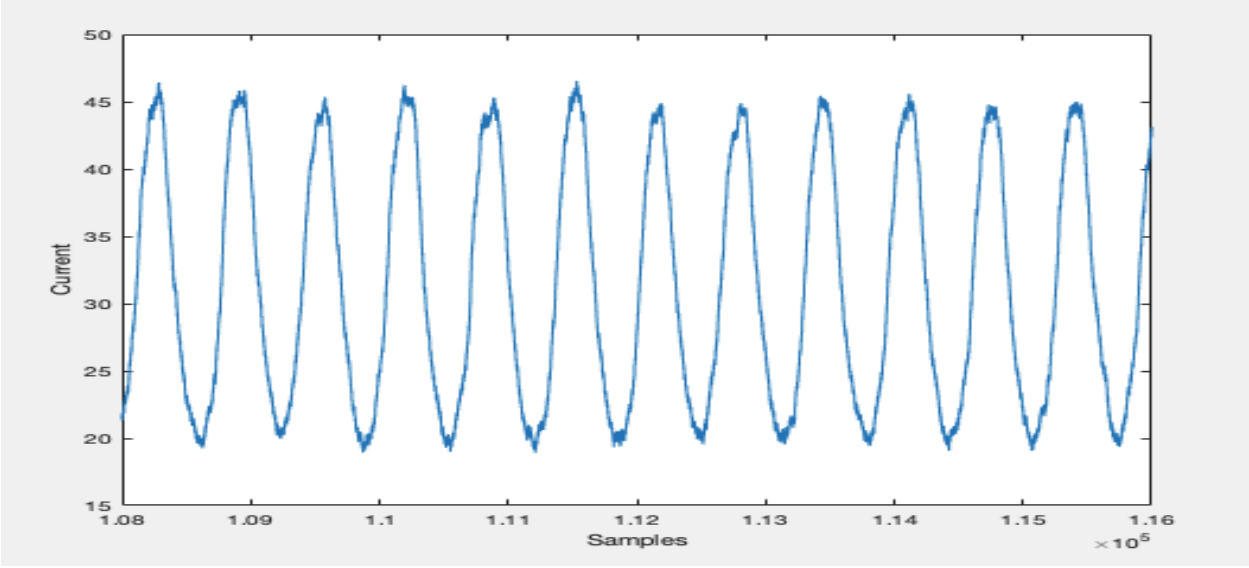

Figure 21: Raw Data of Sub Region 3 (experiment 4, linear)

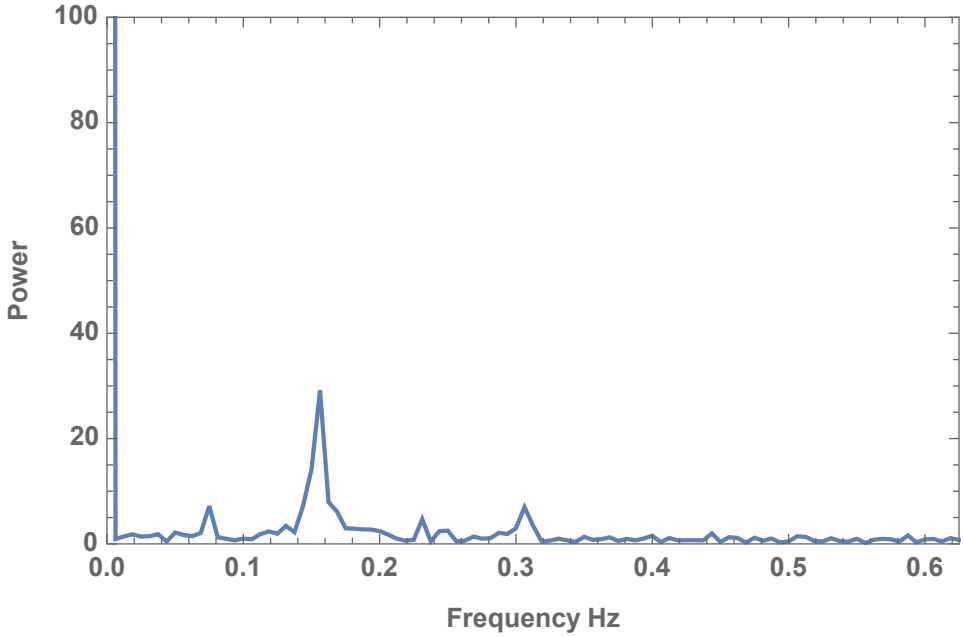

Figure 22: Power Spectral Density of Sub Region 3

Peak is at 0.17 Hz.

Sub region 4 (130,000 to 138,000) plot is given below,

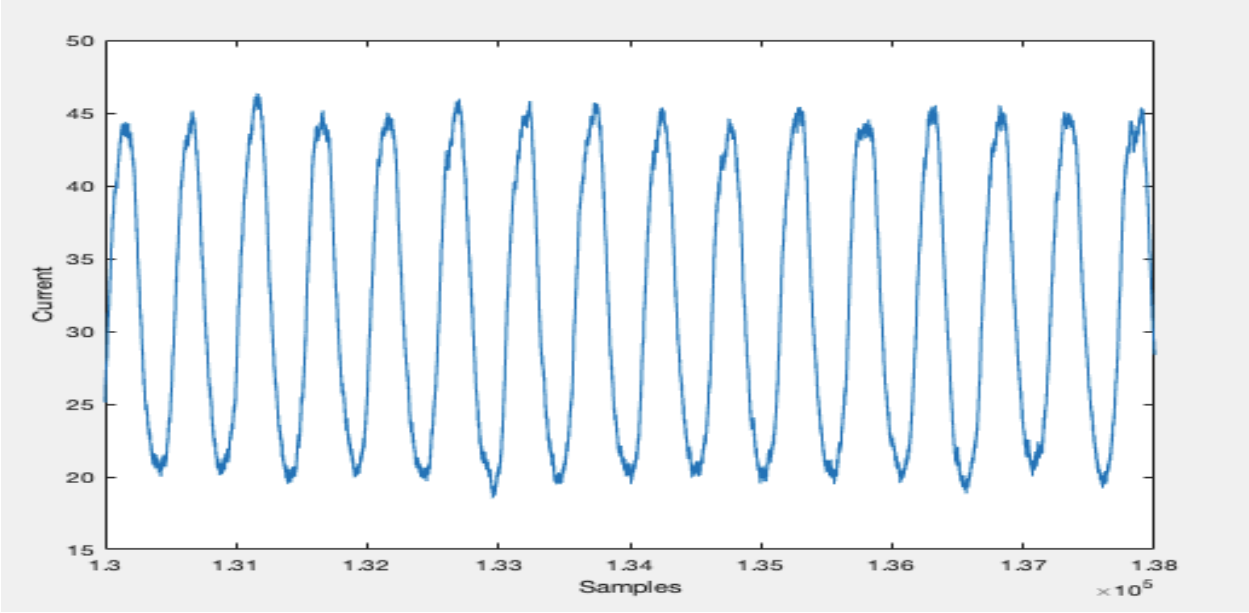

Figure 23: Raw Data of Sub Region 4 (experiment 4, linear)

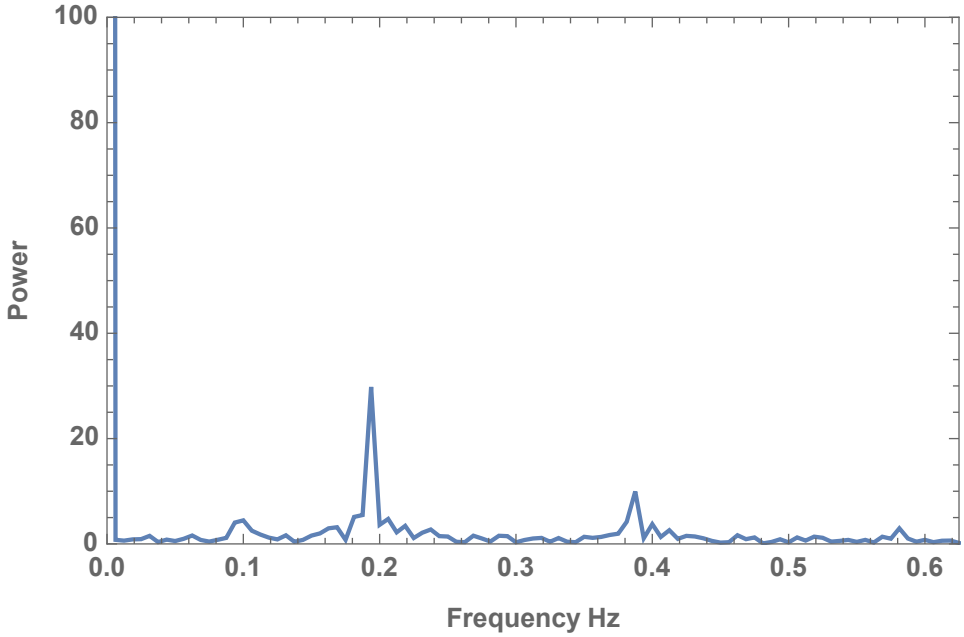

Figure 24: Power Spectral Density of Sub Region 4

Peak at 0.2 Hz.

In the region 2, the reactor was closer to critical with the k approximately 0.980. Region 2 raw data (278,000 to 286,000) graph is provided below,

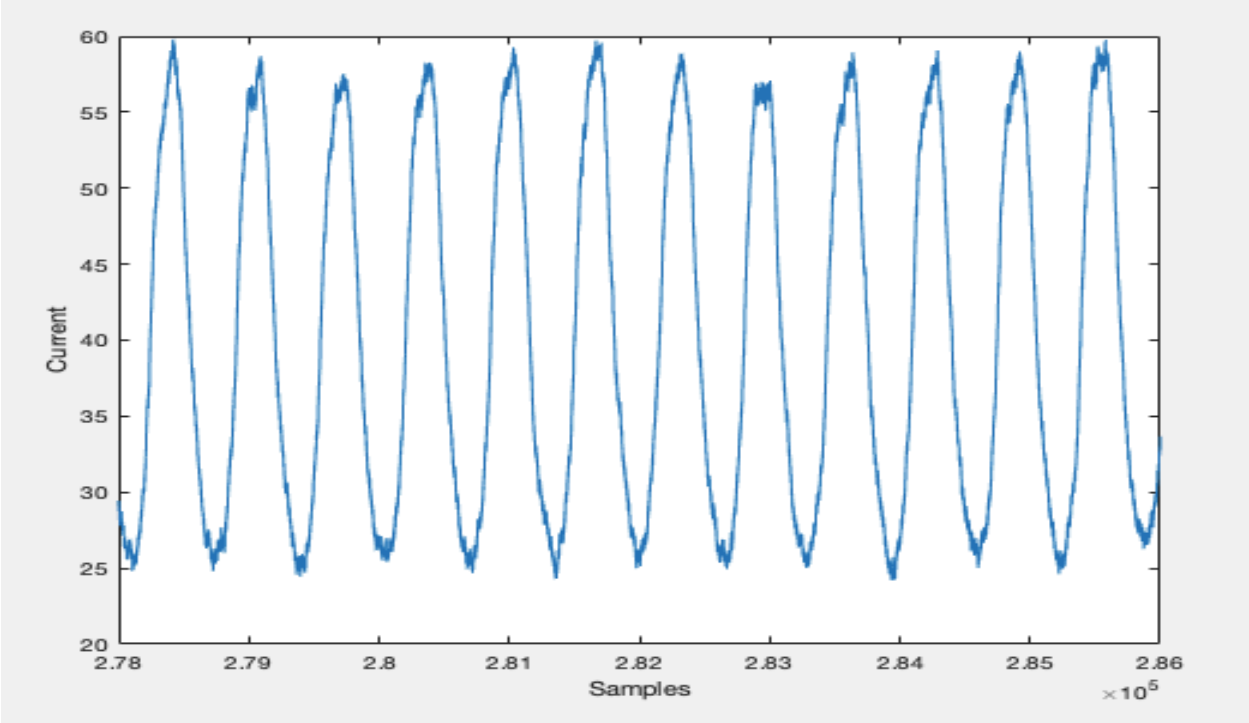

Figure 25: Raw Data of Region 2 (experiment 4, linear)

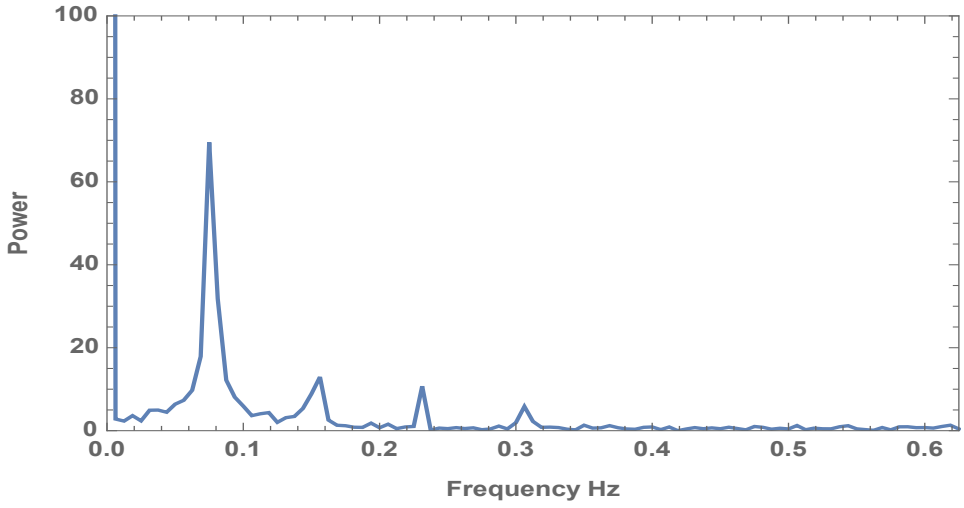

Figure 26: Power Spectral Density of Region 2 (experiment 4, linear)

Peak is at 0.07

The Fourier transform of region 1 and 2 is given in the above figure and it shows the five peaks and some harmonics. In addition, from the inverse kinetics of all sub region and region 2, it can be concluded that the reactivity worth for all of these oscillations were within the same range.

4.6 Inverse Kinetics of Sub Region:

The inverse kinetics was determined from the samples of the sub region. Below inverse kinetics of sub region 2 and 4 is given.

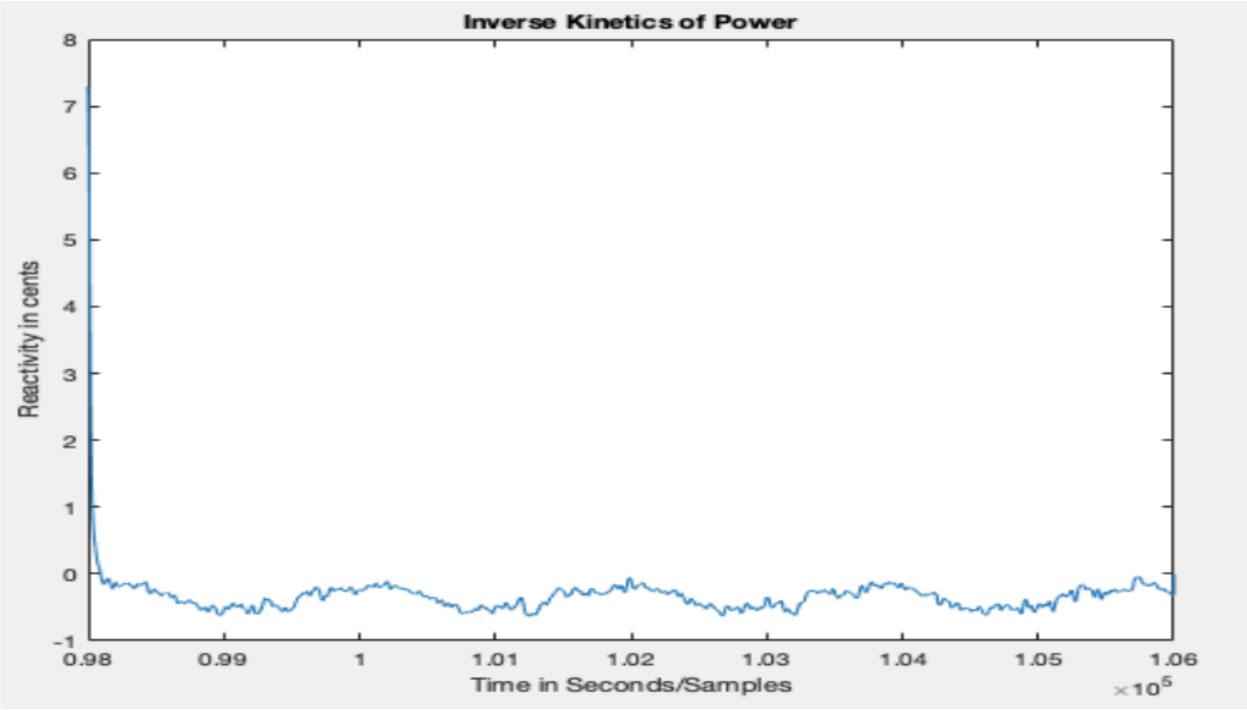

Figure 27: Inverse Kinetics of Sub Region 2

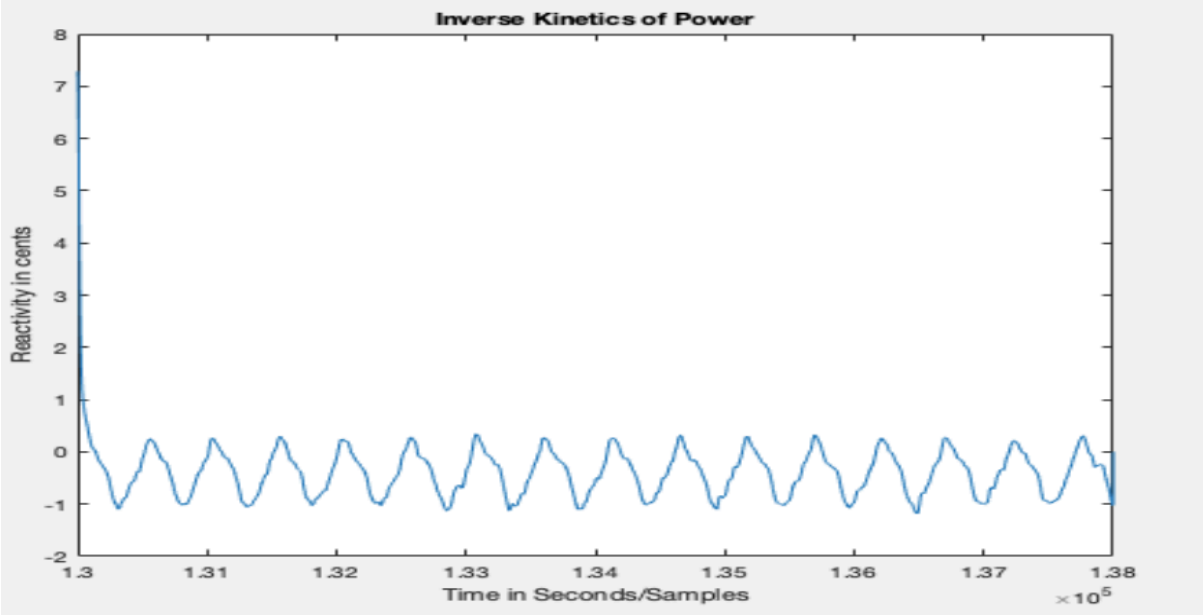

Figure 28: Inverse Kinetics of Sub Region 4

From the inverse kinetics of the sub region 2 and 4 it can be seen that the better signals come from the higher frequency. it can be noted that the swings in reactivity are around 1-2 cents, which is consistent with the analysis in Section 4.2.

4.7 Transfer Function Analysis:

From equation 25 and 26, the transfer function for source and reactivity is given below,

$$\frac{\delta n(s)}{\delta q(s)} = \frac{l}{ls + \beta - \rho_0 - \Sigma_i \frac{\lambda_i \beta_i}{s + \lambda_i}}$$

$$\frac{\delta n(s)}{\delta \rho(s)} = \frac{n_0}{ls + \beta - \rho_0 - \Sigma_i \frac{\lambda_i \beta_i}{s + \lambda_i}}$$

The values of l, β, λ_i, and β_i are obtained from the reactor parameters for AGN 201 reactor, and are given below,

Group	1	2	3	4	5	6
$\lambda' s$	0.0124	0.0305	0.1114	0.3014	1.137	3.014
$\beta' s$	0.00028	0.00158	0.00140	0.00303	0.00095	0.00019

Table 4: Reactor Parameters for AGN-201

β_{eff} is 0.00745 and Λ = 62.2 µsec.

The difference between source and reactivity of the transfer function is only gain. In the reactivity transfer function, the initial power or n_0 is an arbitrary value. For this thesis, n_0 value is considered 2 watts. The reactivity value is assumed to be negative, since k_{eff} is 0.968 with all control rods out. Fission generation time (l) is 62. µs. The response to step input of both source and reactivity are shown and all of the poles are in the left half of the S-plane when the reactor is subcritical.

Source Transfer function analysis is provided below,

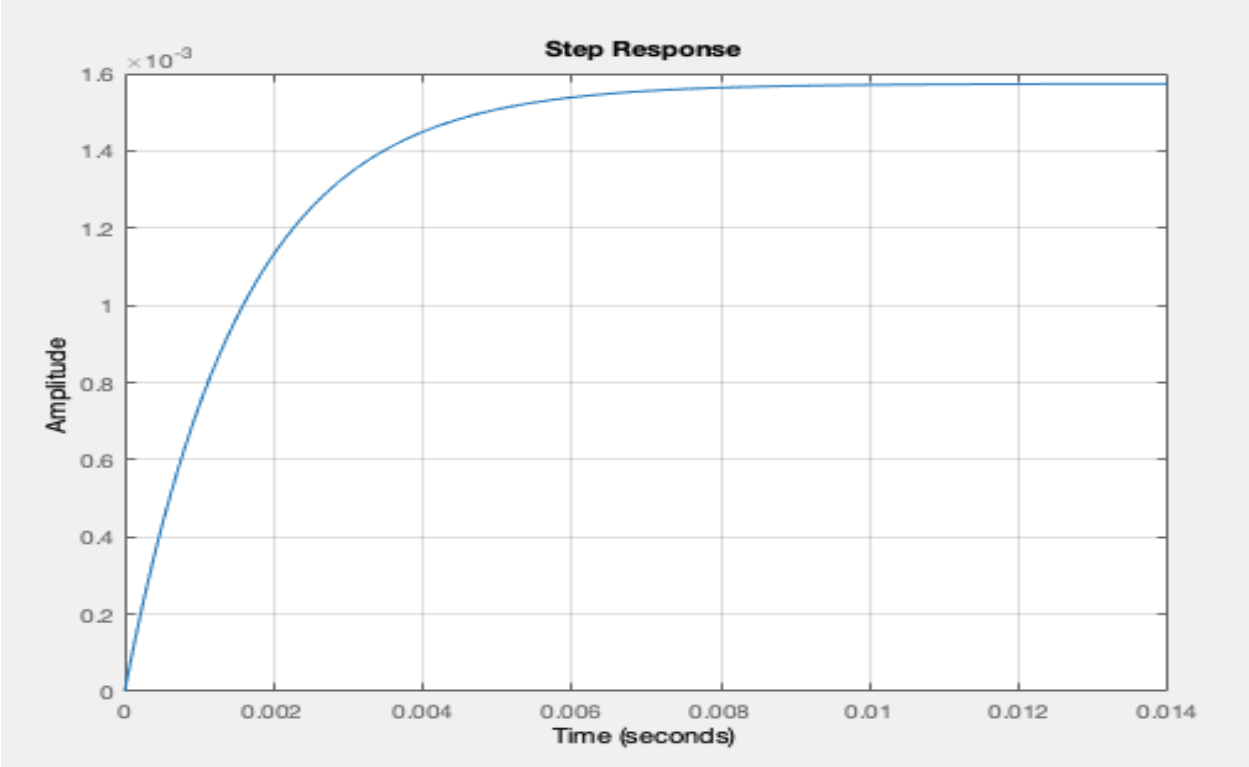

Figure 29: Step Response of Source Transfer Function

From the response to a step source it can be seen that there is virtually no lag to the response (output or measured signal) of the reactor. Therefore, the measured oscillation which are relatively slow is a good representation of the source oscillation.

Reactivity transfer function analysis is provided below,

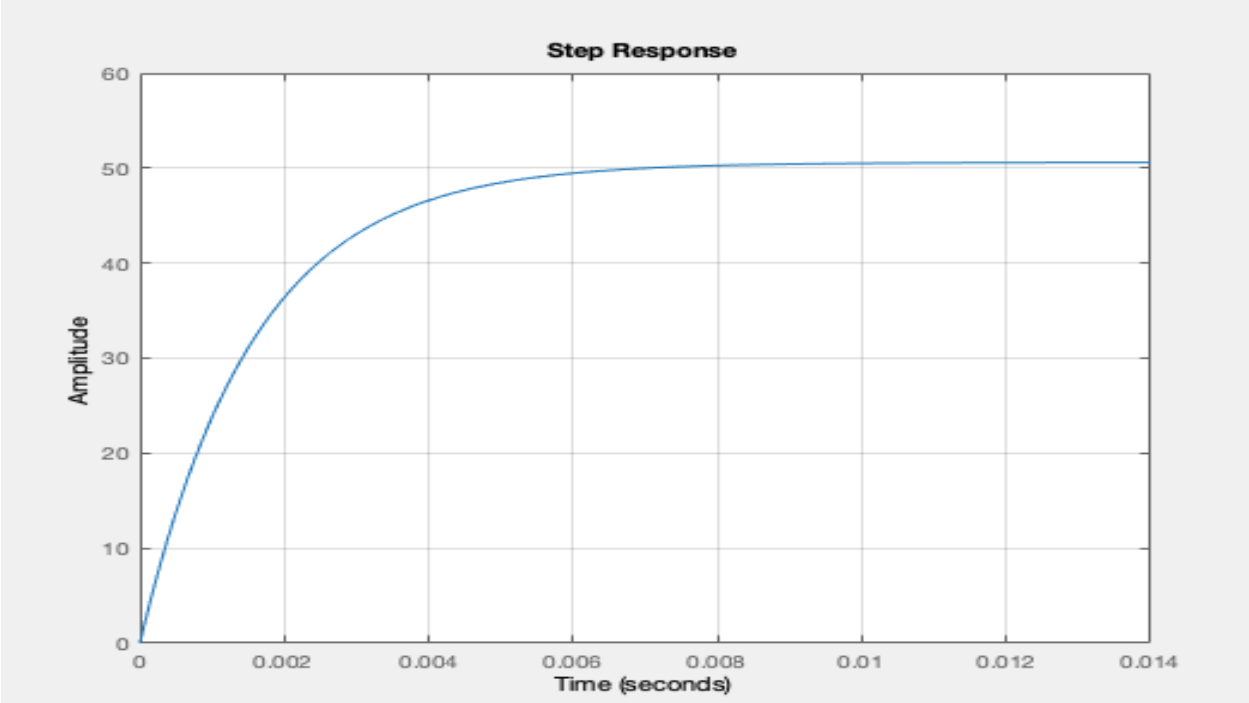

Figure 30: Step Response of Reactivity Transfer Function

Therefore, same conclusion can be drawn from the source step, which is expected since the poles of the transfer functions are same. Only difference between source and the reactivity transfer function is gain.

Chapter 5: Conclusions and Future Work

The purpose of this work was to determine the characteristics of a Californium-252 (Cf-252) in the AGN-201 reactor. A series of four experiments was performed to show the possible source contribution as an independent source, as a parasitic absorber or as fissile material.

Experiment #1 was a testing platform to see the reactor response and set the optimal parameters for the rest of the experiments. Experiment #2 consisted of manually oscillating the Cf-252 source into the reactor core and recording the changes in power and other parameters for posterior calculations. Experiment #3 was done in a same way as experiment two but with a Radium-Beryllium (Ra-Be) source to compare its effects to those of the Cf-252 source. Experiment#4 consisted of oscillating the Cf-252 source in and out of the core by means of a fully automated linear oscillator at different frequencies and recording the data and parameters for its examination. Experiments showed that there is a negative reactivity contribution in the reactor from the Cf-252 source of about -2.34 cents and that there is a greater contribution from the Ra-Be source, of about -8.7 cents. It was also determined that the contribution of the Cf-252 source as fissile material is negligible. From the perturbation theory it was shown that the excess reactivity which appears to be an overestimate (probably due to the overestimate of stainless-steel content in the source) and its encapsulated stainless steel is of about -9.92 cents. From the oscillations of the source into the core at different frequencies it was determined that the reactor was subcritical at all times while conducting experiment #4.

Future improvement is always possible. There are some improvements that can be done in the future. The LabVIEW software can be used to determine oscillation position more accurately as well as the neutron detected current can be recorded with LabVIEW and a picometer. In addition, Sub Critical Assembly or SCA can be used for this experiment, to explore the sub critical region even further. Being able to electronically input the control rod position indicator data into the inverse kinetics program would be desirable.

References

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[2] Lamarsh, John R. *Introduction to Nuclear Reactor Theory*., Addison-Wesley Publishing Company, 1966.

[3] Lamarsh, John R. Baratta, Anthony J. *Introduction to Nuclear Engineering*., 3rd Ed, Prentice Hall Inc., 2001

[4] James J. Duderstadt, Louis J. Hamilton. *Nuclear Reactor Analysis*., John Wiley & Sons, inc., 1976.

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[6] Baker, Benjamin Allen. *Comparison of Open Loop and Closed Loop Reactivity Measurement Techniques on The ISU AGN-201 Reactor*., Pocatello, Idaho : Idaho State University, 2013.

[7] Baker, Benjamin Allen. *Reactor Parameters for the ISU AGN-201 Reactor.*, Pocatello, Idaho : Idaho State University, 2011.

Appendix A

Californium-252 neutron source calculation:

Half-life of Cf-252 is, $t_{1/2}$ = 2.645 years.

Dated activity in the experiment Cf-252 is 112.3 millicurie or 4.155 GBq

1Bq = 1 decay/sec and 1Ci = 3.7010 x 10^{10} Bq

Mass of Cf-252 is given = 209.4 μg = 209.4 x 10^{-6} g

Total emission rate of Cf-252= 2.314 x 10^{12} $ns^{-1}g^{-1}$

Therefore, for 209.4 μg, total emission rate is, 209.4 x 10^{-6} g x 2.314 x 10^{12} $ns^{-1}g^{-1}$

$$= 4.845 \times 10^{8}\ ns^{-1}$$

For the Cf-252 source the data is measured in 1993. Hence, from 1993 to 2019, the time is 26 years. Therefore, $\lambda = \ln 2 / t_{1/2}$ and $N=N_0 e^{-\lambda t} = (4.845 \times 10^{8}\ n\ s^{-1}) x (e^{-\ln 2 \times 26\ years/2.645 years})$

N= 5.342 x 10^{5} n/s

Therefore, Cf-252 can yield 5.342 x 10^{5} neutrons/sec

Radium-Beryllium (RaBe):

In addition, from the ISU AGN-201 reactor specification it is known that Radium-Beryllium (RaBe) can yield 10^{5} neutrons/sec.

Cross-Section Calculation for Cf-252:

Volume of Cf-252, $V_p = \pi r^2 h = \pi(0.635cm)^2(1.905cm) = 2.412\ cm^3$, [subscript "p'' is for perturbed part or number for the perturbation analysis calculation]

Mass of Cf-252 in our experiment = 209.4 x 10^{-6} g

Thus, ρ= (209.4 x 10^{-6} g)/ 2.412 cm^3 = 8.68 x 10^{-5} g/cm^3

Therefore, $N = (\rho N_A)/M = [(8.68 \times 10^{-5}\ g/cm^3)(6.023*10^{23}\ atoms/mol)]/[252.08162\ g/mol]$

N= 2.074 x 10^{17} $atoms/cm^3$ = 2.074 x 10^{-7} $\frac{atoms}{b-cm}$ Since, 1 barn (b)=10^{-24} cm^2

Microscopic fission cross section of Cf-252 is, σ_f = 33.03 b

Microscopic absorption cross section of Cf-252 is, σ_a = 53.75 b

Microscopic total cross section of Cf-252 is, σ_T = microscopic absorption cross section (σ_a) + microscopic scattering cross section (σ_s) = 64.81 b

Again, microscopic scattering cross section (σ_s)= microscopic elastic scattering cross section (σ_{se}) + microscopic inelastic scattering cross section (σ_{si})

Microscopic inelastic scattering cross section (σ_{si}) for Cf-252 is zero.

Hence, microscopic scattering cross section for Cf-252 is, σ_s = 11.066 b

Therefore, macroscopic absorption cross section of Cf-252 is, $\sum_a = N\,\sigma_a = 1.11 \times 10^{-5}$ cm^{-1}

And, macroscopic fission cross section of Cf-252 is, $\sum_f = N\,\sigma_f = 6.85 \times 10^{-6}$ cm^{-1}

Cross-Section Calculation for U-235:

Mass of U-235 in the reactor = 672.9 g

Volume of the reactor, $V_r = \pi r^2 h = \pi(12.8\text{cm})^2(25.8\text{cm}) = 13{,}273$ cm^3

Thus, ρ= (665 g)/ 13,272 cm^3 = 0.0507 g/cm^3

$N = (\rho N_A)/M = [(0.0507\ \text{g/cm}^3)(6.023 \times 10^{23}\ \text{atoms/mol})]/[(235.0439\ \text{g/mol})]$

$N = 1.29 \times 10^{20}$ atoms/cm^3 = $1.29 \times 10^{-4}\ \frac{atoms}{b-cm}$

Microscopic fission cross section of U-235 is, σ_f = 587 b

Microscopic absorption cross section of U-235 is, σ_a = 687 b

Macroscopic fission cross section of U-235 is, $\sum_f = N\,\sigma_f = 0.0757$ cm^{-1}

Macroscopic absorption cross section of U-235 is, $\sum_a = N\,\sigma_a = 0.0886$ cm^{-1}

Cross-Section Calculation for Radium-Beryllium (RaBe):

Mass of Ra in the source = 10 mg

Volume of the Ra-Be source = $\pi r^2 h$ = 2.4 cm^3

Thus, ρ= (10 mg)/ 2.4 cm^3 = 0.00416 g/cm^3

N_{Ra}= (ρN_A)/M = [(0.00416 g/cm^3)(6.023 x 10^{23} atoms/mol)]/[(226.0254 g/mol)]

N_{Ra}= 1.108 x 10^{19} $atoms/cm^3$ = 1.108 x 10^{-5} $\frac{atoms}{b-cm}$

Microscopic fission cross section of Ra-226 is, σ_f = 7.003 μb

Microscopic absorption cross section of Ra-226 is, σ_a = 12.79 b

Macroscopic fission cross section of Ra-226 is, $\sum_f$ = N σ_f = 7.75 x 10^{-11} cm^{-1}

Macroscopic absorption cross section of Ra-226 is, $\sum_a$ = N σ_a = 1.41 x 10^{-4} cm^{-1}

Mass of Be in the source = 10 mg

Volume of the Ra-Be source = $\pi r^2 h$ = 2.4 cm^3

Thus, ρ= (10 mg)/ 2.4 cm^3 = 0.00416 g/cm^3

N_{Be}= (ρN_A)/M = [(0.00416 g/cm^3)(6.023 x 10^{23} atoms/mol)]/[(9.0121 g/mol)]

N_{Be}= 2.78 x 10^{20} $atoms/cm^3$ = 2.78 x 10^{-4} $\frac{atoms}{b-cm}$

Microscopic fission cross section of Be is, σ_f = 0, Hence, $\sum_f$= 0

Microscopic absorption cross section of Be is, σ_a = 0.0092 b

Macroscopic absorption cross section of Be is, $\sum_a$ = N σ_a = 2.5 x 10^{-6} cm^{-1}

Therefore, absorption cross section for Radium-Beryllium (RaBe) is 0.0001435 cm^{-1}

Fission cross section for Radium-Beryllium (RaBe) is $\sum_f$ = 7.75 x 10^{-11} cm^{-1}

Appendix B

Inverse Kinetics Code

This inverse kinetics code is from Dr. Benjamin Allen Baker's PhD dissertation.

```
%%% Inverse Kinetics Code

%Performs inverse kinetics on two vectors
clc; clear all; close all;
format('shorte'); %Changes the number of significant digits matlab uses

% User input for while material file to use

% d = dir;
% str = {d.name};
% str=str(3:end);
% [s,~] = listdlg('PromptString','Select a file:',...
%                 'SelectionMode','single',...
%                 'ListString',str);

% Data Import

%filetitle=str{s};

% Selection of which assemblies.

%prompts the user to set the name of the file it should look for.



%To add the changes for calculating the reactivity modify the file past
%   second line that looks like the one below.
%%%%%%%%%%%%%%%DONT CHANGE ANYTHING AFTER THIS LINE%%%%%%%%%%%%%%%%%%%%

clf %clears the figure

%sr1=csvread(filetitle,0.0); %Reads in the data from the file
sr1=csvread('file name.csv',0.0);

%Pertchoice2 = questdlg('Would you like to convert the LVM data?', ...
%    'LVM convert','Yes','No','No');

Pertchoice2='No';
switch Pertchoice2

    case 'Yes'
        cps=[sr1(:,1) (1/10)*(1./(10.^(2.*sr1(:,2))))];

        x=cps(:,1);
        y=cps(:,2);
        Ylab='linear power / counts';
```

```
    case 'No'

        x=sr1(:,1); %Creates a vector from just the 3rd column of data.
(timestamp)
        %y=sr1(:,2)*10^10; %Create another vector from the 2nd column of data
(Raw data)
        y=sr1(:,2);
        Ylab='(Current from detector)*10^10';

end

%filetitle='lab2finerod.lvm'; %Testing purposes

%The reason the y data is multiplied by 10^10 is to just make the numbers a
%bit easier for matlab to process, it does not change your answer since it
%is only based upon the relative change in data points
filetitle = 'Power';
RawF=figure;
RawAx=axes(RawF);
plot(RawAx,x,y) %Displays the first plot, this shows the raw data gathered
from the reactor.
title(['RAW Data of Power'])
xlabel('Time in Seconds')
ylabel(Ylab)

n=1; %Sets a starting counter for the while loop
yf=zeros(length(y)+2,1); %yf is the filtered data, this creates a new vector
with 2 additional spots

%The reason for the two additional spots is because inorder to filter the
%data it needs the two previous points which the first element in y does
%not have.

%Copies the y vector to yf.

while(n<=length(y));
    yf(n+2,1)=y(n,1);
    n=n+1;
end

%Duplicates the first element twice and places them before the array

yf(1,1)=yf(3,1);
yf(2,1)=yf(3,1);

%Replaces y with yf.
y=yf;


%The following code performs the same actions on the x vector as the y
%vector. The new variables will be xf yf.

n=1;
xf=zeros(length(x)+2,1);
```

```
while(n<=length(x));
    xf(n+2,1)=x(n,1);
    n=n+1;
end

xf(1,1)=xf(3,1);
xf(2,1)=xf(3,1);

%The following code performs the filtering as laid out in the lab manual.
%The equation is the same as the one shown on pg. (insert page reference)


%Since the filtering equation needs to refer to previous points, we start
%the counter at 3 and look back by 2.

%The filtering equation takes in both filtered and unfiltered data, hence
%why everything is written to yf but reads from both yf and y.
n=3;

%The filtering equation

while(n<=length(yf));

    yf(n,1)=1.779*yf(n-1,1)-0.8*yf(n-2,1)+0.00554*y(n,1)+0.01108*y(n-
1,1)+0.00554*y(n-2,1);

    n=n+1;

end

n=1;

%Recreates x and y vectors to eb the same length as yf and xf. The old x
%and y will not be needed anymore.

x=zeros(length(xf)-2,1);
y=zeros(length(yf)-2,1);

%removes the first two extra values

while(n<=length(yf)-2);
    x(n,1)=xf(n+2,1);
    y(n,1)=yf(n+2,1);
    n=n+1;
end

xf=x;
yf=y;

n=2;
i=1;

%Sets the beta i's and the lamdba i's for the AGN reactor.
```

```
B=[0.038; 0.213; 0.188; 0.407; 0.128; 0.026]*0.00745;
L=[1/55.72; 1/22.72; 1/6.22; 1/2.3; 1/0.610; 1/0.23]*log(2);

b=0;

yp=zeros(length(yf),1);


in=zeros(1,6);

% the following 2 while loops are peforming the invese kinetics.

while(n<=length(yf))
    dt=(xf(n,1)-xf(n-1,1));
    a=((yf(n,1)-yf(n-1,1))/dt)*(62.2E-6/yf(n,1))+0.00745;


    while(i<=6)


        in(i)=(in(i)*exp(-L(i,1)*dt))+(yf(n,1)+yf(n-1,1)*exp(-
L(i,1)*dt))*dt/2;

        b=b+B(i,1)*( yf(1,1)*exp(-L(i,1)*xf(n,1)) + L(i,1)*in(i));


        i=i+1;

    end


    yp(n-1,1)=(a-(b/yf(n-1,1)))*(100/0.00745);
    n=n+1;
    i=1;
    b=0;


end

% Refilter the same way as before the inverse kinetics.

n=4;

yfp=zeros(length(yp)+2,1);



yfp(1,1)=yp(1,1);
yfp(2,1)=yp(1,1);
yfp(3,1)=yp(1,1);
```

```
while(n<=length(yfp))
    yfp(n,1)=yp(n-2,1);
    n=n+1;

end

%removes infinities the come up because of the step change in the data
%point values. This is alright to do because we just replace inf with a
%copy of the previous data point.

for n=1:length(yfp)
    if(yfp(n,1)==inf || yfp(n,1)==-inf)
        yfp(n,1)=yfp(n-1,1);
    end
end

for n=1:length(yp)
    if(yp(n,1)==inf || yp(n,1)==-inf)
        yp(n,1)=yp(n-1,1);
    end
end

%Refilter

for i=1:10

    for n=3:length(yp)
        yfp(n,1)=1.779*yfp(n-1,1)-0.8*yfp(n-
2,1)+0.00554*yp(n,1)+0.01108*yp(n-1,1)+0.00554*yp(n-2,1);
    end

end

n=1;

xff=zeros(length(xf)+2,1);

xff(1,1)=xf(1,1);
xff(2,1)=xf(1,1);
xff(3,1)=xf(1,1);

while(n<=length(xf))
    xff(n+2,1)=xf(n,1);
    n=n+1;
end

IKFig=figure('units','normalized','outerposition',[0 0 1 1]);

%This creates all of the subplots the user sees.

subplot(1,2,1,'Parent',IKFig), plot(x,y)

title(['Power Plot for ',filetitle])
xlabel('Time in Seconds')
ylabel(Ylab)
```

```
subplot(1,2,2,'Parent',IKFig), plot(xff,yfp)

title(['Inverse Kinetics of ', filetitle])
xlabel('Time in Seconds')
ylabel('Reactivity in cents')

figure(1)
plot(xff,yfp)

title(['Inverse Kinetics of ', filetitle])
xlabel('Time in Seconds')
ylabel('Reactivity in cents')

%%%%%%%%%%%%DO NOT CHANGE ANYTHING ABOVE THIS LINE%%%%%%%%%%%%%%%%%%

%The filtered inverse kinetics data is the yfp vector and the timestamp
% is xfp.
% The original data is x for time and y for measured data.
```

Flux Volume Integral

```
%%%%% Flux Volume Integral %%%%%
clc; clear all; close all;

R=28.65; %cm
H=15.66; %cm
r=[0:0.5:12.8];
z=[0:0.5:25.8];
[R_,Z_]=meshgrid(r,z);
phi_=besselj(0,2.405*R_/R).*cos(pi*Z_/H);
phi_2=phi_.^2;

figure(1)
surf(phi_2)
xlabel('r(cm)');
ylabel('z(cm)');
zlabel('\phi^2');
title('Surface');

dr=0.01;
dz=0.01;
r_=[0:dr:12.8];
z_=[0:dz:25.8];

temp=0;
I00=0;
I01=0;
I10=0;
I11=0;

for i=1:length(r_)-1
    for j=1:length(z_)-1
        temp=besselj(0,2.405*r_(i)/R).*cos(pi*z_(j)/H);
```

```
        temp=temp.^2;
        I00=I00+temp*r_(i)*dr*dz*2*pi;
        temp=besselj(0,2.405*r_(i)/R).*cos(pi*z_(j+1)/H);
        temp=temp.^2;
        I01=I01+temp*r_(i)*dr*dz*2*pi;
        temp=besselj(0,2.405*r_(i+1)/R).*cos(pi*z_(j)/H);
        temp=temp.^2;
        I10=I10+temp*r_(i+1)*dr*dz*2*pi;
        temp=besselj(0,2.405*r_(i+1)/R).*cos(pi*z_(j+1)/H);
        temp=temp.^2;
        I11=I11+temp*r_(i+1)*dr*dz*2*pi;
    end
end
I00
I01
I10
I11

integral_=mean([I00 I01 I10 I11])
```

Transfer Function Analysis

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Transfer function analysis %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc; clear all; close all;
% Constant
l = 62.2*(10^-6);
b = 0.0065;
p0=-0.03305;
b_i=[0.038; 0.213; 0.188; 0.407; 0.128; 0.026]*0.00745;
lambda_i=[1/55.72; 1/22.72; 1/6.22; 1/2.3; 1/0.610; 1/0.23]*log(2);
n0=2; % 2 watt
s=tf('s');
%%
% dn/dq:
f=0;
for i=1:length(lambda_i)
    f=f+(lambda_i(i)*b_i(i))/(s+lambda_i(i));
end
dN_dQ = l/(s*l+b-p0-f)
figure(1)
impulse(dN_dQ)
grid on;
figure(2)
step(dN_dQ)
grid on;
figure(3)
pzmap(dN_dQ)
grid on;
p_dN_dQ = pole(dN_dQ)
ts=0.02; %20ms sampling rate
dN_dQ_z = c2d(dN_dQ,ts)
%%
% dn/dp:
dN_dP = n0/(s*l+b-p0-f)
```

```
figure(4)
impulse(dN_dP)
grid on;
figure(5)
step(dN_dP)
grid on;
figure(6)
pzmap(dN_dP)
grid on;
p_dN_dP = pole(dN_dP)
ts=0.02; %20ms sampling rate
dN_dP_z = c2d(dN_dP,ts)
%%
```